EXACT SOLUTIONS ON BOUNDARY LAYER FLOW AND HEAT TRANSFER OF CARBON NANOTUBES NANOFLUIDS DUE TO NON-COAXIAL ROTATIONS

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To my beloved husband, Ma, Abah, Along, Izzati, Haziq and Dikna for love and support.

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ABSTRACT

Nanofluid is known as an intelligent engineered fluid which consists of nanometer-sized particles suspended in a conventional base fluid. The development of nanofluid is to enhance the heat transfer capability due to their high thermal conductivity which has attracted numerous researchers' interest. Moreover, the implementation of nanofluid in rotating systems has been applied in various fields such as engineering field in designing advanced cooling and heating systems and medical field in developing the drug delivery in the human body. This thesis presents four problems of boundary layer flow with heat transfer in a rotating non-coaxial carbon nanofluids. The fluid is considered to be an electrically conducting fluid that flows unsteadily through a porous medium over a moving vertical disk. Hence, the effects of magnetohydrodynamic and porosity are taken into account. The first and second problems are discussed on Newtonian fluid model without and with radiation and mass transfer effects. Meanwhile, the third and fourth problems are discussed on Casson fluid model without and with radiation and mass transfer effects. In this research, water as the Newtonian base fluid and human blood as the Casson base fluid are chosen to suspend nanoparticles of single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs). The dimensional governing equations associated with the initial and boundary conditions are converted to the dimensionless form by using appropriate dimensionless variables. By using Laplace transform method, the exact solutions of velocity, temperature, and concentration profiles are obtained. The impact of pertinent parameters such as Casson parameter, Grashof number, modified Grashof number, nanoparticle volume fraction, magnetic field, porosity, radiation, the amplitude of disk, and time on the nanofluid flow, heat and mass transfer are discussed and illustrated graphically. Meanwhile, the skin friction, Nusselt number, and Sherwood number are tabulated in tables. The results show that the fluid with radiation and mass transfer effects has a higher velocity than the fluid without radiation and mass transfer effects. The velocity of Casson nanofluid is higher than Newtonian nanofluid. The flow with a radiation effect has a higher temperature than the flow without radiation. SWCNTs exhibit a lower velocity profile and a higher temperature profile compared to MWCNTs. All the present results are compared to the published results, and the validity of the obtained solutions is confirmed when an excellent agreement is observed. The exactness of the obtained solutions is verified when the comparison of the right-hand side and the left-hand side of the system of equations show an identical value.

ABSTRAK

Bendalir nano dikenali sebagai bendalir pintar yang terdiri daripada zarah-zarah bersaiz nanometer yang terendam dalam bendalir asas konvensional. Pembangunan bendalir nano adalah untuk meningkatkan keupayaan pemindahan haba kerana mempunyai tahap kekonduksian yang tinggi yang telah menarik minat ramai penyelidik. Selain itu, pelaksanaan bendalir nano di dalam sistem yang berputar telah diterapkan dalam pelbagai bidang seperti bidang kejuruteraan dalam membina sistem penyejukan dan pemanasan yang canggih dan bidang perubatan dalam mencipta sistem penghantaran ubat di dalam tubuh manusia. Tesis ini membentangkan empat permasalahan aliran lapisan sempadan dengan pemindahan haba dalam bedalir nano yang berputar secara bukan sepaksi. Bendalir ini dianggap sebagai bendalir pengalir elektrik yang mengalir secara tidak mantap melalui medium yang berliang di atas cakera menegak yang bergerak. Oleh itu, kesan hidrodinamik magnet dan keliangan diambil kira. Permasalahan pertama dan kedua membincangkan model bendalir Newtonan tanpa dan dengan kesan radiasi dan pemindahan jisim. Manakala, permasalahan ketiga dan keempat membincangkan model bendalir Casson tanpa dan dengan kesan radiasi dan pemindahan jisim. Dalam kajian ini, air sebagai bendalir asas Newtonan dan darah manusia sebagai bendalir asas Casson telah dipilih untuk merendam nanopartikel jenis tiub nano karbon dinding tunggal (SWCNTs) dan tiub nano karbon dinding pelbagai (MWCNTs). Persamaan menakluk berdimensi yang dikaitan dengan syarat awal dan sempadan ditukar kepada bentuk tak berdimensi dengan menggunakan pemboleh ubah tak berdimensi yang sesuai. Dengan menggunakan kaedah penjelmaan Laplace, penyelesaian tepat bagi profil halaju, suhu, dan kepekatan diperoleh. Kesan parameter yang berkaitan seperti parameter Casson, nombor Grashof, nombor Grashof yang diubah, pecahan isipadu nanopartikel, medan magnet, keliangan, radiasi, amplitud cakera, dan masa terhadap aliran bendalir nano, pemindahan haba dan jisim dibincangkan dan digambarkan secara bergraf. Geseran kulit, nombor Nusselt, dan nombor Sherwood dijadualkan dalam jadual. Hasil kajian menunjukkan bahawa bendalir dengan kesan radiasi dan pemindahan jisim mempunyai halaju yang lebih tinggi berbanding dengan bendalir tanpa radiasi dan pemindahan jisim. Halaju bendalir nano Casson lebih tinggi daripada bendalir nano Newtonan. Aliran dengan kesan radiasi mempunyai suhu yang lebih tinggi berbanding dengan aliran tanpa radiasi. SWCNT menunjukkan profil halaju rendah dan profil suhu tinggi berbanding MWCNT. Semua keputusan yang ada dibandingkan dengan keputusan yang diterbitkan dan kesahan penyelesaian yang diperoleh adalah disahkan apabila penyesuaian yang sangat baik diperhatikan. Ketepatan penyelesaian yang diperoleh adalah disahkan apabila perbandingan pada sebelah kanan dan kiri sistem persamaan menunjukkan nilai yang sama.

