MIXED CONVECTION FLOW OF A TWO-PHASE DUSTY FLUID AND DUSTY JEFFREY FLUID DUE TO AN EXPONENTIALLY STRETCHING SHEET

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DEDICATION

Specially dedicated to all, who have stuck with me through this whole epic roller coaster of feels;

Dear abah and ma
I hope this achievement will complete the dream that you had for me all those many years ago when you chose to give me the best education you could.

Dear siblings
Kaklong, Khairi, Aminin, Elly, Anik, Adik za
thank you for everything-

Dear husband Ku Aznan
and our children Ku Afwan and Ku Afeeya
i am truly thankful for having you in my life-

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ABSTRACT

Multiphase flows (either two or more phases) are signified as the flows of different matter types. Physically, they represent the carrier phase consisting of the particulate phase of any random chemical component. The carrier phase is either gas, liquid, or solid. The solid particle is assumed to be in the form of ash, soot, and dust. Interest in studying fluid behaviour free from all impurities (clear fluid) has increased substantially over the past decades due to many engineering applications. It is well known that pure fluid is rarely available in numerous practical situations. In this thesis, a study had been conducted to investigate the influence of magnetic fields on mixed convection flow with heat and mass transfer by considering Newtonian and non-Newtonian fluid in the presence of dust particles. A mathematical model of two-phase flow was developed based on Saffman's model. Specifically, the primary governing equation for the two-phase flow model considered the distribution of both fluids (as a carrier phase) and solid (dust) particles. The two-phase flow model was governed by modifying the single-phase model. The total fluid-particle interaction force was also considered into the momentum equation. Correspondingly, some physical assumptions for dust particles were made. The governing equations were converted to non-dimensional form and solved numerically via the Keller-box method, programmed in FORTRAN software. The effects of parameters, namely Prandtl number, fluid-particle interaction for velocity, fluid-particle interaction for temperature, fluid-particle interaction for concentration, mixed convection, magnetic field, Deborah number, a ratio of relaxation to retardation times, chemical reaction and Schmidt number on velocity, temperature, and concentration profiles had been analyzed based on their graphical behaviours. Also, the expressions of Sherwood number, Nusselt number, and skin friction had been evaluated and displayed in tabular forms. Numerical results showed that the dust particles in both types of fluids offer more resistance to the flow. The fluid motion was also influenced by the mixed convection parameter. Comparative results between the two kinds of fluids demonstrated that the velocity profiles are higher than those without mass transfer. Meanwhile, the dusty Jeffrey fluid velocity is higher than the velocity of the dusty Newtonian fluid. The presence of chemical reaction in the dusty Jeffrey fluid generate more physical properties than the dusty Newtonian fluid, which affected the fluid flow concentration.

ABSTRAK

Aliran multifasa (sama ada dua atau lebih fasa) dikenali sebagai aliran dari jenis bahan yang berbeza. Secara fizikal, ianya mewakili fasa pembawa yang terdiri daripada fasa partikulat dari sebarang komponen kimia rawak. Fasa pembawa terdiri daripada gas, cecair, atau pepejal. Zarah pepejal dianggap berada dalam bentuk abu, jelaga, dan debu. Minat dalam mengkaji tingkah laku bendalir bebas dari segala kekotoran (bendalir jernih) telah meningkat dengan ketara sejak beberapa dekad yang lalu kerana kewujudan pelbagai aplikasi dalam kejuruteraan. Diketahui bahawa bendalir tulen jarang ditemui dalam kebanyakan situasi praktikal. Dalam tesis ini, kajian telah dijalankan untuk menyelidiki pengaruh medan magnet pada aliran perolakan campuran dengan pemindahan haba dan jisim dengan mempertimbangkan bendalir Newtonan dan bukan Newtonan dengan kehadiran zarah debu. Model matematik aliran dua fasa dibangunkan berdasarkan model Saffman. Khususnya, persamaan menakluk utama untuk model aliran dua fasa mempertimbangkan pengedaran kedua-dua bendalir (sebagai fasa pembawa) dan zarah pepejal (debu). Model aliran dua fasa diatur dengan mengubah model fasa tunggal. Kekuatan interaksi bendalir-zarah juga dipertimbangkan dalam persamaan momentum. Sejajar dengan itu, beberapa andaian fizikal untuk zarah debu dilakukan. Persamaan menakluk diubah menjadi persamaan tak bermatra dan diselesaikan secara berangka melalui kaedah Keller-box, yang diprogramkan dalam perisian FORTRAN. Kesan parameter, iaitu nombor Prandtl, interaksi bendalir-zarah bagi halaju, suhu dan kepekatan, perolakan campuran, medan magnet, nombor Deborah, nisbah kelonggaran dengan masa penundaan, tindak balas kimia dan nombor Schmidt pada halaju, suhu, dan profil kepekatan telah dianalisis berdasarkan tingkah laku grafik parameter tersebut. Juga, ungkapan nombor Sherwood, nombor Nusselt, dan geseran kulit telah dinilai dan dipaparkan dalam bentuk jadual. Hasil berangka menunjukkan bahawa zarah debu di kedua-dua jenis bendalir mempunyai daya tahan yang lebih tinggi terhadap aliran. Aliran bendalir juga dipengaruhi oleh parameter perolakan campuran. Hasil perbandingan antara kedua-dua jenis bendalir menunjukkan bahawa profil halaju lebih tinggi daripada bendalir tanpa pemindahan jisim. Sementara itu, halaju bendalir Jeffrey yang berdebu lebih tinggi daripada halaju bendalir Newtonan yang berdebu. Kehadiran tindak balas kimia dalam bendalir Jeffrey yang berdebu menghasilkan lebih banyak sifat fizikal daripada bendalir Newtonan yang berdebu, yang menjejaskan kepekatan aliran bendalir.

