

MODELLING OF MULTIVARIABLE SYSTEMS USING ADVANCE FUZZY  
ARITHMETIC

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## **ABSTRACT**

Fuzzy arithmetic has received little attention among researchers regarding its ability to solve real world problem. However, in recent years, fuzzy arithmetic which is based on transformation method (TM) has been used to solve many optimization problems in engineering. In this study, state space modelling of several multivariable systems using fuzzy arithmetic based on the TM are presented. The TM is used to evaluate the state space model equations and to quantify the influence of each parameter of a given system. As a result, the value of its gain factors is calculated to allow the estimation of the relative measures of the uncertainty. A software, called, FAMOUS (Fuzzy Arithmetical Modelling of Uncertain Systems) is customized to illustrate the effectiveness of fuzzy arithmetic based on the TM. Moreover, the analytical solutions of state equations of these systems are obtained. The effectiveness of the method is shown by implementing it to the state space model of components in power plants (steam turbine and boiler) and nuclear power plants (steam generator and pressurizer). The computation time is reduced once the analytical solutions are determined for the systems. Furthermore, the influences of the uncertain parameters are obtained by simulations.

## ABSTRAK

Aritmetik kabur kurang mendapat perhatian di kalangan penyelidik mengenai keupayaannya untuk menyelesaikan masalah dunia sebenar. Walau bagaimanapun, dalam beberapa tahun kebelakangan ini, aritmetik kabur yang berasaskan kaedah transformasi (TM) telah digunakan untuk menyelesaikan beberapa masalah pengoptimuman dalam bidang kejuruteraan. Dalam kajian ini, pemodelan ruang keadaan untuk sistem bagi beberapa multipembolehubah menggunakan aritmetik kabur berdasarkan TM telah dibentangkan. TM digunakan untuk menilai persamaan model ruang keadaan dan untuk mengukur pengaruh setiap parameter di dalam sistem yang diberikan. Hasilnya, nilai faktor dapatannya dikira untuk membenarkan anggaran ukuran relatif ketidakpastian. Satu perisian, iaitu FAMOUS (Pemodelan Aritmetik Kabur Sistem Tidak Menentu) telah disesuaikan untuk menggambarkan keberkesanan aritmetik kabur berdasarkan TM. Tambahan pula, penyelesaian analisis daripada persamaan keadaan sistem telah diperolehi. Keberkesanan kaedah telah ditunjukkan dengan melaksanakannya kepada model ruang keadaan bagi komponen dalam loji tenaga (turbin stim dan dandang) dan loji tenaga nuklear (penjana stim dan penekan). Masa pengiraan telah dikurangkan sebaik sahaja penyelesaian analitik ditentukan untuk sistem. Tambahan lagi, pengaruh parameter yang tidak menentu telah diperolehi daripada simulasi.

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## LIST OF ABBREVIATIONS

TM	-	Transformation Method
FAMOUS	-	Fuzzy Arithmetical Modelling of Uncertain Systems
FFSSM	-	Feedback Fuzzy State Space Model
PWR	-	Pressurized Water Reactor
MATLAB	-	Matrix Laboratory
FSSM	-	Fuzzy State Space Model
UTM	-	Universiti Teknologi Malaysia
HP	-	High Pressure
IP	-	Intermediate Pressure
LP	-	Low Pressure

## LIST OF SYMBOLS

$\eta_i^{(j)}$	-	Gained value
$\tilde{q}$	-	Fuzzy value output
$n$	-	Number of parameters
$X_i^{(j)}$	-	Interval
$m$	-	Interval of length
$A/B$	-	Tensor division of $A$ and $B$
$A.B$	-	Tensor multiplication of $A$ and $B$
$A - B$	-	Tensor minus of $A$ and $B$
$A + B$		Tensor addition of $A$ and $B$
$t_i$		The $i^{th}$ of time
$A \otimes B$		Tensor product of $A$ and $B$
$\mu$		Fuzzy relation
$b_i$		Upper interval
$a_i$		Lower interval
$\alpha$		Alpha
gfn*		quasi-Gaussian functions
gfn		Gaussian function
$\sigma_r$		Sigma on right side
$\sigma_l$		Sigma on left side
$k_i$		Standardized mean gain factor
$X_o$		Steam energy per unit volume
$w_{ou}$		Outlet steam flow
$\rho_{ou}$		The steam density
$e$		Exponential
$\bar{x}$		Modal Value
$\rho_i$		Relative Measure of Influence
$\tilde{p}$		Fuzzy Number
$\mu$		Membership value
$x_i'$		The Dynamic of the $i^{th}$ Variable