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LIST OF ABBREVIATIONS

CNTs	-	Carbon nanotubes
SWCNTs	-	Single-wall carbon nanotubes
MWCNTs	-	Multi-wall carbon nanotubes
MHD	-	Magnetohydrodynamic
ESD	-	Electrostatic discharge
LHS	-	Left-hand side
RHS	-	Right-hand side
Cu	-	Copper
Al_2O_3	-	Alumina
TiO_2	-	Titanium oxide
Fe_3O_4	-	Iron oxide

LIST OF SYMBOLS

Roman Letters

B_0	-	Magnitude of applied magnetic field (Wb m^{-2})
$\mathbf{b}^*$	-	Induced magnetic field
$\mathbf{B}$	-	Total magnetic field
$\mathbf{B_0}$	-	Magnetic field vector
C_p	-	Specific heat (J kg^{-1} K^{-1})
C	-	Concentration of nanofluid (kg m^{-3})
C_w	-	Wall concentration (kg m^{-3})
C_∞	-	Free stream concentration (kg m^{-3})
D	-	Mass diffusivity
D_{nf}	-	Mass diffusivity of nanofluid (m^2 s^{-1})
D_f	-	Mass diffusivity of fluid (m^2 s^{-1})
$\frac{D}{Dt}$	-	Substantial derivative
f	-	Dimensional non-coaxial velocity in x-direction
$\mathbf{E}$	-	Electric field
$\hat{e}_{ij}$	-	$(i,j)^{th}$ component of deformation rate
$erfc$	-	Complementary error function
exp	-	Exponential function
F	-	Complex velocity (m/s)
$\mathbf{F}^*$	-	Body force
g	-	Dimensional non-coaxial velocity in y-direction
g_x	-	Gravitational acceleration in x-direction
$\mathbf{g}$	-	Gravitational acceleration vector
Gm	-	Mass Grashof number
Gr	-	Thermal Grashof number
h	-	Dimensional non-coaxial velocity in z-direction

i	-	Imaginary unit
$\mathbf{i}$	-	Unit vector in x-direction
$\mathbf{I}$	-	Identity tensor
$\mathbf{j}$	-	Unit vector in y-direction
$\mathbf{J}$	-	Current density
$\mathbf{k}$	-	Unit vector in z-direction
k	-	Thermal conductivity
k_1	-	Permeability of porous medium (m^2)
k_{nf}	-	Thermal conductivity of nanofluid ($m^2\ s^{-1}$)
k_f	-	Thermal conductivity of fluid ($m^2\ s^{-1}$)
k_{CNTs}	-	Thermal conductivity of carbon nanotubes ($m^2\ s^{-1}$)
K	-	Dimensionless porosity parameter
M	-	Dimensionless magnetic parameter
Nu	-	Nusselt number
p	-	Pressure
p	-	Modified pressure gradient
p_d	-	Dynamic pressure
p_h	-	Hydrostatic pressure
p_y	-	Yield stress
Pr	-	Prandtl number
q	-	Laplace transform parameter
q_r	-	Radiative heat flux
$\mathbf{r}$	-	Vector of radius
$\mathbf{R}$	-	Darcy's resistance for porous medium
Rd	-	Radiation parameter
Sc	-	Schmidt number
Sh	-	Sherwood number
t	-	Time (s)
t^*	-	Dimensionless time
T	-	Temperature of nanofluid (K)

T_w	-	Wall temperature (K)
T_∞	-	Free stream temperature (K)
$\mathbf{T}^*$	-	Cauchy stress tensor
u	-	Velocity in x-direction
U	-	Amplitude of the moving plate
U_0	-	Characteristic velocity
v	-	Velocity in y-direction
$\mathbf{V}$	-	Velocity vector field
w	-	Velocity in z-direction
x	-	Coordinate axis of the plate
y	-	Coordinate axis of the plate
z	-	Coordinate axis normal to the plate
z'	-	Coordinate axis for non-coaxial rotation
z^*	-	Dimensionless in z-direction

Greek Letters

∇	-	Del operator
$\nabla\cdot$	-	Divergence
β_T	-	Thermal expansion coefficient
$(\beta_T)_{nf}$	-	Thermal expansion coefficient of nanofluid
β_C	-	Mass expansion coefficient
$(\beta_C)_{nf}$	-	Mass expansion coefficient of nanofluid
γ	-	Casson parameter
μ_{nf}	-	Dynamic viscosity of nanofluid
μ_f	-	Dynamic viscosity of fluid
μ_P	-	Plastic dynamic viscosity of the non-Newtonian fluid
π^*	-	Product of the component of deformation rate
π_c	-	Critical values of this product based on non-Newtonian model
ρ	-	Density

ρ_{nf}	-	Density of nanofluid
ρ_f	-	Density of fluid
ρ_{CNTs}	-	Density of carbon nanotubes
ρ_∞	-	Density of fluid in free stream
σ	-	Electrical conductivity
σ_{nf}	-	Electrical conductivity of nanofluid
σ_f	-	Electrical conductivity of fluid
σ_{CNTs}	-	Electrical conductivity of carbon nanotubes
σ_{xx} , σ_{yy} , σ_{zz}	-	Normal stress
τ	-	Skin friction
τ^*	-	Non-dimensional skin friction
τ_{ij}	-	Component of stress tensor
τ_{xy} , τ_{xz}	-	Shear stress in x -direction
τ_{yx} , τ_{yz}	-	Shear stress in y -direction
τ_{zx} , τ_{zy}	-	Shear stress in z -direction
ϕ	-	Solid volume fraction of nanoparticles
ν	-	Kinematic viscosity
ν_{nf}	-	Kinematic viscosity of nanofluid
Ω	-	Constant angular velocity
$\mathbf{\Omega}$	-	Angular velocity vector

Subscripts

w	-	condition on the wall
∞	-	free stream condition
f	-	based fluid
nf	-	nanofluid
$CNTs$	-	carbon nanotubes

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Preface

This introductory chapter discusses the overview of the study. Section 1.2 explains some remarks for the background of the research topic on heat and mass transfer, nanofluids, carbon nanotubes, fluids, boundary layer theory, rotating flow, types of effect, dimensionless parameters and the method of Laplace transform involved in the research. In Section 1.3, the research problems in conducting this study are highlighted and this study is guided by the given objectives in Section 1.4. The scope of research is presented in Section 1.5 and the significance of study is pointed out in Section 1.6. Lastly, thesis organization are discussed in Section 1.7.

