MODELLING OF MULTIPLE CONSTRAINTS PORTFOLIO OPTIMIZATION USING MODIFIED PARTICLE SWARM OPTIMIZATION

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DEDICATION

This thesis is dedicated to my late beloved father ***Zaheer Uddin*** and father in law ***Muhammad Arif*** for their love, concern and support to make sure I achieve higher targets. This thesis is dedicated to my beloved mother ***Naheed Jahan*** and mother in law ***Razia Begum***. This thesis is a testimony of the efforts of my parents, how they prayed and struggled for me to get the best things of both worlds. It is also dedicated to my beloved four little princesses ***Aairah Binte Kashif, Rahmah Binte Kashif, Ashna Kashif and Mirha Binte Kashif*** for their love and dua. Last but not the least my wife ***Amber Nehan Kashif*** without her love, support and presence i would never be able to complete this work, she helped me a lot to overcome the most difficult period.

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ABSTRACT

In finance, the portfolio is the set of investment in the assets. Meanwhile, its optimization leads towards the best selection and diversification of investments. Portfolio optimization involves the objectives (mean, variance or Sharp Ratio (SR)) and constraints (budget, short sell, outliers, cardinality, lot and transaction cost, liquidity), as well as some others which makes it more complex, dynamic and intractable. The SR function is considered to be the measuring tool for best portfolio selection and optimization. At present, the area of portfolio optimization lacks in having multiple constraints with the SR as the objective function. This research focuses on a two-stage portfolio selection, diversification, and optimization. The normality tests have been performed from the data considered and it is found that the data is nonlinear and stochastic. The two selection criterion (mean and variance) have been introduced in this research. Furthermore, several constraints have been considered for the problem of Multiple Constraints Portfolio Optimization (MCPO). A metaheuristic technique needs to be developed with the financial toolbox in MATLAB and the Particle Swarm Optimization (PSO) for portfolio construction, diversification, and optimization, namely, the Modified PSO (MPSO). The simulation on the benchmark model for restriction on the short sale was performed. Also, the diversification phenomenon for having the 10, 50 and 150 assets collection has been observed. The obtained results for the benchmark model are 42.51% and 84.20% increment in Maximum of Maximum Sharp Ratio (MMSR), whereas 39.88% and 84.30% increment in Average of Maximum Sharp Ratio (AMSR). The results of the models having mean of return selection criteria have increments of 2.58%, 21.10%, 16.41%, 11.67%, and 6.42%; whereas, models M3 and M4 for MMSR values have decrement of 3.52% in comparison with the model having the variance of return selection criteria. This research will be beneficial for those involved such as in mathematical finance modeling, asset portfolio optimization and financial model optimization using metaheuristic techniques.