$\epsilon$	Epsilon Variant
$\times$	Multiplication Sign
$\mathbb{R}$	Real Number
$\leq$	Less than or equal to
$<$	Less than
$>$	Greater than
$\rho_{EG}$	The density of exhaust gas from the boiler
$X_{F1}$	The enthalpy and density of exhaust gas from the boiler
$Q_{ir}$	Heat transferred to the risers in J/s
$Q_{is}$	Heat transferred to the superheater in J/s
$Q_{rs}$	Heat transferred to the reheater in J/s
$Q_{es}$	Heat transferred to the economizer in J/s
$P_G$	Furnace air pressure in Pa
$T_{rh}$	Reheater metal tube temperature
$X_{rh}$	Specific enthalpy of outlet steam and steam density
$P_{ro}$	Outlet steam pressure
$T_r$	Reheated steam temperature

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# CHAPTER 1

## INTRODUCTION

### 1.1 Introduction

Mathematical modelling of multifaceted systems is extraordinarily important in various fields of science, medical and engineering. A new move towards mathematical modelling is much influenced by competition, energy efficiency and the demand for low cost operation. For example, a complex system power generation plants must function optimally in order to remain competitive. However, a small enhancement in energy efficiency can show the way to substantial cost savings.

A combined cycle is a large scale electrical power generation plant in which electricity is obtained from both gas and steam turbines. Energy is transferred in the form of heat or gas which flow all the way through each turbine to produce electricity. Additionally, electricity is produced using wasted heat which exist in exhaust gases from turbines or other parts of the system. Efforts have been taken to reduce losses of energy. A steam turbine has complex features and a bundle of layer steam expansion in order to increase thermal efficiency. This complexity is the reason why nonlinear analytical model has been developed to study transient dynamics of steam turbine.

In order to increase boiler system proficiency, it is essential to understand the characteristics of the combustion process. For example, a furnace operating at 2400 F with 25% excess air could experience 24% fuel savings by reducing excess combustion air to 10% (Morris, 2007). Moreover, the conventional way of evaluating a crisp model is not practical since boiler parameters are subjected to uncertainty because they reveal the inconsistency depending on the quantity of input. A boiler and a steam turbine system have complex features that affecting thermal

efficiency. In this regard, no mathematical model is capable to correctly explain such a complicated process. Inaccuracy is typical in a developed model due to uncertain recognition of parameters. Therefore, deficiency in accurate modelling causes many complications in control strategies.

A mathematical model is developed to understand and describe a system. Two components of a pressurized water reactor (PWR) of a nuclear power plant, steam generator and pressurizer, are chosen for this study. The model is a set of partial differential equations, in which the nonlinearity arises due to convection of force in the flow, variable properties and radioactive. Conversely, rough calculation and suggestions are used to make these equations simpler, resulting in ordinary differential and algebraic equations for production.

The transformation method is proposed as an appliance for simulation and analysis in this study. This method is available in two forms, namely in a general form which can be used for simulation and analysis of arbitrarily non-monotonic modelling and in a reduced form, which can reduce the computing time.

## **1.2 Research Background**

Khan *et al.* (2012) developed the inverse Feedback Fuzzy State Space Model (FFSSM) for a steam turbine. The developed algorithm was used to maximize the power generation with optimal input parameters. The inverse modelling approach was implemented to increase the output and maintain the desired conditions.

Harish *et al.* (2010) modelled a boiler as a graph. The boiler system consists of subsystems such as the furnace, superheater, drum, riser and reheater. These subsystems were presented as vertices and the interconnections as edges of the graph. Initially, the input-output variables for the subsystems are identified using a state space approach. The graphical representation of the boiler system acts as an initial stage for their simulations.