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LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamics
CFD	-	Computational Fluid Dynamic
PDE	-	Partial Differential Equation
ODE	-	Ordinary Differential Equation
RKF45	-	Runge-Kutta-Fehlberg Techniques
HAM	-	Homotopy Analysis Method
PEHF	-	Prescribed Exponential Order Heat Flux
PEST	-	Prescribed Exponential Order Surface Temperature
PHF	-	Prescribed Power Law Heat Flux
PST	-	Prescribed Power Law Surface Temperature

LIST OF SYMBOLS

$\mathbf{a}$	-	Acceleration
$\mathbf{b}_1$	-	Induced magnetic field
$\mathbf{B}$	-	Total magnetic field
$\mathbf{B}_0$	-	Applied magnetic field
c_f	-	Specific heat of fluid
c_p	-	Specific heat of dust particle
C	-	Concentration of the fluid
C_w	-	Concentration of the fluid at wall
C_f	-	Skin friction coefficient
C_∞	-	Fluid concentration in free stream
C_p	-	Dust particle concentration
D_m	-	Mass diffusivity
d/dt	-	Material time derivative
De	-	Deborah number
$\mathbf{E}$	-	Electric field
Ec	-	Eckert number
Gm	-	Mass Grashof number
Gr	-	Thermal Grashof number
f_x	-	x-component of body force
f_y	-	y-component of body force
f_z	-	z-component of body force
$\mathbf{F}$	-	Net force
$\mathbf{F_D}$	-	Total fluid-particle interaction force
F_{D_x}	-	Total fluid-particle interaction force along x-direction
F_{D_y}	-	Total fluid-particle interaction force along y-direction
$\mathbf{F_s}$	-	Body force

F_{s_x}	-	Body force along x-direction
F_{s_y}	-	Body force along y-direction
$\mathbf{g}$	-	Gravitational acceleration vector
G	-	Mass concentration of dust particles
Gm	-	Mass Grashof number
Gr	-	Thermal Grashof number
H	-	Buoyancy ratio
i	-	Imaginary number
$\mathbf{i}$	-	Unit vector in x-direction
$\mathbf{I}$	-	Identity vector
$\mathbf{j}$	-	Unit vector in y-direction
$\mathbf{J}$	-	Current density
$\mathbf{J} \times \mathbf{B}$	-	Lorentz Force
K	-	Constant drag coefficient of spherical dust particle
K_1	-	Chemical reaction coefficient
Kr	-	Chemical reaction parameter (non-dimensional)
L	-	Reference length
m	-	Mass
M	-	Magnetic parameter
N	-	The total density of dust particle per unit volume
Nu_x	-	Nusselt number
p	-	Pressure
p_h	-	Hydrostatic pressure
Pr	-	Prandtl number
Q_D	-	Total thermal interaction between fluid and dust particle
q_w	-	Wall heat flux
q_s	-	Mass wall flux
r_p	-	Radius of dust particle
Re	-	Local Reynolds number