ABSTRAK

Dalam kewangan, portfolio adalah set pelaburan dalam aset. Sementara itu, pengoptimumannya mengarah kepada pilihan yang terbaik dan kepelbagaian pelaburan. Pengoptimuman portfolio melibatkan objektif (min, varians atau Nisbah Tepat (SR)) dan kekangan (belanjawan, jualan pendek, data terpencil, kardinaliti, lot dan transaksi kos, kecairan), serta lain-lain yang menjadikan ia lebih kompleks, dinamik dan sukar dikawal. Fungsi SR dipertimbangkan sebagai alat pengukur untuk pemilihan portfolio terbaik dan pengoptimuman. Pada masa ini, bidang pengoptimuman portfolio kurang dalam mempunyai pelbagai kekangan dengan SR sebagai fungsi objektif. Kajian ini memberi tumpuan kepada dua peringkat pemilihan portfolio, kepelbagaian, dan pengoptimuman. Ujian kenormalan telah dijalankan dari data yang dipertimbangkan dan didapati bahawa data adalah tak linear dan stokastik. Kedua-dua kriteria pemilihan (min dan varians) telah diperkenalkan di dalam kajian ini. Tambahan pula, beberapa kekangan telah dipertimbangkan untuk masalah Kekangan Pelbagai Portfolio Pengoptimuman (MCPO). Satu teknik metaheuristik perlu dibangunkan dengan kotak alat kewangan dalam MATLAB dan Particle Swarm Optimization (PSO) bagi pembinaan portfolio, kepelbagaian, dan pengoptimuman, iaitu PSO terubah suai (MPSO). Simulasi model penanda aras untuk sekatan ke atas jualan singkat dijalankan. Juga, fenomena kepelbagaian yang mempunyai aset koleksi 10, 50 dan 150 telah diperhatikan. Keputusan yang diperolehi untuk model penanda aras adalah 42.51% dan 84.20% kenaikan dalam Maksimum bagi Nisbah Tepat Maksimum (MMSR), manakala 39.88% dan 84.30% kenaikan dalam Purata bagi Nisbah Tepat Maksimum (AMSR). Keputusan model yang mempunyai min kriteria pemilihan pulangan mempunyai kenaikan 2.58%, 21.10%, 16.41%, 11.67%, dan 6.42%; manakala, model M3 dan M4 untuk nilai MMSR mempunyai pengurangan sebanyak 3.52% berbanding dengan model yang mempunyai varians kriteria pemilihan pemulangan. Kajian ini akan memberi manfaat kepada mereka yang terlibat seperti dalam pemodelan matematik kewangan, pengoptimuman portfolio aset dan pengoptimuman model kewangan menggunakan teknik metaheuristik.

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LIST OF ABBREVIATIONS

ABC	-	Artificial Bee Colony
ACO	-	Ant Colony Optimization
AIS	-	Adaptive Investment Strategy
APAES	-	Adaptive Pareto Archived Evolution Strategy
APO	-	Assets PO
APSO	-	Adaptive PSO
AR	-	Appraisal Ratio
BCAPSO	-	Binary CAPSO
BR	-	Burke Ratio
CCPOP	-	Cardinality Constrained Portfolio Optimization Problem
CCMPO	-	Cardinality Constraint Markowitz PO
CPOP	-	Constrained Portfolio Optimization Problem
CPSP	-	Constraint Portfolio Selection Problem
C-PSP	-	Complex Portfolio Selection Problem
CPSO	-	Compact PSO
CAPSO	-	Centripetal Accelerated PSO
CRSP	-	Center for Research in Security Prices
CSE	-	Colombo Stock Exchange
CoSR	-	Conditional Sharp Ratio
CR	-	Calmar Ratio
CVaR	-	Conditional Value-At-Risk
DEMPO	-	Differential Evolution Multi–objective PO
DPSM	-	Dynamic Portfolio Selection Model
DPSO	-	Drift PSO
EEMOPOS	-	Efficiently Encoded Multi Objective Portfolio
	-	Optimization Solver

ELPSO	-	Example Learning based PSO
EPSO	-	Extended Particle Swarm Optimization
eRAR	-	Extended RAR
FBPSO	-	Floating Boundary PSO
FTB	-	Financial Tool Box
FTB-PSO	-	Financial Tool Box Particle Swarm Optimization
GA	-	Genetic Algorithm
GCPSO	-	Guaranteed Convergence PSO
HKSM	-	Hong Kong Stock Market
HPSO	-	Hybrid Particle Swarm Optimization
IR	-	Information Ratio
ISC	-	Initial Selection Criteria
MABC	-	Modified Artificial Bee Colony
M-CABC	-	Multi-objective Co-variance guided Artificial Bee Colony
MCPO	-	Multiple Constraint PO
MCPOM	-	Multiple Constraint Portfolio Optimization Models
MCPSO	-	Multiple Constraints Portfolio Selection and Optimization
MCPOP	-	Multi Constrained Portfolio Optimization Problem
MIR	-	Modified Information Ratio
MiSE	-	Milan Stock Exchange
MOEAs	-	Multi Objective Evolutionary Algorithms
MPSO	-	Modified Particle Swarm Optimization
MoPSO	-	Median-oriented PSO
M-PSO	-	Migration PSO
MPT	-	Modern Portfolio Theory
MR	-	Martin Ratio
MSR	-	Modified Sharp Ratio
MSPO	-	Multi Stage PO
MV	-	Mean–Variance

