MODIFIED LOGISTIC MODEL FOR MORTALITY RATES IN MALAYSIA

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DEDICATION

Alhamdulillah;

I thank Allah for always shower me with HIS blessing and mercy.

This thesis is dedicated to my father and mother for their unconditional love and support. I also dedicate this thesis to my husband, my baby Dahlia, my sisters and brother who is always patient and be there for me.

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ABSTRACT

Rapid improvement and changes in mortality trends signify the growing percentage of older people in the population of a country. However, this remains challenging to study due to data limitations, particularly the lack of data for all ages and the sparseness characteristics in the data due to number of death compared to the living at older ages are small. Besides this, the current changes in the mortality curvature have caused the existing mortality models to be accurate at a certain age range only. This research improves an old-age mortality model namely Wilmoth model by combining it with Akima spline to produce smooth rate estimates at younger ages. This approach allows mortality estimates to be calculated for the oldest ages while accounting for uncertainty in the age and gender parameter estimates. This research also introduces a threshold age where the transition between Akima spline and the extended mortality model is optimal. The proposed model can also extrapolate the oldest age mortality rate beyond the available age. This research applies the proposed model to Malaysian mortality data from the years 2010 to 2016 and compares it with six other mortality models, namely Gompertz, Makeham, Beard, Kannisto, Wilmoth, and Heligman Pollard models. The comparison is done using root mean squared error and mean absolute percentage error via a cross-validation method as well as simulation study using bootstrap method. The results show that the proposed logistic model significantly improves Malaysian mortality estimation for all ages in terms of accuracy and prediction performance, as well as its ability to capture the important mortality features such as accident hump, mortality crossover, and deceleration of mortality at old ages. Moreover, the simulation study has also proved that the proposed model is unbiased and consistent. In summary, the proposed model better fits Malaysian mortality rates when compared to the other six mortality models.

ABSTRAK

Pengurangan kadar kematian dan perubahan pesat dalam trend kematian menunjukkan peningkatan peratusan orang tua dalam populasi sesebuah negara. Namun, isu ini sangat mencabar untuk dikaji kerana keterbatasan data, terutamanya tiada data tersedia untuk setiap usia dan ciri data yang jarang kerana jumlah individu yang mati berbanding hidup pada usia tua terlalu sedikit. Selain itu, perubahan semasa dalam graf kematian telah menyebabkan model kematian yang sedia ada hanya tepat pada kumpulan umur yang tertentu sahaja. Penyelidikan ini menambahbaik model kematian usia tua yang dinamakan model Wilmoth dengan menggabungkannya dengan gelugur Akima untuk menghasilkan anggaran kadar yang tepat pada usia muda. Pendekatan ini membolehkan anggaran kematian dikira untuk umur tertua di samping mengambil kira ketidakpastian pada anggaran parameter umur dan jantina. Penyelidikan ini juga memperkenalkan umur ambang di mana peralihan antara gelugur Akima dan model kematian yang dicadangkan adalah optimum. Model yang dicadangkan ini juga boleh mengekstrapolasi kadar kematian umur tertua melepasi umur sedia ada. Penyelidikan ini menggunakan model yang dicadangkan untuk data kematian rakyat Malaysia daripada tahun 2010 sehingga 2016 dan membandingkannya dengan enam model kematian lain, iaitu model Gompertz, Makeham, Beard, Kannisto, Wilmoth dan Heligman Pollard. Perbandingan dilakukan dengan menggunakan ralat punca min kuasa dua dan ralat peratusan min mutlak melalui kaedah pengesahan bersilang serta kajian simulasi menggunakan kaedah butstrap. Hasil kajian menunjukkan bahawa model logistik yang dicadangkan dapat meningkatkan ketepatan dan prestasi ramalan anggaran kematian usia yang lebih tua di Malaysia serta dapat menangkap ciri-ciri penting kematian seperti bonggol kemalangan, keratan kematian, dan kadar perlambatan kematian pada usia tua. Tambahan lagi, kaedah simulasi juga telah membuktikan bahawa model yang dicadangkan adalah tidak pincang dan konsisten. Ringkasnya, model yang dicadangkan memberikan penyuaian yang lebih baik terhadap kadar kematian rakyat Malaysia dibandingkan dengan enam model kematian yang lain.