$\mathbf{R}_1$	-	First Rivlin-Ericksen tensor
$\mathbf{S}$	-	Extra stress tensor
Sc	-	Schimdt number
Sh_x	-	Sherwood number
t	-	Matrix transpose
T	-	Fluid temperature
T_p	-	Dust particle temperature
T_w	-	Sheet temperature
T_∞	-	Ambient fluid temperature
T_0	-	Reference temperature
u	-	Velocity component along x-direction
u_p	-	Dust particle velocity component along x-direction
v	-	Velocity component along y-direction
v_p	-	Dust particle velocity component along y-direction
$\mathbf{V}$	-	Velocity vector of flow field
$\mathbf{V}_p$	-	Velocity vector for dust particle
W_p	-	Mass diffusion per unit volume
x	-	Coordinate axis of the plate
y	-	Coordinate axis normal to the plate
σ	-	Electric conductivity of fluid
$\boldsymbol{\tau}$	-	Cauchy stress tensor
τ_w	-	Local wall shear stress
τ_v	-	Relaxation time of dust particle
τ_T	-	Thermal equilibrium time
τ_c	-	Concentration equilibrium time
$\tau_{xx}, \tau_{xy}, \tau_{yx}, \tau_{yy}$	-	Component of Cauchy stress tensor
ρ	-	Density of the fluid
ρ_p	-	Density of dust particle
ρ_∞	-	Ambient fluid density

β^*	-	Thermal expansion coefficient
β	-	Fluid concentration expansion coefficient
β_v	-	Local fluid-particle interaction for velocity
β_T	-	Local fluid-particle interaction for temperature
β_c	-	Local fluid-particle interaction for concentration
δ	-	Boundary layer thickness
κ	-	Fluid thermal conductivity
Ψ	-	Stream function
η	-	Similarity variable
γ	-	Specific heat ratio
μ	-	Dynamic viscosity
λ	-	Mixed convection parameter
λ_1	-	Ratio of relaxation to retardation times
λ_2	-	Retardation times
$\Delta\eta$	-	Step size
θ	-	Fluid temperature (non-dimensional)
θ_p	-	Dust particle temperature (non-dimensional)
ϕ	-	Fluid concentration (non-dimensional)
ϕ_p	-	Dust particle concentration (non-dimensional)
Δ	-	Del operator

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CHAPTER 1

INTRODUCTION

1.1 Introduction

This section discusses the key study areas for Newtonian fluids and non-Newtonian fluids, particularly on two-phase dusty fluid and dusty Jeffrey fluid, along with some basic and significant physical effects of boundary layer flows. It introduces the research background, statement of the problem, research objectives, scope of research and the study's significance.

1.2 Research Background

Fluid mechanics is the field of science that focuses on the behaviour of fluid. Gases and liquid are treated as fluid when its related to the fluid mechanics field. Fluid mechanics are generally subdivided into three parts, which consist of:

i Fluid static: the study of fluids at rest.

ii Fluid kinematic: the study of fluids in motion that disregard the pressure forces.

iii Fluid dynamics: the fluid's research in operation in view of the pressure force.

Fluid mechanics is fundamental and so essential in daily life. The study of blood moving through the vessels in human bodies and the motion of the air that goes into the lungs are simple examples of fluid mechanics ideas. This proves that life is surrounded by fluid flows. Specifically, a fluid, for instance, water, oil or air, is described as a substance that will continuously deform (flow) as long as there is shearing stress, $\tau = F/A$, applied on it of any size. Accordingly, to solve the problems in fluid mechanics in terms of physic and the mathematics of the fluid, it must satisfy:

i Conservation of Mass (the continuity equation);

ii Newton's second law of motion (Momentum principle);

iii First and Second Law of Thermodynamics (Energy equation).