1.2 Research Background

1.2.1 Heat and Mass Transfer

Heat energy transfer has become a significant process in industry applications due to rising demand in manufacturing. Due to that, an essential knowledge subjected to heat transfer is required to understand well in heat transfer performance for many applications. The knowledge of heat energy transfer includes the methods to enhance heating and cooling processes. Enhancing the heating and cooling processes in industries will save energy, reduce the processing time, increase the thermal rate, and increase the equipment's lifespan. Heat transfer, also known as flow of heat, is a process by which the thermal energy is transferred from one region to another region because of the difference of temperature occurred. Truly of heat flow phenomenon, all are worked in pervasive way where it can be taken placed between any materials or substances that own the unbalance of temperature and the flow is driven from higher

temperature to a lower temperature. The heat flow results from the cooling of the sun become the greatest instance of the process and we have naturally experienced it. Even, the earth's surface is warmed by the cooling of its core and also by the radiation from the distant stars. Furthermore, the heat transfer can be classified into three types of mode as shown in Figure 1.1 which are conduction, convection and radiation. Conduction is defined as transferring of heat exist due to direct contact without molecular motion of the substances and the temperature gradient causes the heat to flow from a hot to a cold region until thermal equilibrium is achieved. Meanwhile, convection is the movement of heat due to the molecular motion of non-uniform temperature fluid. This convection heat transfer can only take place between the fluids or within a fluid and solid but not between the solids. Another mode of heat transfer is radiation by which the heat is transferred through electromagnetic waves without the presence of any matter. The temperature gradient is unrequired when the heat is transmitted through radiation. Similar concept is adopted in mass transfer where it defines as the transportation of mass from one place to another place due to the concentration gradients appeared. Just as the heat transfer, the movement of mass also diffuses from high concentration region to low concentration region. The significance of heat transfer is seemed to be widely explored especially among the emergence industries where it can be imposed in producing various heat transfer equipment such as heat exchanger, fin fan cooler, cooling towers and radiators.

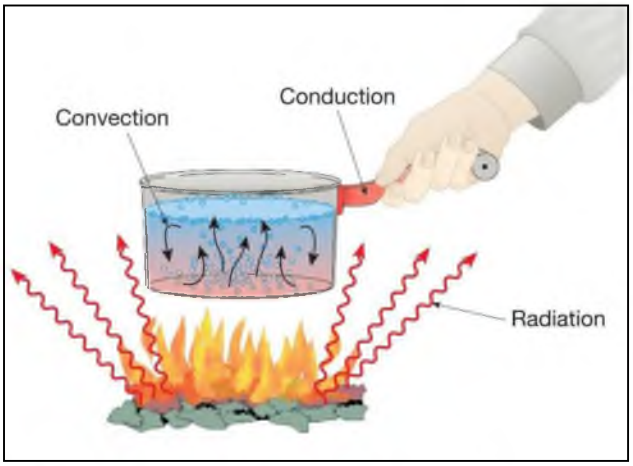

Figure 1.1 Mode of heat transfer

Basically, the heat and mass can be transferred from one region to another region by utilizing different types of convective mechanism which are free, forced and

mixed convection. In this study, free convection is the main focus, and heat and mass are transmitted by this type of mechanism. As we know, free convection also known as natural convection, refers to the fluid motion which induced by natural means, buoyancy forces and gravity without any external forces like a suction device, fan or pump. The arising of buoyancy force in a fluid is driven by the existence of density gradients which usually caused by difference of temperature in the fluid. Moreover, the velocity of fluid by natural convection is relatively slow compared to another two types of convection and thus cause a low heat transfer. However, the transport mechanism is called as forced convection when the fluid flow is maintained by imposing the external forces due to the friction between the surfaces. The alternate situation occurs when the fluid motion is affected by free and forced convections simultaneously and thus called as mixed convection.

1.2.2 Nanofluids

Recently, the advancement of heat transfer system among emergence industries has created a significant demand for having a new technology, which able to improve their heat transfer process. The implementation of nanofluid is an alternative for industries to have an efficient heat transfer system. The development on nanofluid as a smart heat transfer fluid by dispersing nanoparticles in conventional base fluid was initiated by Choi and Eastman (1995). It is good to know that nanofluid is a fluid that containing suspended nanoparticles in a base fluid, and it experiences two-phase system of a colloidal dispersion which occurs between nanoparticles (solid phase) and conventional base fluid (liquid phase). Normally, the conventional base fluid such as water, ethylene glycol, kerosene and engine oil have relatively low thermal conductivity compared to thermal conductivity of solids, making this fluid to has a limited capability of heat transfer. Motivated from this fluid deficiency, the nanoparticles, nano-sized material with unique chemical and physical properties like metals, metal oxides or carbon, are immersed in this fluid so that the nanofluid thermal conductivity enhances, and thus, the process of conduction and convection be more effective in transferring heat (Sivashanmugam, 2012; Feng and Kleinstreuer, 2010; Khan *et al.,* 2012). The schematic diagram represented the production of nanofluid is given in Figure 1.2. The nanofluid is realized to has great properties and play a vital

role to improve system in transferring heat from one place to another place. In many industries, having a low thermal conductivity of fluid become a primary limitation. Therefore, nanoparticles are added into the base fluid and this is the most recent techniques applied in various areas of industry. As a result, the high thermal conductivity of fluid is exhibited and lead to the enhancement of heat transfer.