Ashaari *et al.* (2015a) also presented works in pressurized water reactor (PWR) system as a graph. The steam generator is a bridge between the primary and secondary components as well as for stage transformation from water into steam. The objective of their work was to form and examine the performance of steam generator using Fuzzy State Space Model (FSSM). Moreover, FSSM of component pressurizer of nuclear power plant was proposed in Ashaari *et al.* (2015b). A function of pressurizer is to organize pressure and temperature in a nuclear power plant. Fuzzy state space approach was used to model the pressurizer.

A paper by Hanss (2002) described the functioning of fuzzy arithmetic to avoid the recognized effect of overestimation which frequently arises when fuzzy arithmetic is used. The efficiency of the transformation method of a friction between sliding surfaces of a bolted joint connection is shown. However, Hanss and Oliver (2001) show the improvement of parameter identification for complex biomedical models on the basis of fuzzy arithmetic. The models are analyzed with determining the influence of each parameter by reducing the high-dimensional identification procedure to a lower-dimensional optimization problem.

A new uncertainty investigation for the transformation method (TM) was proposed by Gauger *et al.* (2008). It allocates the estimation of the relative measures of uncertainty. These methods then allow to quantify the influence of uncertainty of the inputs and outputs. Hanss and Klimke (2004) reported the influence measure of the transformation method can be compared to a classical approach. A major advantage of TM is that the degree of influence for each uncertain parameter can be determined easily.

Fuzzy numbers and their combined mutual arithmetic are the subject of some text books and many publications such as Duboids and Prade (1980), Kaufman and Gupta (1985) and Klir and Yuan (1995). Beginning their organization, fuzzy sets are employed in diverse engineering and previous applications. Wood *et al.* (1989) measure the probability calculus and fuzzy set calculus for conducting the vagueness on uncertainty. They over and done with fuzzy set calculus was more appropriate

than the probability calculus in managing the vagueness characteristic of uncertainty in the initial design phase.

Fuzzy arithmetic is founded on the expansion principle (Zadeh, 1973), which authorizes the source of membership functions of fuzzy numbers. Still, it is important to employ the extension principle directly. However, fuzzy number can put into practice via two techniques: parameterized functions from Duboids and Prade (1980) or the discretized fuzzy number from Dong and Wong (1987). Both of these techniques are about estimation with decrease in the computational difficulty and increase in valuable results.

### **1.3 Problem Statement**

Standard fuzzy arithmetic is only concerned with the optimization of the system by customizing the input to get the desired output and vice versa. For that reason, this approach still lacks the study of fuzzy arithmetic since the result of the problem only illustrates the general influence of all uncertain parameters collectively. However, in general the percentage influence for the different uncertain parameters of the system on its output is definitely not equal. Thus, how can the degree of influence of uncertain model parameters of a steam turbine, boiler and nuclear power plant be determined?

### **1.4 Research Objectives**

The aims of this research are complete as follows:

- (a) To enhance the state space modelling of a steam turbine and boilers systems.
- (b) To simplify the state space equations by model reduction of a steam turbine, boiler and nuclear power plant.

- (c) To simulate the uncertain model parameters of a steam turbine, boiler and nuclear power plant.
- (d) To measure the degree of influence of uncertain model parameters of a steam turbine, boiler and nuclear power plant.

### **1.5 Significance of Research**

The new method and implementation in this study is expected to contribute significantly to the fields of mathematics and engineering. The work will contribute in the direction of:

- (a) The enhancement the state space modelling of steam turbine and boiler systems which give more valuable result.
- (b) The simplification of the state space equations by model reduction of a steam turbine, boiler and nuclear power plant can reduce the computing time.
- (c) Simulation of the uncertain model parameters of a steam turbine, boiler and nuclear power plant describes the fuzzification of model parameter.
- (d) The degree of influence is computed for the different uncertain model parameters of a steam turbine, boiler and nuclear power plant.
- (e) The beneficiary of this study will able to justify the most influential parameter on the system and can reduce the model parameter by ignoring the uncertain model parameters, whose influence turns out to be negligible.

### **1.6 Scope of Research**

The scopes of this study include accurate details of two areas of study, which is engineering and mathematics. The study of engineering area included three

systems, which is a steam turbine, boiler and nuclear power plant. These include the process of the working system and the uncertain model parameters. The mathematics part of this research is the study of analytical solution, state space and advanced fuzzy arithmetic. The whole technique in mathematical study will be implemented in real world application in a steam turbine, boiler and nuclear power plant (see Figure 1.1).