In addition, the fluid density, $\rho(\text{rho}) = m(\text{mass})/v(\text{volume})$ and the fluid specific weight, $\gamma(\text{gamma}) = \rho g$ are the fluid properties use to measure the mass and weight of a fluid, respectively. However, water and oil are the two different fluids that have grossly different characteristics. Hence, fluid viscosity is listed as another property to describe the 'fluidity' of the fluid. Fluid can be classified into two categories, notably Newtonian and non-Newtonian fluids. Fluids that satisfied Newton's law of viscosity and follows the behaviour explained by this law are called Newtonian fluid. Newton's law of viscosity defined that the shear stress in a flowing flow is directly proportional to the shear strain rate. Mathematically, Newton's law of viscosity is expressed as

$$\tau = \mu \frac{du}{dy}, \tag{1.1}$$

where μ is the constant of proportionality and is called the dynamic viscosity. Additionally, the shear strain rate, du/dy should be linear to the dynamic viscosity. Most commonly, liquids and gases are examples of Newtonian fluid. The other type of fluid is the non-Newtonian fluid, which is fluids that do not follow Newton's law of viscosity. The shearing stress for this type of fluid is not linear to the shear strain rate. Non-Newtonian fluids are classified based on how their apparent viscosity varies with shear rate. Verily, these non-Newtonian fluid differential systems are substantially more complicated than Newtonian fluid due to the higher order of non-linear governing equations. In particular, blood is one of the non-Newtonian fluid that is crucial for human life. The blood suspended in the plasma, consisting of red blood cells,has a slightly higher density than the water. However, its viscosity is more elevated than water at a constant temperature.

Aside from that, different non-Newtonian fluid models such as Casson fluid, Maxwell fluid, and viscoelastic fluid have been acknowledged by previous researchers. All of these fluids typically have their uniqueness of fluid flow properties. Therefore, no single constitutive equations developed in the literature akin to the Newtonian fluid model have the simplest rheological fluid model governed by the Navier-Stokes equation. Due to that, a few constitutive equations have been proposed to clarify the dynamic flow of non-Newtonian fluids with different rheological effects. Hence, analyses on non-Newtonian fluid in various aspects from analytical and numerical methods have significantly increased, majorly due to the rapid development in many industrial applications, especially in polymer and food industries. Some foodstuffs utilise fluids that exhibit the non-Newtonian behaviour, which promotes the flow characteristic of non-Newtonian significantly.

The term viscoelastic indicate a non-Newtonian fluid that have both properties; viscous and an elastic material. Here, the resistance of fluid flow to changing form is referred to as viscosity, where the factor subjected to stress is due to the strain rate. Meanwhile, the factor stress for the term elasticity is because of strain field. Elasticity refers to the ability of a solid material to strain first after being stretched at a certain limit. Then, the material will resume to its earliest shape once the stress is discarded. However, when pressure is applied to a liquid, it deforms continuously. The Jeffrey fluid is one of the relatively most specific types of viscoelastic fluid, which displays both characteristics of relaxation and retardation times (Turkyilmazoglu and Pop, 2013) has been chosen to represent the non-Newtonian fluid flow. This model is capable of describing the stress properties of non-Newtonian fluid, which the usual viscous fluid model cannot describe, where it was found that Jeffrey fluid model can describe well their properties such as relaxation and retardation time.

Besides, magnetohydrodynamics (MHD) effect is another physical phenomenon that influences the fluid behaviour. MHD is generally an interaction between a fluid flow in electrically conducting fluids and magnetic properties that affects the fluid flow. Plasmas, salt water, electrolytes and liquid metals are among the examples. MHD effect occurs when a conductor and a magnetic field move relative to each other, where an electric current is induced in the conductor. The polarisation occurs in fluid when an electric field is distorted, which consequently affects the magnetic field. Interest in the MHD flow arose in 1918 when Hartmann created the electromagnetic pump. Later, in 1942, Hannes Alfven was the first to initiate the term MHD. Without hard work on MHD effect, there is no success for Hannes Alfven, who won the Nobel prize in 1970.

According to Mohyud-Din *et al.* (2018), when a magnetic field is applied, the boundary layer of separation in various physical phenomena can be controlled. Furthermore, magnetic fields effects significantly affects the temperature profiles (Yasmin *et al.*, 2020). Due to this circumstance, MHD has become a primary concern in many studies.

Heat transfer is defined as an exchange of thermal energy within an object due to differences in temperature. The temperature difference act as a driving force that causes the heat to flow, which is created by the movement of particles. Boiling water or baking a cake in an oven are common daily life examples, known as thermal transfer. Heat transfer can be completed in three processes, specifically convection, radiation, and conduction. Conduction heat transfer refers to a thermal energy transfer from more energetic to less energetic particles due to their interaction (molecules collide in random motion). The conduction generally takes place in solids and liquids. As a conductor, metal is one example of a substance that can transfer thermal energy effectively. Meanwhile, wood or paper acts as insulators where the implications incapably transfer the heat.