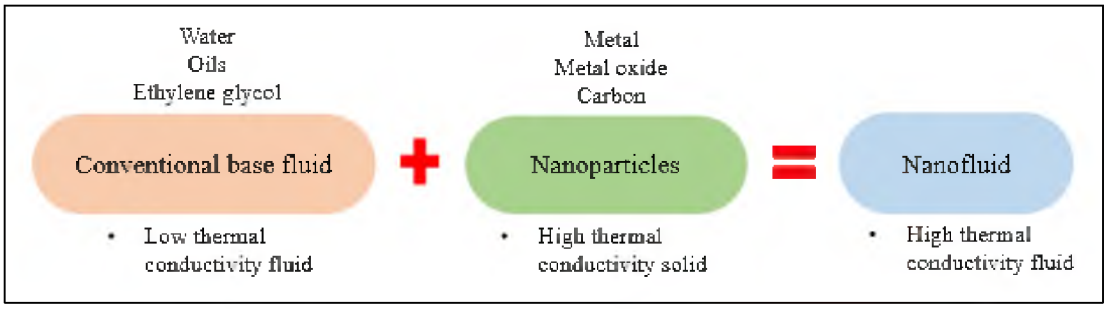

Figure 1.2 Production of nanofluids

1.2.3 Carbon Nanotubes

One of the greatest discoveries in material science history is carbon nanotubes (CNTs), which was discovered by a Japanese researcher in the beginning of the 1990s. Since the discovery, due to the unique electronic structural and mechanical characteristics, CNTs are found as valuable nanoparticles, especially in nanotechnology field. CNTs are great conductance which is highly sought in medical applications. They have been used as drug carriers, biomedical imaging, and have benefited cancer therapy treatments. Mainly, carbon nanotubes can be found in two types, which are single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs). Specifically, SWCNTs is known as the cylindrical tubes that made up by a single layer of 0.4-3nm diameter of graphene sheet, while MWCNTs are structured by a group of nested tubes with multi-layer of graphene sheet and have diameter in range of 0.4 to 30nm (Ellahi *et al.*, 2015; Nasir *et al.*, 2019). The overview of their structure is depicted in Figure 1.3. The most interesting part of CNTs is that they have an extremely strong form of molecular interaction, which chemically bonded each other. This unique feature comes with the van der Waals forces roping them together and providing the opportunity to develop ultra-high strength, low-weight materials that have highly conductive electrical and thermal properties. All these properties make carbon nanotubes as the most suitable materials for electronic devices

like sensor, transistor, lithium-ion batteries, electrostatic discharge (ESD) and electrical-shielding applications.

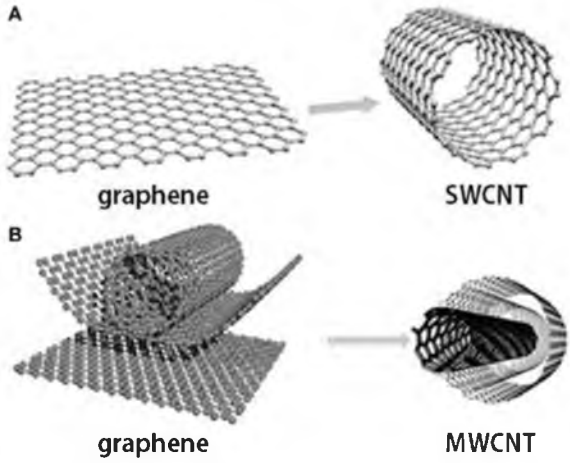

Figure 1.3 Structure of single-wall carbon nanotubes (SWCNTs) and multi-wall carbon nanotubes (MWCNTs) (Vidu *et al.*, 2014)

1.2.4 Fluids

Fluid is the most crucial phase that brings a huge responsibility in convective heat transfer and also where the motion of flow will be affected on. This fluid can be found in two types of fluid which are Newtonian and non-Newtonian fluids. Newtonian fluid is known as fluid that follows the Newton's Law of viscosity. The equivalent saying to this fact is that its flow behavior can be described with a simple linear relation between shear stress and shear rates. Some examples of Newtonian fluids are water, benzene, hexane and organic solvents. Normally, the viscosity of this fluid only depends on the temperature and pressure where it can be seen that its proportionality constant is being the coefficient of viscosity. This directly implies that the viscosity of Newtonian fluid remains unchanged and does not influence by the amount of the force applied on the fluid. However, the opposite fact is found in non-Newtonian fluid where its viscosity is an applied stress and force dependent variable. The non-Newtonian fluid viscosity can change either to be more liquids or more solids which all depend on the amount of force applied. This kind of physical behavior is commonly exhibited by substances that have an unfixed coefficient of viscosity such as cornstarch, paint, honey, tomato sauces, blood and polymer solution.

Casson fluid is one of the fluid models for non-Newtonian fluid and it behaves as viscoplastic fluid as shown in Figure 1.4. The flow behavior of this fluid is characterized by the effect of shear thinning and the existence of a yield stress. The shear thinning fluid is described by a non-uniform of viscosity (the slope of shear stress versus shear rate curve) where the slope (viscosity) is seemed to decrease when the shear rates increase as in Figure 1.5. A clear illustration of viscosity versus shear rates for different types of fluid is also demonstrated in Figure 1.6. The yield stress in Figure 1.4 is defined as the minimum shear stress for a solid to undergo the permanent deformation or plastic flow. Many researchers have viewed the yield stress for a viscoplastic fluid as the marking point for the transition from solid to plastic behavior (the reaction does not return to their original when the applied forces are removed). More precisely, the viscoplastic fluid tends to exhibit both solid and plastic properties which are greatly depend on the applied shear stress. The fluid behaves as a likely solid phase (high viscosity) when the yield stress is greater than applied shear stress. Meanwhile, the fluid starts to flow when the applied shear stress is greater than yield stress (Irgens, 2014). It is more exciting when Casson fluid is found as the most relevant model to describe the characteristics of human blood. Due to its unique characteristic, it becomes the most interesting part of our nature to be explored. When the human blood flows through a large diameter of arteries with a high shear rate, this phenomenon indicates that the blood exhibits the non-Newtonian fluid's properties. Sometimes, the blood also behaves as the Newtonian fluid and this happened when it flows through small diameter of arteries with low shear rates. In addition to the characteristics of human blood, the composition of high volume (91%) of water together with the protein, hormones and glucose in human body has caused the blood to behave like Newtonian properties at the certain human body temperature (Earl and Mohammadi, 2018). Due to the incredibly high-water content, the viscosity of the blood is highly affected by the hydration level in human body. Once the body dehydrated, the blood becomes more viscous and the non-Newtonian properties is almost exhibited.