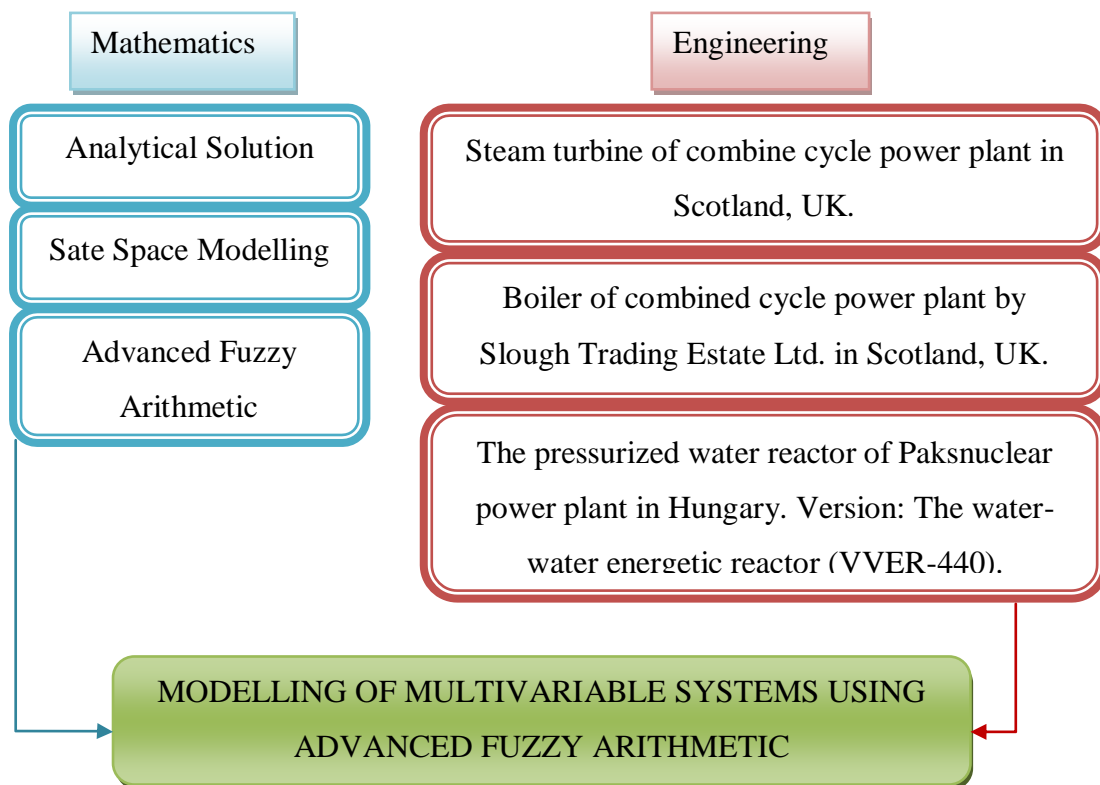


Figure 1.1 An overview of the research.

## 1.7 Research Outline

This thesis is organized into seven chapters. Each chapter includes an introduction and a summary. This chapter presents a clear description of the background and rationale in the new modelling method, statement of the problem, objectives, significance and overall scope of the study. An overview of the study is also presented in this chapter.

Chapter 2 is an overview of mathematical backgrounds. It includes classification of uncertainties, fuzzy number, arithmetic operation, state space modelling and analytical solution. Chapter 3 contains concepts and principles in advanced fuzzy arithmetic based on the transformation method. The discussion includes a detail explanation of the formulation for general and reduced transformation method.

The major involvements of this study are presented in the next three chapters. Chapter 4 elaborates the algorithm for advanced fuzzy arithmetic based on the transformation method and the algorithm for FAMOUS programming. Chapter 5 describes the improvement in the state space of steam turbine and boiler (furnace and reheater) systems. A detailed description of the analytical solution by using integrating factor is illustrated. The method is applied to simplify the state space equation to a linear equation in order to reduce computing time. The simulation results by using the developed algorithm are presented in Chapter 6. Finally, Chapter 7 contains the conclusion and recommendation for advance research work. The summary of the study is illustrated in Figure 1.2.