MVCCPO	-	Mean–Variance Cardinality constrained PO
MVPO	-	Mean Variance PO
MVPOM	-	Mean-Variance Portfolio Optimization Model
MVPS	-	Mean–Variance Portfolio Selection
MVPSM	-	Mean–Variance Portfolio Selection Model
NAR	-	Non-parametric Autoregressive
NM	-	Nelder-Mead
NM-PSO	-	Nelder-Mead Particle Swarm Optimization
NPGA	-	Niched Pareto Genetic Algorithm
NPGA2	-	Niched Pareto Genetic Algorithm 2
NSGA-II	-	Non-dominated Sorting Genetic Algorithm II
NSGA	-	Non-dominated Sorting Genetic Algorithm
ORP	-	Optimal Risky Portfolio
OmR	-	Omega Ratio
OSR	-	Omega Sharp Ratio
PAES	-	Pareto Archived Evolution Strategy
PESA	-	Pareto Envelope-based Selection Algorithm
PGM	-	Probe Guided Mutation
PLM	-	Polynomial Mutation
PO	-	Portfolio Optimization
PR	-	Pain Ratio
PrR	-	Prospect Ratio
PSO	-	Particle Swarm Optimization
PSO-LVIW	-	PSO with Linearly Varying Inertia Weight
PSO-TVAC	-	PSO with Time Varying Acceleration Coefficients
PSOAG	-	PSO with Age Group
PSORDS	-	PSO with Random Dimension Selection
PSODDS	-	PSO with Distance based Dimension Selection
PSOHDS	-	PSO with Heuristic Dimension Selection

PSP	-	Portfolio Selection Problem
RAR	-	Random Assortment Recombination
SoR	-	Sortino Ratio
SPEA	-	Strength Pareto Evolutionary Algorithm
SPEA2	-	Strength Pareto Evolutionary Algorithm 2
SR	-	Sharp Ratio
SSE	-	Shanghai Stock Exchange
StR	-	Sterling Ratio
StCR	-	Sterling Calmar Ratio
STSPSO	-	Simple Two Stage Portfolio Selection and Optimization
TC	-	Transaction Cost
TCA	-	Transaction Cost Analysis
TEF	-	True Efficient Frontier
TR	-	Treynor Ratio
TVPSO	-	Time Variant PSO
VaR	-	Value-At-Risk
VAR	-	Vector Autoregressive

LIST OF SYMBOLS

p	-	Portfolio
t	-	Time
D	-	Day
M	-	Month
Y	-	Year
i or j	-	Number of the Assets
d	-	Number of the Days
CP_{di}	-	Closing Price of the Day
$CP_{(d-1)i}$	-	Closing Price of the Previous Day
r_{di}	-	Daily Return of Asset
N	-	Number of Assets
$\bar{\mathrm{R}}_i$	-	Mean of Daily Asset Return
V_i	-	Variance of Daily Asset Return
w_{di}	-	Weight/Proportion Invested in Asset i
SR	-	Sharp Ratio
R_{pd}	-	Mean Return of Portfolio
V_{pd}	-	Variance of Return of Portfolio
σ_{ij}	-	Covariance of Daily Returns
R_f	-	Risk Free Rate of Return for p
R^*	-	Desired Mean Return of p
λ	-	Risk Aversion Parameter
$StdDev(p)$	-	Standard Deviation of the Returns for Portfolio
$Q_{1\tau}$	-	First Quartile
$Q_{2\tau}$	-	Second Quartile
$Q_{3\tau}$	-	Third Quartile
IQ_τ	-	Interquartile Range