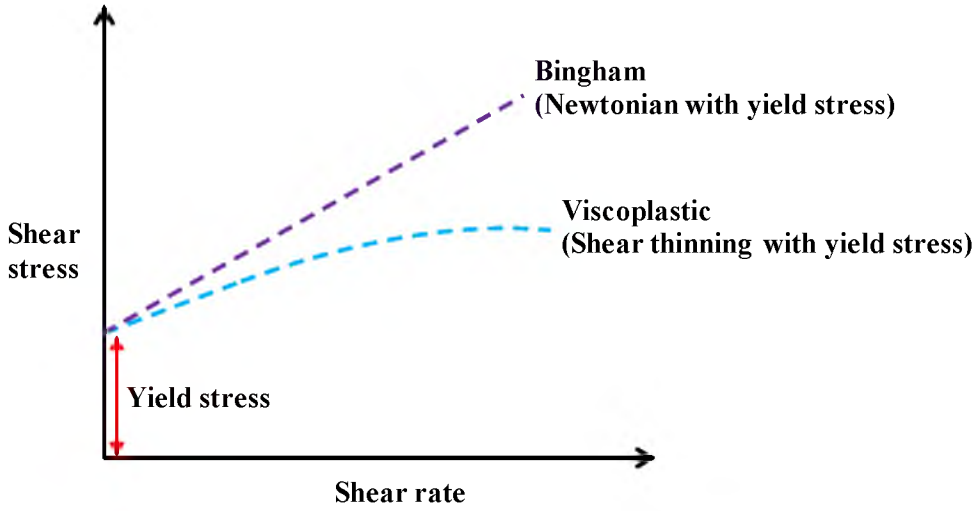

Figure 1.4 Rheology behavior of shear stress versus shear rate for fluid with yield stress

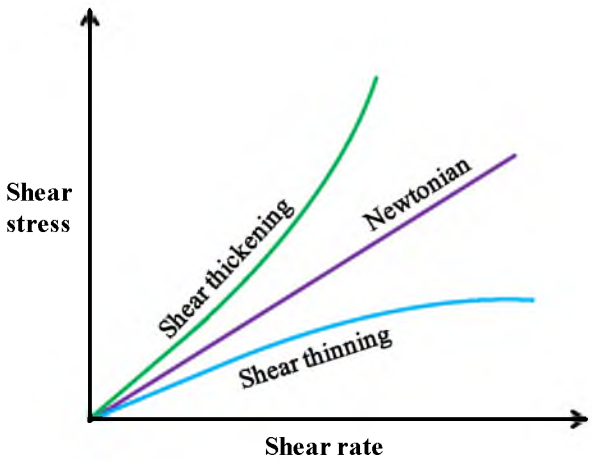

Figure 1.5 The shear stress of fluids as a function of shear rate

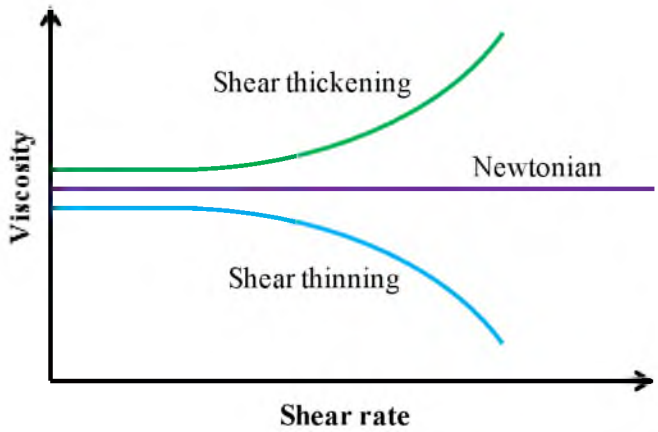

Figure 1.6 The viscosity of Newtonian, Shear thickening and Shear thinning fluids as a function of shear rate

1.2.5 Boundary Layer Theory

The mathematical problems associated with the solutions of the equations of motions, energy and concentration have promoted researchers to develop concepts that lead to the simplification of these equations. Hence, the boundary layer concept is most successful in achieving simplification of the equation of motion, energy and concentration and has been applied to large variety of practical situations. In boundary layer concept, the flow over a body is divided into two regions: i) a very thin layer in the neighborhood of the body, called the boundary layer, where the velocity and temperature gradients are steep; and ii) the region outside the boundary layer, called external flow region, where the velocity, temperature and concentration gradients are small. In general, the boundary layer concept provides a good description of the velocity, temperature and concentration fields.

1.2.6 Rotating Flow

Apart from that, when discussing the subject of fluid flow and heat transfer, a rotating fluid is an attractive phenomenon that can affect the fluid motion in transferring heat. Basically, the theory on rotating fluid is adapted critically on understanding and predicting the flow of phenomena on earth surface, especially at large scale atmospheric and oceanic flows. Rotating fluid theory is important to explain and describe the mathematical models of rotating flow which are used to study the fluid motion. Even, this model can be widely explored in various field of science and engineering, providing model in rotating geophysical flow which usually depend on the earth rotation and also producing an axial flow of turbo fan jet engine. Moreover, it is very useful to study on fluid motion of rotating fluids as they have numerous physical features in common, and the understanding of this features can often be aided by the use of a common set of mathematical techniques. For instances, the vortices produced in a flow along the channel, the secondary flows induced for flow around a bend and wing-tip vortices caused the downstream of a wing. When discussing on rotating fluid models, the Coriolis force arises when equations of motion are written in rotating system and it is certainly important in comparison of inertial and viscous forces. Besides that, Coriolis force in fluid also plays a significant role in

determining the differences between dynamics of non-rotating and rotating fluids. An apparent deflection of moving objects within an opposite direction of rotating frame is called as the effect of the Coriolis force and the objects are seem to deviate from its path due to the motion of frame. Results from literature review, the rotation of fluid can be found in two types which are coaxial rotation and non-coaxial rotation. Coaxial rotation is defined as the fluid that sharing the same axis of rotation on a straight line. Meanwhile, non-coaxial rotation is known as the two fluid flows which rotating on their respective axes separated by a distance noted as length.