## MODELLING OF MULTIVARIABLE SYSTEMS USING ADVANCE FUZZY ARITHMETIC

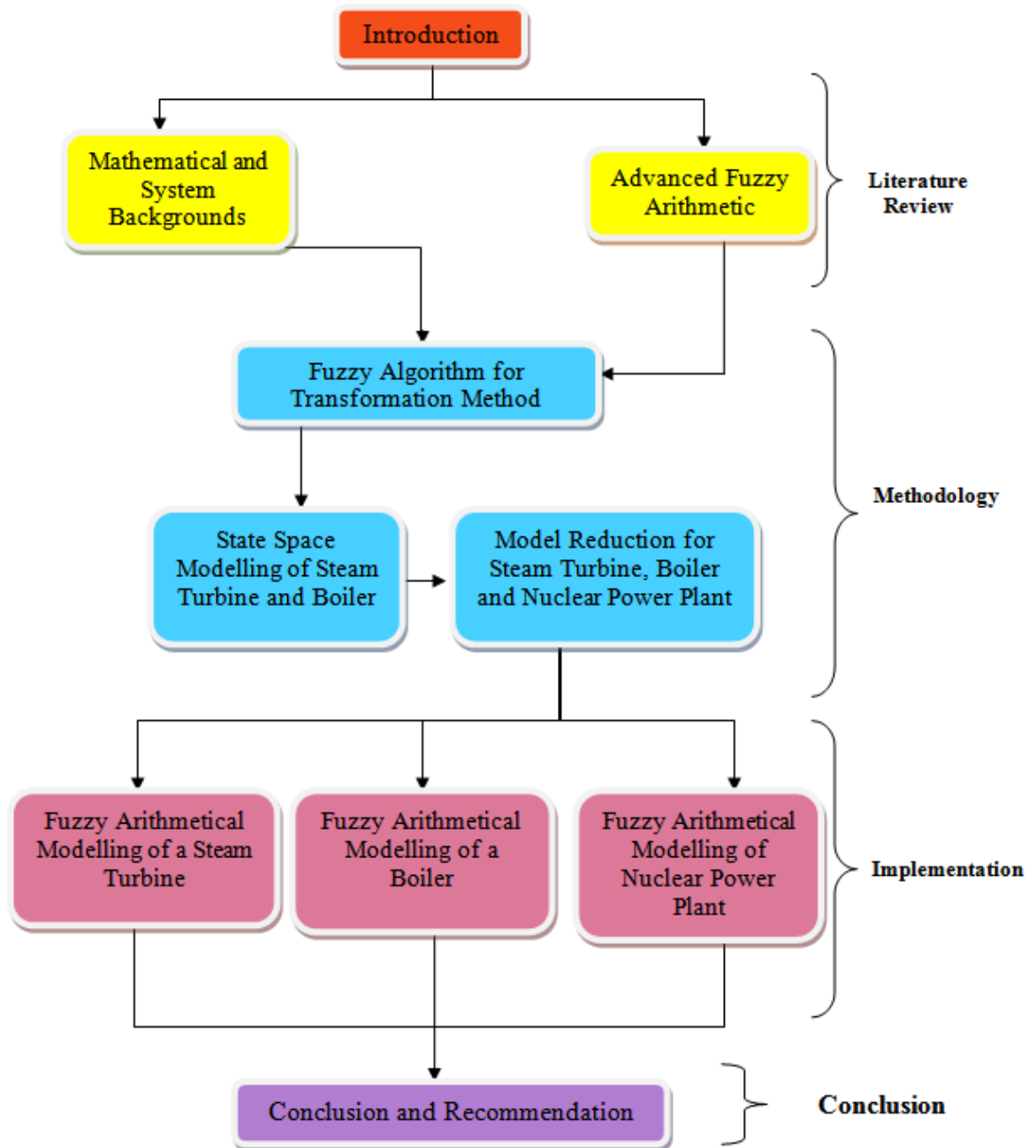


Figure 1.2 Research Frameworks.



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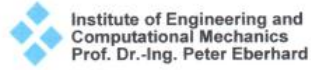
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# Appendix A Licence Form FAMOUS for Non -Commercial Use



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## Subject: Non-commercial use of FAMOUS

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