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LIST OF ABBREVIATIONS

ASEAN	–	Association of Southeast Asian Nations
BVAR	–	Bayesian Vector Auto Regression.
CALGO	–	Computing Machinery Collected Algorithm.
CV	–	Cross-Validation.
DOSM	–	Department of Statistics Malaysia.
EPF	–	Employee Provident Fund.
GAM	–	Generalised Additive Model.
GLM	–	Generalised Linear Model.
HP	–	Heligman Pollard.
LS	–	Least Square.
LIAM	–	Life Insurance Association of Malaysia.
M9903	–	Mortality Table in the year 1999 to 2003.
MAPE	–	Mean Absolute Percentage Error.
NLS	–	Non Least Square.
OECD	–	Organisation for Economic Co-operation and Development.
RMSE	–	Root Mean Squared Error.
VAR	–	Variance.
WHO	–	World Health Organization.

LIST OF SYMBOLS

x	–	Age
$m_{x,t}$	–	Mortality at age x and year t
$t = 2010, \ldots, 2016$	–	Year
$m_{x,t}$	–	Mortality rates for a person aged x in year t
$m'_{x,t}$	–	First derivative for a person aged x in year t
$w'_{x,t}$	–	Weight of a person aged x in year t
V	–	Sum of square of deviation from a straight line of the LS fit
D	–	Distance factor
N	–	Total number of observations
T	–	Total number of years
x_0	–	Threshold age
ξ	–	Degree of 'wriggliness'
α	–	Asymptote of the proposed logistic model
β	–	Inflection point of the proposed model
j	–	Iterative number
k	–	Iterative number
$s(x, m, t)$	–	Akima spline function
S	–	Sum of square of LS
S_b^*	–	B^{th} Bootstrap sample
J	–	Jacobian
$\Delta\boldsymbol{\theta}$	–	Vector of increments
$\widehat{m}_x$	–	Estimate of mortality rate for a person aged x
q_x	–	Conditional probability of death for a person aged x
$b = 1, \ldots, B$	–	Number of samples generated for simulation purposes
L	–	Log of the loss function
L^*	–	Bootstrap statistics

B^*	–	Bias of the bootstrap statistics
V	–	Variance
ω	–	Maximum age
b_0	–	Intercept of the LS line
b_1	–	Slope coefficient of the LS line or regression coefficient
$\mathbb{Z}$	–	Longevity factor
${}_nq_x$	–	Probability of dying between age x and $x+n$
${}_nd_x$	–	Number of deaths between age x and $x+n$
${}_nL_x$	–	Survivors at exact age x
l_x	–	Number of person-years lived between age x and $x+n$
${}_nS_x$	–	Survival ratio
T_x	–	Total number of person years lived after exact age x
e_x	–	Life expectancy

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CHAPTER 1

INTRODUCTION

1.1 Background of Study

Population for every country is inevitably ageing. Population ageing is defined as an increasing proportion of older people out of the total population (United Nations, 2015). As Malaysia is still a developing country, older people are those above the age of 60 years as opposed to the advanced and developed nations, which their elderly are those above 65 years because of better educational, health and economic level as well as services and thus, longer life expectancy. In Malaysia, the elderly are mostly baby boomers who were born between 1946 and 1964. The number of older people is projected to accelerate soon, more so for "oldest- old" persons (individual aged 80 years or over) where the number will increase from 0.3 million in 2017 to approximately 14 million by 2040 and the majority of this age group are females (Ibrahim *et al.*, 2017). By this year, the ratio of young and older population is almost equal with the young (18.6%) and older people (14.5%) and there will be three older individuals for every 20 people. In fact, a 65 years old male could expect to live an additional 14.8 years, on average (Department of Statistics of Malaysia, 2019). As reported by United Nations (2015), Malaysia had become an ageing nation in 2015 when the proportion of elderly were 9.2% of the whole population.