1.2.7 Types of Effect

1.2.7.1 Porosity

In addition to the flow regimes of a fluid and heat transfer, the fluid motion is also highly affected by the condition of the passed medium. In this study, the porous condition of a medium is considered. A porous medium is defined as the medium that containing pores and its skeleton is usually be in a solid phase. The characteristic of a porous medium varies, depend on the size of the pores, the porosity and the compositions of medium itself. The porosity is known as the ratio of the volume occupied by the effective voids to the total volume of the medium. Moreover, the porous medium is also influenced by its permeability, whereby it will affect the ability of a fluid to flow passing through the pores. In other words, the more permeable of the porous medium, then the flow of fluid passing through its pores be easier. The study of fluid flow subjected to porous medium has received significant attention from number of researchers due to its broad applications in scientific and engineering fields. The model of flow in porous medium plays an important role in petroleum engineering to study the motion of natural gas, water and oil through the oil reservoirs as well as in chemical engineering for the purpose of filtration and water purification process. In light of its importance, a cluster of studies subjected to this field has been done, including Ali *et al.* (2016), Chaudhary and Jain (2010), Shah *et al.* (2019), Krishna *et al.* (2020) and Krishna and Chamkha (2019).

1.2.7.2 Magnetohydrodynamics (MHD)

The other factor influencing the movement of fluid flow is magnetic fields. They are commonly found in many natural and man-made flows. Their applications also encountered in many industries for the purpose of heating, pumping, stirring and levitating liquid metals. There is the terrestrial magnetic field which is maintained by fluid motion in the earth's core, the solar magnetic field which generates sunspots and solar flares, and the galactic magnetic field which is thought to influence the formation of stars from interstellar clouds. The study of these flow is called as magnetohydrodynamics (MHD) (Davidson, 2002). Specifically, the word magnetohydrodynamics is actually come from the combination of three terms, which are *magneto* means magnetic field, *hydro* means water and *dynamics* means movements. The MHD flow is the magnetic fields which can induce currents in a moving conductive fluid. Liquid metals, plasmas and electrolytes are all important examples of MHD fluids. In MHD flows, there exists Lorentz force which has been used to regulate the variety of flow regimes. The applications of MHD flow can be found in propulsion systems, energy generators, smart spacecraft landing gear systems, hydrogen production with solar MHD plants, plasma fusion technology, nuclear thermal control systems, MHD chemical reactor processing and biomagnetic reactors (Das and Jana, 2014).

1.2.8 Dimensionless Parameters

1.2.8.1 Prandtl number

Prandtl number (Pr) is a dimensionless number, named after its inventor, a German engineer Ludwig Prandtl, who also identified the boundary layer. The Prandtl number is defined as the ratio of momentum diffusivity to thermal diffusivity. The momentum diffusivity, or as it is normally called, kinematic viscosity, denoted the material's resistance to shear-flows in relation to density. The Prandtl number is given as

$$\Pr = \frac{\nu_f}{\alpha_f} = \frac{\mu_f / \rho_f}{k_f / \left(\rho C_p\right)_f} = \frac{\nu_f \left(\rho C_p\right)_f}{k_f}, \tag{1.1}$$

where ν_f is the kinematic viscosity (momentum diffusivity) and α_f is the thermal diffusivity of the fluid. Meanwhile, μ_f, ρ_f, k_f and $\left(\rho C_p\right)_f$ are the dynamic viscosity, density, thermal conductivity and heat capacitance of the fluid.

1.2.8.2 Schmidt number

Schmidt number (Sc) is a dimensionless number, named after the German engineer Ernst Heinrich Wilhelm Schmidt (1892-1975). The Schmidt number is defined as the ratio of momentum diffusivity (kinematic viscosity) and mass diffusivity, and is used to characterize the fluid flows in which there are simultaneous momentum and mass diffusion convection processes. The Schmidt number describes the mass momentum transfer and the equation is given as

$$Sc = \frac{\nu_f}{D_f} \tag{1.2}$$

where D_f is mass diffusivity of the fluid.

1.2.8.3 Grashof number and modified Grashof number

Grashof number (Gr) is a dimensionless number, named after Franz Grashof and it is defined as the ratio of the buoyancy force to viscous force acting on a fluid in the velocity boundary layer. Its role in natural convection is the same as that of the Reynold number in forced convection. In heat and mass transfer, natural convection is caused by a change in density of a fluid due to a temperature and concentration change or gradient. The Grashof number (Gr) and modified Grashof number (Gm) are respectively defined as

$$Gr = \frac{(\beta_T)_f \, g_x}{\Omega^2 \ell}(T_w - T_\infty) \tag{1.3}$$

$$Gm = \frac{(\beta_C)_f \, g_x}{\Omega^2 \ell}(C_w - C_\infty) \tag{1.4}$$

where is $(\beta_T)_f$ thermal coefficient expansion, g_x is gravitational due to an acceleration, Ω angular velocity, ℓ distance, T_w and T_∞ are wall and free stream temperature, $(\beta_C)_f$ is concentration coefficient expansion, C_w and C_∞ are wall and free stream concentration.

1.2.8.4 Magnetic field parameter

The magnetic field parameter (σ_f) is investigated in this study due to consideration of MHD effect and it can be mathematically defined as

$$M = \frac{\sigma_f B_0^{\,2}}{\Omega \rho_f}, \tag{1.5}$$

where σ_f is electrically conductivity of fluid, $B_0^{\,2}$ is the strength of the magnetic field, Ω is uniform angular velocity and ρ_f density of fluid.

1.2.8.5 Porosity parameter

The porosity parameter (K) is investigated due to the consideration of porous medium and can be defined as

$$\frac{1}{K} = \frac{\nu_f}{k_1 \Omega}, \tag{1.6}$$

where ν_f is the kinematic viscosity of fluid, k_1 is the permeability of porous medium and Ω is uniform angular velocity.