$L_{if\tau}$	-	Lower Inner Fence
$L_{of\tau}$	-	Lower Outer Fence
$U_{if\tau}$	-	Upper Inner Fence
$U_{of\tau}$	-	Upper Outer Fence
τ_i	-	Distance of the Particle for Asset i
ξ_i	-	Outlier Constraint for Asset i
$\mathbb{R}$	-	Real Numbers
γ_i	-	Cardinality Constraint
ϵ_i and δ_i	-	Lower and Upper Proportion
μ_i	-	Lot Constraint
y_i	-	Price of a share before μ_i
η_{fi} and η_{vi}	-	Fixed and Variable Net Transaction Cost
I_{fi} and I_{vi}	-	Fixed and Variable % Brokerage Charges
κ_{fi} and κ_{vi}	-	Fixed and Variable Decimal Values for Transaction Cost
w_{fi} and w_{vi}	-	Total Amount Including Transaction Cost
$\mathbb{N}$	-	Natural Numbers
x_i^k	-	Initial Position of the Particle
x_i^{k+1}	-	Updated Position of the Particle
v_i^k	-	Velocity of the Particle
w	-	Inertia Weight
c_1 and c_2	-	Acceleration Coefficients
R_1 and R_2	-	Random Numbers Generated in PSO
P_{best}	-	Personal Best Position of Particle i
G_{best}	-	Global Best Position of Particle i
k	-	Iteration Index
M1	-	Model 1
M2	-	Model 2
M3	-	Model 3
M4	-	Model 4

M5	-	Model 5
M6	-	Model 6
M7	-	Model 7
M8	-	Model 8
M_{MSR}	-	Maximum of Maximum Sharp Ratio
A_{MSR}	-	Average of Maximum Sharp Ratio
α	-	Reference Value
β	-	Other Value
γ	-	Percentage Increase or Decrease

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CHAPTER 1

INTRODUCTION

1.1 Background

Financial markets are around since the inception of mankind. Early men trade their crops to barter the commodity with their need. In modern times, they are found at every corner of the world, creating an open and regulated system for companies. A financial market is actually a place comprises of entities that can be exchanged on the basis of different commodities. People trade commodities, financial securities and other similar assets at prices that are determined by pure supply and demand principles. These principles are dynamic, and take different values over the course of time. These changes are event driven and mainly reflect upon the seasonal variation and nature of commodity. A financial market can be highly organized, or may serve some basic functions like purchase and sale of commodities.

In general, a financial market can be thought of an organization that serves setting and maintaining of prices, allocation of assets, providing opportunities to gaining profit, and setting up a feasible environment for transaction while managing the risk in above mentioned functions. Financial markets are of different types and facilitate various types of investors, furnishing upon their interests. Some of the most commonly markets are physical asset markets, financial asset markets, spot markets, futures markets, money markets, capital markets, primary markets, secondary markets, private markets and public markets; to mention a few. The investors mainly fall into two categories, individual investor and institutional investor. Their role corresponds to building up a portfolio by investing for themselves versus an organization. In general, an investor needs to gain profit with their investment by having minimum loss. That's why it is important to investigate the behaviour of portfolio and its association with profit optimization and risk aversion. In particular, portfolios that are related to stock and money market, that corresponds to the narrower spectrum of the financial market.

A portfolio is defined as a group of financial assets such as shares, stocks, bonds, mutual funds, also the collection of currencies as foreign exchange at a particular time.

The stock and money market is governed by certain norms used to regulate the behaviour of market, considering various constraints in to account. These norms are classified as:

- Restriction on short sale, whether the stock market allows it or not. It is basically a controlling component of market, most of the times it is not allowed in stock market where as in money market it is allowed.
- Limit on minimum and maximum number of assets/stock in the stock market known as the lot.
- Brokerage fee as transaction cost fixed and variable, also the fixed overnight charges.
- Liquidity of the assets, due to the regulations of the stock market.