This growing older population presents both fiscal and monetary pressure to nearly all sectors including labour, health care and well-being systems, financial systems such as pensions and social protection providers as well as the demand for goods such as housing and transportation. The government also needs to improve the environment and facilities to be more elderly -friendly. In Malaysia, the government has increased the statutory ages at retirement to ages 60 and considered to switch to contribution-based (Employee Provident Fund, EPF) from the benefit-based pension system as well as promoting the private retirement scheme to prolong the labour force participation of older people and improve the financial sustainability of retirement system. Furthermore, the number of healthy life years lost with a higher life expectancy due to critical illness and disabilities. As an example, the average healthy life years for Malaysian are calculated around 63 to 65 years before a Malaysian person is diagnosed with critical diseases or disabilities (United Nations, 2015). Thus, Malaysia has increased the awareness and treatment facilities on these morbidities. The practice of a healthy lifestyle has also been adapted tremendously by all age range. The World Health Organization (WHO) highlights that the government does not need to formulate and plan for excessive increases in the health budgets. Indeed, the technology advancement in the medical and health care, growth in personal wealth and cultural norms and attitudes surrounding end-of-life care are far more powerful than the population ageing (United Nations, 2015).

1.2 Motivation of Study

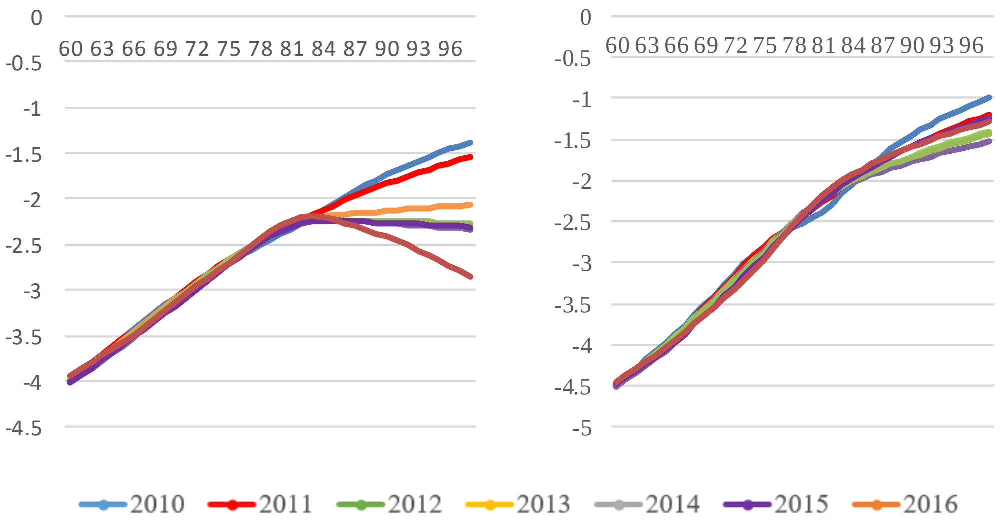
The challenges mentioned in Section 1.1 can be addressed by anticipating the coming demographic shifts. The increasing longevity mainly causes shifts or changes in the population age structure. The issue of longevity risk has gained prominence as mortality rates which is simply the division of deaths number by the population number. Upon examination of past mortality trends, two inherent structures can be distinguished. The mortality rates initially progress with no increment and then the rates follow an accelerating path and decelerating improvements. Finally, the mortality rates revert again to a course of no improvements (Kingdom, 2019). WHO stated that in early non-industrial phase, the mortality rate was high at all age. Therefore, there are only a few people reached old age. Nowadays, most people live above middle age and risk of death is higher at older generations (United Nations, 2011; Li, 2015). On the contrary, life expectancy improves due to the low mortality rate for infants. Yet, the mortality rate for the whole population ironically increases because of the increasing number of deaths from an older population.

Such increases will certainly escalate the future costs for both pensions and care for the elderly. Therefore, major attention should be paid to the modelling of the old-age mortality. In consequences of the wrong estimation of mortality and life expectancy, the retirement and pension provider such as Employee Provident Fund (EPF) will incur losses due to prolonged claim payment and individuals will suffer from insufficient retirement income (Dowd *et al.*, 2010; Villegas, 2015).