1.2.8.6 Radiation parameter

In this study, the transportation of heat is also induced by the thermal radiation and this leads to the investigation of radiation parameter. Its mathematical expression is defined as

$$Rd = \frac{16\sigma^* T_\infty^{\,3}}{3k^* k_f}, \tag{1.7}$$

where is σ^* the Stefan-Boltzmann constant, $T_\infty^{\,3}$ is the free stream temperature, k^* is the mean absorption coefficient and k_f the thermal conductivity of fluid.

1.2.9 Laplace Transform Method

Various analytical methods are available for the exact solutions. Amongst them, the Laplace transform method is beneficial for linear differential equations with constant coefficient subject to given conditions called initial and boundary conditions. Exact solutions are considerably important as this could be a verification and benchmark for other solution obtained by numerical and approximate methods. The Laplace transform is a powerful tool in applied mathematics and engineering which particularly useful in solving linear ordinary differential equations that involve the functions with respect to time. Since the present problems will be involved this function, therefore the Laplace transform method (A French mathematician namely Pierre-Simon de Laplace was the inventor of this method) is chosen to solve these problems. This method transforms a given problem to one that easier to be solved by converting the ordinary differential equations in time t domain into algebraic equations in the q domain. Mathematically,

$$\begin{aligned} \mathcal{L}\{f(t)\} &= \int_0^\infty e^{-qt} f(t)dt, \\ &= F(q), \end{aligned} \tag{1.8}$$

where is Laplace transform parameter. After that, the solutions are transformed back to the function of t by using the inverse Laplace transform as $F(t) = \mathcal{L}^{-1}\{f(q)\}$. Furthermore, in some cases, it is difficult to find the inverse Laplace transform for a complicated transformed function. In such situation, the formulae used to inverse back the Laplace solution is presented in the Appendix A.

1.3 Problem Statement

The main work of this research is to study the behavior of heat and mass transfer in free convection flow of carbon nanotubes on nanofluid induced by non-coaxial rotation of moving vertical disk. Focus of the research is to investigate the impact of carbon nanotubes on the nanofluid motion and heat transfer, where have been specifically conducted on two types of base fluid which are Newtonian and Casson fluids. Conducting this research will explore the following questions.

(i) How does the mathematical model behave in the problem of unsteady free convection flow of carbon nanotubes on Newtonian and Casson nanofluids in non-coaxial rotation?

(ii) How can the exact solutions for complicated free convection flow for the proposed nanofluid model be obtained?

(iii) How do the physical parameters embedded in the nanofluid flow models affect the behaviors of velocity, temperature and concentration profiles as well as skin friction, Nusselt and Sherwood numbers?

1.4 Objectives of the Research

The main objectives of this study are:

(i) To construct the mathematical model by extending appropriate governing momentum, energy and mass equations of non-coaxial rotation nanofluid subjected to the suitable initial and boundary conditions,

(ii) To obtain the exact solutions for the velocity, temperature and concentration profiles by using Laplace transform method,

(iii) To analyze graphically the impact of difference physical parameters on the behavior of the velocity, temperature and concentration profiles,

(iv) To compute Nusselt number, skin friction and Sherwood number for the problem,

(v) To compare the obtained solutions with published results and numerical coding in order to verify the validity of results.

for the following problems

1. Non-coaxial rotation of MHD Newtonian nanofluid flow in a porous medium past a moving vertical disk with heat transfer effect.

2. Radiative non-coaxial rotation of MHD Newtonian nanofluid flow in a porous medium past a moving vertical disk with heat and mass transfer effects.

3. Non-coaxial rotation of MHD Casson nanofluid flow in a porous medium past a moving vertical disk with heat transfer effect.

4. Radiative non-coaxial rotation of MHD Casson nanofluid flow in a porous medium past a moving vertical disk with heat and mass transfer effects.

1.5 Scopes of the Research

This research focusses on the heat and mass transfer effect of non-coaxial rotation in unsteady free convection flow for nanofluid which specifically synthesize in two different types of based fluid. In this problem, carbon nanotubes are going to be the nanoparticles that suspended in Newtonian and Casson fluids as the base fluid. The first two problems of Newtonian carbon nanotubes are focused on fluid motion induced by the heat transfer as well as heat and mass transfer together with the radiation, magnetic and porosity effects. While, the next two problems tackle the same focus as the first two problems but considering of Casson carbon nanotubes. The model of this problem is modelled by governing non-coaxial momentum, energy and mass equations for nanofluid with associated initial and boundary conditions based on proposed model. This system of equations is transformed into a set of dimensionless partial differential equation by introducing suitable dimensionless variables. The resulted dimensionless system of equations is then solved by using Laplace transform method. The skin friction, Nusselt number and Sherwood number are also evaluated. The exact solutions of temperature, velocity and concentration profiles are graphically plotted by using Mathcad coupled with comprehensive discussion. In order to check for accuracy, the present results will be compared with the published work in the literature.

1.6 Significance of the Research

The results obtained from this project are significant because of the following reasons.

(i) To build a better understanding on the rheological behavior of non-coaxial rotation of fluid flows in carbon nanotubes,

(ii) To enhance the knowledge on the heat and mass transfers characteristics in rotating nanofluid,

(iii) To give insight on the physical behavior of non-coaxial rotation of fluid flows affected by free convection phenomenon,

(iv) To introduce new knowledge of theoretical study that can be a good reference to researchers, engineering applications and education,

(v) These exact solutions can be used as a check of correctness for the solutions of more complex mathematical models obtained through numerical schemes.

1.7 Thesis Organization

This thesis consists of eight chapters which begins with the introductory chapter, Chapter 1. This chapter highlights some definitions for the terms related to the research topic and followed by the problem statements, objectives of the research, scopes of the research, significance of the research and thesis organization. Chapter 2 discusses on the literature reviews that related to the research topic. The following Chapter 3 provides the research methodology and derivation of governing momentum, energy and concentration equation for the non-coaxial rotation of unsteady Casson nanofluid flow with heat and mass transfer phenomenon. A boundary condition for moving disk together with the effects of radiation, magnetic field and porosity are taken into consideration in deriving the formulation.