On the other hand, constraints of the stock and money market can be determined as:

- Limitation of the budget for an individual and institutional investor as budget constraint.
- Restriction on investment in the area, sector, stock and currency, from an individual and institutional investor as cardinality constraint.
- Market uncertain behaviour as outlier constraint.

On the basis of aforementioned norms and constraints, a portfolio is dictated by a number of factors. Therefore, optimization of such portfolio asks to include multiple constraints in single portfolio. As such, optimization is performed on a multivariable parameter space, keeping into account the effect of covariance among the parameters. This area in the field of Financial Mathematics is relatively less studied due to inherent complexity in the optimization problem. However, optimizing multiple constraints in a portfolio is significant area of study because of the following reasons, viz, (i) as a multivariate and dynamic mathematical problem; it is an interesting region to explore in its own right, (ii) the continuing variant nature of the stock market, conjoining with a large amount of available data makes the optimization problems demanding the generic statistical distribution of the assets. This distribution helps us finding the optimal values of the assets which, in turn, would encourage the investor by minimizing risks, (iii) the parameters implemented through regulatory bodies draw the interest of both investor and the market. Assessing their effect as constraints in a portfolio could lead us pose research questions to yield better insights in portfolio optimization.

1.2 Modelling of Multiple Constraints Portfolio Optimization

On the basis of the issues that pertain to the intractability and complexity of the system discussed above, there is much need to model a multiple constraints portfolio that has efficient selection and optimization algorithm. For portfolio selection, we consider the mean or variance of the portfolio as criteria for portfolio selection. For efficient portfolio optimization, an objective function for optimization needs to be selected. This objective function may be mean or variance; both as single objective, or sharp ratio or any other ratio. In addition, the inclusion of the norms and constraints of the stocks and money markets acts as multiple constraints. Historically, the Modern Portfolio Theory (MPT) has been established through the benchmark in 1952 namely, Mean-Variance (MV) model for portfolio optimization. In this model the author has considered the variance of the portfolio as an objective function, whereas the mean of the portfolio and two more basic parameters are taken as constraints. Later on, the researchers have developed different relationships between the mean and variance as single objective function, naming them as efficient frontier, treynor ratio and sharp ratio; to name a few. However, they still lack the ability to capture the market reality in terms of multiple market constraints taken simultaneously.

The realistic market constraints are budget, short sale, cardinality, outlier, tax, transaction cost, overnight holding charge, lot and liquidity. It could be rather simple to incorporate the behaviour of them taken one at a time than a combination of them. The nature of data available to portfolio optimization using multiple constraints mostly leads to misleading results, setting another barrier in addition to the complex nature of the problem. This makes mathematical modelling becomes intractable, especially for the large amount of the data available from the financial market. It could be worth mentioning that the criteria for selection of portfolio at several levels play a vital role in portfolio optimization. Also, considering sharp ratio as an objective function leads better results in multiple constraints portfolio optimization. Here, modelling of such multiple constraints portfolio optimization can be classified into two stage, viz, (i) initial selection criteria having mean or variance of the portfolio, and (ii) the later selection and optimization having the constraints and objective function. This two stage portfolio optimization gives the better control in obtaining the solution of mathematical models.

1.3 Metaheuristic Techniques

One way to obtain the solution for mathematical models of multiple constraints portfolio optimization is to apply metaheuristic techniques. Here, meta stands for beyond or an upper level, heuristic stands for to find, now metaheuristic are the strategies that guide the search process or the goal to efficiently explore the search space in order to find optimal/near solution. Metaheuristic algorithms are approximate and usually non-deterministic. These techniques are of nature-inspired phenomena and have the capability to capture the inherent stochastic nature of the problem. These kind of techniques have been developed by observing the behaviour and characteristics of natural phenomena like the birds flocking, fish schooling, and the movement of ant, bee and cuckoo in search of food. These techniques use insects, fish and birds as agents or particles. Many metaheuristic techniques, like Genetic Algorithms (GA), tabu search, Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Artificial Bee Colony (ABC) optimization, cuckoo search optimization and simulated annealing etc. have been developed for the complex optimization problems.