Nonetheless, the main challenge in modelling the mortality rates of Malaysian people is how to deal with the random variations which exist due to the low number of deaths and people survive at old ages. This becomes the main reason why most studies on the Malaysian mortality only cover the age below 80 years old such as studies by Ibrahim and Siri (2011), Husin, Zainol and Ramli (2016), and Nor, Yusuf and Bahar (2018). Besides, there is also a problem with the quality of data of older people in terms of inaccuracy in the reporting of age due to the age heaping (Yue, 2002; Ibrahim and Siri, 2011).

The procedure that has been applied so far is to select one mortality model based on the considerations of past practice, the goodness of fit, and other reviews. Many researchers suggest modelling the older age mortality rates using the exponential-based models (Gompertz, 1825; Makeham, 1867; Heligman and Pollard, 1980). In Malaysia, Department of Statistics of Malaysia which is the governing body to publish the Malaysian mortality rates applied the Gompertz exponential model in modelling the rates (Department of Statistics Malaysia, 2019).

As more and better mortality data becomes available, studies have discovered mortality deceleration in the adult life spans. Hence, logistic-based models have been proposed to models these mortalities such as Beard (1963), Tatcher *et al.* (1998) and Wilmoth *et al.* (2007). Furthermore, starting from 2010, instead of increasing with increasing rate, both older age male and female mortality curves appear to grow at a decreasing rate. Hence, their shapes can no longer be modelled using the existing exponential models. For a visual illustration of the problem, consider the mortality rates in Malaysia from 2010 until 2016, obtained from the Department of Statistics of Malaysia, depicted in Figure 1.1 below.



(a) male (b) female

Figure 1.1 Older age mortality rates for (a) male and (b) female in Malaysia from 2010 to 2016

Overall, the mortality models are expected to answer key questions in the ageing population: Does human longevity increase indefinitely or is there some age limit at which people survive? This research not only answers this question by proposing a model that best reflects the Malaysian mortality scenario but also, aim to solve the problem of the sparseness of mortality data at these ages.

It should be highlighted that up to this point, as far as our knowledge reaches, the study of mortality modelling in Malaysia focuses on the ages before 80 years old because of the limitation of data. Certainly, our study offers a new path of research in Malaysia and provides a significant and coherent solution to the problem relating to old ages mortality.

1.3 Problem Statement

The mortality rates for every country typically follow the same curve for overall ages except for the older age mortality rates. Statisticians, actuaries, demographers and insurers normally have to decide the nature of the distribution of the death risk for the age covered.

Nevertheless, the main problem in modelling the old ages mortality rates is the sampling issue. More often than not, the population, death and mortality rates data are abridged data where the data is stored by age groups, such as a group of five to ten years old people, a group of 10 to 15 years old people until a maximum age group (usually the age end at age 80) (Department of Statistics of Malaysia, 2020). The main reasons for providing mortality data in abridged form are mainly related to the misstatement of age, age heaping and small death and risk exposure for the old ages (Yue, 2002; Ibrahim and Siri, 2011; Dodd e*t al.*,2018).

Therefore, mortality models are often used to estimate the mortality rates at older ages. Several models have been formulated such as Gompertz, Makeham, Heligman Pollard, Beard, Kannisto and Wilmoth. These models, however, have several weaknesses. Firstly, there is a shift in the mortality curve in which studies has observed downward deviations from the Gompertz model and the improvisation of it, Makeham model (Richards, 2010). Secondly, although the Kannisto, Beard, Heligman

Pollard and Wilmoth models have accounted for this deceleration of mortality rates, but these models are constructed for the developed countries. Developed countries have a different meaning of the older age (their older age are five years older than Malaysia) and their citizens can live up to 120 years old (United Nations, 2015; Department of Statistics of Malaysia, 2020). Hence, all of the established models can overestimate the Malaysian mortality rates.

Besides that, as these models are limited to the oldest-old (age 80 years and above) mortality rates, they are poor estimator of younger ages (Bebbington, Lai and Zitikis, 2007; Bebbington *et al.*, 2009; Richards, 2010). Since then, a combination of numerical methods and parametric mortality models have been introduced so far to deal with the mortality rates across full ages. Furthermore, due to the data constrained in Malaysia (where there is a limited record of amount of calendar year and age range), it is very important to research on proper methodology in mortality estimation.

To solve these problems, this research proposes a modification of the existing logistic mortality model to accurately estimate and extrapolate the older age mortality rates beyond age 95 as well as the methodology to obtain the full age ranges mortality rates.