Furthermore, heat transfer by radiation happens in a vacuum of space from one place to another by electromagnetic waves. The transfer of energy by radiation is presentable in any medium and solids, liquids and gases. Hence, it was considered different from conduction and convection. Besides, radiation does not require any form of matter to be transferred. Some simple examples to describe the radiation emitted or absorbed are the heat that warms the Earth by the Sun and the heat that warms a human body due to the light of fire. Makinde (2005) and Siddiqa *et al.* (2012) concluded that the presence of thermal radiation parameters in their studied increase along with the heat transfer rate. Therefore, thermal radiation is applied in a variety of engineering fields and industrial settings.

Next, heat transfer by convection occurs when there is a fluid flowing over a surface due to the heat movement (warm matter). Convection is a natural phenomenon that happens when warmer air or liquids rest underneath a cooler layer. This occurs during boiling water process, where heat from the burner is transferred to the pot, heating the water at the bottom. The heated water underneath rises, where colder water replaces it, resulting in a circular motion.

Additionally, there are three forms of mechanism convection: free, forced, and mixed convection. Free convection is a natural phenomenon that happened naturally and not by any external sources. The fluid motion is generated only by buoyant force, in which the fluid density difference induces it due to the temperature gradients. Free convection is used to warm up a cold drink in a warmer environment and chill a boiled egg in a colder climate. In forced convection, the flow occurs when fluid is forced to move because of moving devices such as a pump or any suction device. For example, a fan that acts as an external forces (device) creates the air movement, and consequently, the fan's spinning causes the air to flows. The most common application for forced convection is to enhance the heat exchange rate.

Meanwhile, mixed convection occurs when convection free and forced flows concurrently, leading to heat transfer. These drive the mixed convection flow to buoyancy force and externally generated flows. A basic example in an electrical fields is when a fan is needed to provide component free convection. Another example is the diesel generator used to cool the nuclear reactor during shutdown of emergency power facilities, called mixed convection.

Although much research work has been done on heat transfer, the mass transfer phenomenon also acquired attention due to its importance in our daily life and industry. Mass transfer is defined as an exchange of thermal energy within a mass movement from one particular place to another owing to the concentration difference. Mass can be transferred in many ways, such as absorption, adsorption, drying or distillation. Mass transfer occurs between two phases through an interface where it is assumed that changes in concentration exist near the boundary. For instance, all gases and liquids are appropriately mixed during gas and liquid absorption processes except within the gas and liquid phases (gas-liquid interface). One of the most prevalent industrial uses is in petrochemical refining. One type of mass transfer process is fractional distillation, which separates crude oil components. Almost everystep in the refining system relies on mass transfer.

In particular, several scholars investigated different cases when fluid flow is affected by the chemical reaction on different geometries. When the chemical reaction occurs, the fluid flow encounters two types of reaction processes. The first is endothermic, where the energy is absorbed, while the other one is exothermic, where energy is released in the form of heat (Ibrahim *et al.*, 2008). As a result, the fluid flows are significantly affected. The existence of contaminants between fresh water and the air is acknowledged. Due to this, in some cases, chemical reaction can be categorised as heterogeneous and homogeneous processes. Both mixture processes usually occur either at an interface or a single-phase volume reaction (Muhaimin *et al.*, 2009). The heat and mass transfer's research containing chemical reactions is beneficial in the hydrometallurgical and chemical industry.

Consequently, a chemical reaction in a dusty Newtonian or dusty non-Newtonian fluids model is needed due to its potential, which can estimate the performance of the reactor. Therefore, the understanding of effects of convective heat and mass transfer in the flow is vital for a wide range of situations. All the various processes from which raw material is converted to final product must undergo heat and mass transfer in one way.

Overall, a multiphase flow includes a solid (dust) and fluid phase on a variety of chemical elements. Dusty fluid flows are defined as fluid flow with particulate suspension, including the suspended matter such as liquid droplets, gas bubbles, solid particles, or a mixture of these. In addition, it is also labelled as two phase flows since they required a composite of two materials or two phases with distinct differences. Particularly, one phase is the fluid medium, which is a continuous phase, while the particulate suspensions is the other phase dispersed throughout the fluid medium. In nature, fluid flows containing suspended material particles abound, where typical examples are water contamination and air pollution. The atmosphere of the Earth has a mostly gaseous envelope of air that surrounds the planet. It also contains liquid droplets and solid particles. Furthermore, it is continually contaminated by a variety of dust particles, such as hazardous substances, which are unavoidable implications and natural by-products of highly evolving industrialisation.