Despite excellent modelling and prediction power, some metaheuristic techniques have a drawback of sticking in the local optimum, hence unable to reach the global optimum solution of the problem. For choosing the best technique for optimization, there is a need to select technique which is robust and doesn't stick in the in the local optima. The Particle Swarm Optimization (PSO) is an excellent candidate to serve this purpose. To control the robustness of the technique while creation and presentation of the portfolio, it requires modification in the algorithm for multiple constraints two-stage portfolio selection and optimization problems.

1.4 Problem Statement

The basic constraint model for portfolio optimization in the financial market is available with the single objective of mean and variance of return which covers only the budget and short sale (allowed and not allowed) as constraints. Since the real world is more complexed and holds several constraints, also the initial selection criteria for portfolio optimization play a vital role for the individual and institutional investor. Few of the constraints and criterion are presented here, (i) it is found that there is a lack of initial criteria for portfolio selection. It could be the return or risk of the stock/asset, considering the stocks/assets of maximum return or minimum risk. (ii) The extraordinary events and fluctuation of the financial market should be considered as a constraint namely outlier. (iii) The investorâĂŹs inclination towards the inclusion or not inclusion of the stock/asset should be taken into account as constraint namely cardinality. (iv) Making a transaction in the financial market needs some fee charged by the regulator of the market. This charged fee is called the transaction cost, which affects the overall profit gain or loss. This transaction cost should be considered as a constraint. (v) The restriction imposed by the company or firm on the trade of single share. The minimum number of shares should be traded in the form of a lot, which could be 100 or more share per lot. This lot should be taken into account as a constraint.

The involvement of all the aforementioned constraints at the same time in the financial market of the real world make the modeling of the portfolio and its optimization more difficult and complex. There is a need to construct the multiple constraints portfolio optimization model. The lack of having the capable and compact algorithm leads towards the development of an efficient algorithm. After having the multiple constraints portfolio optimization models, there is a need to develop an algorithm for the construction, visualization and obtaining the solution of an efficient portfolio model.

1.5 Objectives of the Thesis

The main objective of the thesis is to construct the multiple constraints portfolio optimization models. The purpose of the modeling of such models is to incorporate the real world complexity for stock portfolio optimization in the financial market. In addition, such models should provide the optimal way of investment with regulations imposed by the financial market, the incorporation of robustness and behaviour of the financial market. To achieve the aim, the specific objectives include;

1. To design a multiple constraints model with six real world constraints namely; budget, short sale, outlier, cardinality, lot and transaction cost .
2. To measure the performance of the designed models, considered the sharp ratio as optimal function.
3. To improve the efficiency measure, considered the mean and variance of returns as initial selection criterion.
4. To construct, visualize and obtain the optimal solution of the Multiple Constraints Portfolio Optimization (MCPO) models, modify the PSO as the modified particle swarm optimization technique namely; Modified PSO (MPSO).

In this work, the performance of MCPO depends on the mean and variance as an initial selection criterion, efficient budgeting, restriction on a short sale, an outlier, cardinality, transaction cost and lot as constraints. The comparison of the two different initial selection criterion with two different transaction cost is presented.

1.6 Scope of the Thesis

In this thesis, modeling of the multiple constraints portfolio optimization is restricted to the stock market which is one of the well defined financial markets. Also in terms of multiple constraints, there are few like the basic budget and short sale constraints with outlier, cardinality, lot and transaction cost (fixed and variable) constraints moreover, the regulations of the financial market. The financial market considered here is the Shanghai Stock Exchange 50 index (SSE50 index) for the period of November 13^{th}, 2017 to January 7^{th}, 2011 on daily basis (1665 days) also considered here the various 172 stocks from different areas and sectors.