1.4 Research Questions

The research questions are as follow:

1. Which mortality model is the most fitted model to estimate the mortality rates of the older ages in Malaysia?
2. Which model is the most relevant to estimate and to smooth the single age mortality curve (age 1,2,…,85) from the five-year abridged rates (1-4,5-9,…,80-85) in Malaysia?
3. Which model is the best to extrapolate the mortality rate beyond age 95?
4. What methods should be employed to validate the accuracy, parameter consistency, bias and variance?

1.5 Objectives

The objectives of the study are based on the research problems stated above:

1. To modify the logistic mortality model namely Wilmoth model which can estimate the mortality rates for people age 60 years and above in Malaysia.
2. To estimate and smooth the single age mortality rate (age 1,2,…,85) from the five-year abridged mortality rate (1-4,5-9,…,80-85) using the Akima spline method.
3. To extrapolate the mortality rate for age 96 and above using the modified logistic mortality model.
4. To validate the accuracy, consistency, bias and variance of the modified logistic mortality model using cross-validation method and simulation study.

1.6 Scope of Study

This research focuses on the mortality and survival experience of the whole Malaysian population from 2010 until 2016 but not specific to any races, religions and states. These data are also provided from age 1 until age 98 in two forms; abridged and single age data. The mortality data for the infant however, is not studied in this research as infants are more susceptible to all kinds of diseases due to low immune systems. Thus, their mortalities have their distinct features. Furthermore, despite the obtainable of mortality data for developed countries such as Singapore, Japan, United Kingdom and Australia, these data are not considered in this research as the mortality curves and their definition of older age are different from Malaysia. This research does consider the mortality data of developing countries in South East Asia such as Thailand but the single age mortality rates are difficult to acquire and there is a limitation of time. As for the abridged data, the population, death and mortality rates are stored in a five-year age group except for the age over 85 where the data are aggregated in an open group 85+. Hence, there is a total of 18 age groups for both males and females.

1.7 Significance of Study

The composition in term of age and the size of the Malaysian population continue to change in tandem with the changing times. As the ageing issue has gained prominence as mortality rates, study on the mortality trends and modelling these trends is significant so that the government and private sectors can prepare immediately and adequately the specific needs for the aged such as the health care and retirement plan.

Existing studies on Malaysian mortality are limited because the data of the oldest-old mortality are not readily available. Furthermore, the current methods are established for the developed nation such as Australia, United States and United Kingdom where their people live longer than Malaysian people (up to 120 years of age) and have a different definition of older age (five years older than Malaysian people). Thus, the existing mortality models no longer applicable to the current situation of Malaysian mortality. This study contributes to extending the Wilmoth mortality model that best reflects the Malaysian mortality experience. Besides, the method proposed in this study embedded Akima function into the proposed model and thus, able to produce single-age mortality rates. This is certainly beneficial as the mortality data in Malaysia is only available in the abridged form and the oldest mortality data (age 80 and above) are scarce and not readily available. Furthermore, this research also introduces threshold age, at which the transition between Akima and the proposed mortality model is optimal. The extrapolation property of the proposed model also not only useful in statistical research but also the financial businesses and healthcare businesses.

1.8 Organization of Thesis

This thesis is composed of six themed chapters:

1. **Chapter 1** provides an introduction to the current scenarios and issues in Malaysian mortality.

2. **Chapter 2** reviews the development related to the study on mortality, starting from the world and Malaysian mortality experiences to the models adopted to explain these trends.

3. **Chapter 3** describes the methodologies adopted to solve the problems that have been identified in Chapter one. This section presents the existing and proposed mortality models as well as the validation tests and simulation study to assess the models.

4. **Chapter 4** discusses the results obtained using the methodologies in Chapter 3. The results are presented in graphical and numerical manners.

5. **Chapter 5** summarises this research and briefly discusses future development related to the mortality study.

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LIST OF PUBLICATIONS

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1. **Khaliludin, N. I. A**., Khalid, Z. M., and Rahman, H. A. On estimate of Malaysian mortality rates using interpolation methods. *MATEMATIKA*. 2019. 35(2): 177-186.