Chapter 4 presents on the non-coaxial rotation of MHD Newtonian nanofluid flow in a porous medium past a moving vertical disk with heat transfer effect. Water is considered as the Newtonian base fluid and both types of carbon nanotubes (SWCNTs and MWCNTs) are chosen as the dispersing nanoparticles. The dimensional governing equations with their initial and boundary conditions are transformed to the dimensionless form by using suitable dimensionless variables. The exact solutions for this problem are solved analytically using the Laplace transform method. The impacts of embedded parameters on the velocity and temperature profiles are illustrated graphically using Mathcad software. The skin friction and Nusselt number are computed. Two ways of validation are conducted by which the previous

study published by Mohamad *et al.* (2016) and numerical Gaver-Stehfest algorithm are used as the benchmarks to check the accuracy of the obtained solutions.

Chapter 5 provides an extension to the previous problem in Chapter 4, where the effects of radiation, heat and mass transfer are included. Following the similar procedure, new velocity, temperature and concentration profiles are obtained and plotted graphically in order to investigate the effects of related parameters. The skin friction, Nusselt number and Sherwood number are evaluated based on the resulting solution. The similar validation is also carried out. After that, Chapter 6 presents the similar problem as in Chapter 4 but different in the type of fluid model. This chapter has considered Casson fluid model and uses human blood as the base fluid. Meanwhile, the involved nanoparticles and the method use to solve the problem remain the same. A new expression of skin friction for Casson nanofluid is defined. Chapter 7 discusses on the similar problem in Chapter 5 but replacing the Newtonian nanofluid model to Casson nanofluid model. The last chapter is Chapter 8 where the summary of the research together with the future research suggestions are presented. References and appendixes are listed at end of this thesis.

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Journal with Impact Factor

1. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie, Ilyas Khan, Lim Yeou Jiann and Mohd Rijal Ilias. (2021). Non-coaxial Rotation Flow of MHD Casson Nanofluid Carbon Nanotubes Past a Moving Disk with Porosity Effect. *Ain Shams Engineering Journal*, *12*(4), 4099-4110. https://doi.org/10.1016/j.asej.2021.03.011. **(Q2, IF: 3.18)**. *Problem 3.*

Indexed Journal

1. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie, Ilyas Khan and Lim Yeou Jiann. (2020). Radiative non-coaxial rotation of MHD Newtonian carbon nanofluid flow in porous medium with heat and mass transfer effects. *Journal of Nanofluids*, *9*(4), 321-335. doi:10.1166/jon.2020.1754. **(Indexed by ISI and SCOPUS)**. *Problem 2.*

2. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie and Ilyas Khan. (2021). Unsteady free convection flow of water-based carbon nanotubes due to non-coaxial rotations of moving disk. *Journal of Applied Science and Engineering*, 25*(3)*, 401-410. http://dx.doi.org/10.6180/10.6180/jase.202206_25(3).0005. **(Indexed by ISI and SCOPUS)**.

3. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie and Lim Yeou Jiann. (2021). Radiative viscous fluid flow under non-coaxial rotation of accelerated disk with MHD and porosity effects. Sains Malaysiana. **(Q4, IF: 1.009)**. The full paper is **accepted.**

4. Ahmad Qushairi Mohamad, **Wan Nura'in Nabilah Noranuar**, Sharidan Shafie, Abdul Rahman Mohd Kasim, Mohd Rijal Ilias, Lim Yeou Jiann and

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Non-indexed Journal

1. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie and Lim Yeou Jiann. (2021). Heat and mass transfer in non-coaxial rotation of radiative MHD Casson carbon nanofluid flow past a porous medium. *Data Analytics and Applied Mathematics*. *2*(2), 37-51. https://doi.org/10.15282/daam.v2i2.6832. *Problem 4.*

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1. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie and Ilyas Khan. (2021). Accelerated Non-Coaxial Rotating Flow of MHD Viscous Fluid with Heat and Mass Transfer. In *IOP Conference Series: Materials Science and Engineering*. (pp. 012044). IOP Publishing. doi:10.1088/1757-899X/1051/1/012044

Book Chapter

1. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Ilyas Khan, Sharidan Shafie, Mohd Rijal Ilias and Lim Yeou Jiann. (2021). Analysis of heat transfer in Newtonian carbon nanofluid flow due to non-coaxial rotation with MHD and porosity effects. In: IntechOpen. Nanostructured Materials – Physics, Chemistry, Classification, and Emerging Application in Industry, Biomedical, and Agriculture. https://doi.org/10.5772/intechopen.100623. ***Problem 1.***

2. **Wan Nura'in Nabilah Noranuar**, (2021). Heat transfer analysis on accelerated flow of MHD Casson nanofluid in porous medium. The full paper is **submitted**.

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1. **Wan Nura'in Nabilah Noranuar,** Ahmad Qushairi Mohamad and Sharidan Shafie. 2020. *Thermal Radiation Effect on Non-Coaxial Rotating Flow of Viscous Fluid over an Accelerated Disk*. 8th Internantional Graduate Conference on Engineering, Science and Humanities **(IGCESH 2020)**, organized by UTM Postgraduate Student Society (PGSS-UTM) in collaboration with School of Graduate Studies UTM, held on 18th – 19th August 2020 at Universiti Teknologi Malaysia (UTM).

2. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Sharidan Shafie and Ilyas Khan. *Accelerated Non-Coaxial Rotating Flow of MHD Viscous Fluid with Heat and Mass Transfer*. 6th Malaysia-Japan Joint International Conference 2020 **(MJJIC 2020)** in conjunction with 5th International Conference on Advanced Technology and Applied Sciences **(ICaTAS 2020)**, organized by Malaysia-Japan International Institute of Technology (MJIIT), held on 7th – 9th October 2020 at Kuala Lumpur, Malaysia.

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Competition

1. **Wan Nura'in Nabilah Noranuar**, Ahmad Qushairi Mohamad, Ilyas, Khan and Sharidan Shafie, Free Convection Flow of Carbon Nanotubes due to The Non-Coaxial Rotation. **Poster Pitching Competition. Faculty of Science, Research Month 2020**. Universiti Teknologi Malaysia. 10th - 25th August 2020.

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