Since the data considered form the stock market is for the adjusted closed prices. It is robust in nature. Modeling and optimization of such complex and robust natured data require the special kind of solution technique. An algorithm for the modified metaheuristic technique has been developed. It is specially designed for the optimization in the field of the financial market, namely modified particle swarm optimization (MPSO). Also, the stochasticity of the particle swarm optimization has been controlled in this MPSO. In this, algorithm and modeling an approach of the selection criteria at the initial stage has been introduced using the mean and variance of the return as initial selection criteria. This thesis doesn't cover the liquidity and short sale allowance as constraints.

1.7 Contribution and Significance of the Research

The main contributions of the work are the modelling of multiple constraints portfolio optimization. This MCPO model allows the incorporation of the several financial market complex constraints in the model at the same time. Also, it takes into account the inclination and behaviour of the investor towards the area, sector and stock. It captures the uncertainty and stochasticity of the financial market. Moreover, it controls the robustness and uncertainty of the system. Here, the better visualization and selection of the initial and final portfolio has been made easier. The specific contribution includes.

1. Initial selection criterion with mean and variance have been introduced.
2. Multiple constraints portfolio optimization models have been constructed, depending upon the initial selection criterion.
3. Algorithm for modified metaheuristic technique has been developed, namely Modified PSO (MPSO).

The broader significances for this study are to model the multiple constraints portfolio optimization by incorporation of the real financial world complex constraints in the models. The comparison of the significance of the proposed multiple constraints portfolio optimization models with the existing model has been made. The visualization and uncertainty control of the portfolio have been made. This study shows the priority for the initial selection criterion, on the basis of the result reported. Also, the importance of the constraint selection has been proposed.

This study shows the significance by expending the knowledge in the field of MCPO. It will enable the researchers, institutions, and investors to see the effects and behaviors of the selection criterion and multiple constraints. Also, it will provide an effective and simple mathematical formulation for optimization. Moreover, it will increase a new dimension to the mathematics of finance especially in the field of portfolio optimization. This study will also contribute to the area of intractable and complex problems of the real world. It is hoped that the knowledge can be translated into practical application in the world of financial mathematics.

1.8 Thesis Outline

This thesis is consists of five chapters. Chapter 1 comprises of a background, modelling of multiple constraints portfolio optimization, metaheuristic techniques, problem statement, objectives of the thesis, scope of the thesis, research contributions and significance of the research. The contents of remaining five chapters are outlined as follows:

Chapter 2 studies the literature review as introduction; portfolio selection, diversification and optimization; portfolio optimization with cardinality constraint; portfolio optimization with transaction cost constraint; portfolio optimization with lot and liquidity constraints; heuristic and metahuristic techniques in optimization and conclusion.

Chapter 3 studies the introduction; portfolio structure; mean and variance of daily return; portfolio mean and variance of return; budget constraint; short sale constraint; Markowitz mean-variance model; models as single objective function; the efficient frontier model; the sharp ratio model; model using sharp ratio without initial selection criteria; nonlinear representation of data; outlier constraint; cardinality constraint; lot constraint; transaction costs constraint; standard PSO with linear functions; modified PSO; initial selection criteria; two-stage portfolio selection and optimization models; sharp ratio models with initial selection criteria; multiple constraint model with fixed transaction cost; multiple constraint model with variable transaction cost and conclusion.

Chapter 4 studies the introduction to simulation results and their discussion under the benchmark model without and with initial selection criteria. Finally, the chapter concludes having a conclusion.

Chapter 5 studies the introduction to simulation results and their discussion under the multiple constraints models with the initial selection criteria having the fixed and variable transaction cost as constraints. Finally, the chapter concludes having a conclusion.

Chapter 6 studies the conclusion, significant achievements and directions for future work.

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