Sewage solids are being thrown into oceans and rivers, which leads to the contamination of the natural waterways (sea and rivers) around us. Many waste materials are released into flowing waterways as a results of industrial activities including food processing, chemical manufacturing, and pulp and paper manufacture. Therefore, understanding the behaviour of two-phase flows is required to reduce and control these massive water and air pollution issues. Similarly, two-phase flows are a barrier in a variety of industrial and engineering applications. Fluids containing dust particles are utilised in gas cooling chambers to improve heat transfer processes in heat transfer technology.

Furthermore, in the petroleum sector and crude oil purification, understanding two-phase flows are also essential. Due to these facts, dusty fluid flow, exclusively related to Saffman's model, has increased the study of modelling, solving, and analysing the particles suspension in the fluid flow amongst many researchers. Although extensive research works have been devoted to heat and mass transfer in dusty Newtonian fluids, recently, research in dusty non-Newtonian fluid flow in various aspects from both analytical and numerical methods has increased dramatically among researchers as well. Theoretical study on these fluids is more exciting and attention-grabbing due to the complexity of their constitutive equations.

The study on free, forced and mixed convection under the influence of different physical conditions past a continuous stretching sheet surface has gained much attention due to its numerous industrial application. Examples of such industrial manufacturing processes are the purification of crude oil, the glass-fiber or paper production and the plastic sheet extraction. Wide varieties of this problem deal with heat and mass transfer involving boundary layer concept. The study found that during some stretching sheet problems, the stretching motion and the simultaneous heating/cooling are two main factors that affected the quality of the final products due to the direct impact on heat transfer rate. Besides, the boundary layer is usually defined as a thin layer of fluid near the neighbourhood of the solid boundary when a real fluids flows past a body. The idea of the boundary layer concept over a moving continuous solid surface (sheet issuing) was very well documented by Sakiadis (1961). The author derived the approximated and exact solutions for the laminar velocity field. Motivated by Sakiadis's work, the analytical and experimental approach of the heat transfer characteristics effects on a continuous moving surface was analysed Tsou *et al.* (1967). Works on finding the solution to linear stretching sheet problems have also been done by Crane (1970).

However, there were also investigations on the boundary layer flow employing a non-standard stretching, such as exponentially, quadratically, and nonlinearly stretching sheet. A paper by Gupta and Gupta (1977) on the mass and heat transfer on stretching sheet surface may be referred. The authors pointed out that this geometry may not always be linear. This assumption has been agreed by Kumaran and Ramanaiah (1996). In this paper, the author considered quadratics stretching sheet for solving the boundary layer problem. Magyari and Keller (1999) presented a similar solution using analytical and numerical heat and mass transfer of a flow over an exponentially stretching sheet. Also, Sajid and Hayat (2008) solved the problem of viscous fluid analytically in boundary layer flow and heat transfer by considering radiation effect. The research of viscous fluid flow over an exponentially stretching sheet has been extended by Ishak *et al.* (2006) and Bidin and Nazar (2009).

The study of fluid and how forces influence them is called fluid dynamics. As an instance, fluid dynamics is applied to understand how to fly an aeroplane. For that reason, partial differential equations (PDEs), which govern the flow, are often modelled to describe the physical phenomenon of fluid dynamics. Once the mathematical model has been formulated, including the continuity, momentum, energy and concentration equations of any heat and mass transfer problems, the analytical or numerical methods will be used in finding the solutions. In this situation, the analytical method yields the exact solution, while the numerical method generates the approximate solution. One drawback to this analytical method is that not all mathematical equations can be implemented. However, this method is significant for comparison with the numerical method, which gives efficient and excellent results of any problems. Thus, as one of the numerical methods, Keller-box, which was invented by Keller and Cebeci (1971), is a suitable method to be applied to all the issues proposed in this thesis. Motivated by Keller's work, the idea of this method very well-exposed by Cebeci and Bradshaw (1984), which became a general approach for solving boundary layer equations. The Keller-box method consists of a four-step approach. The first step reduces the non-linear governing equations into first order differential equations system, domain discretization step by writing equations in the finite-difference form using centred-difference derivatives, express the equations in the matrix-vector form by linearising the resulting non-linear system using Newton's method and lastly solve the linear system by the block tri-diagonal elimination technique (Zokri *et al.*, 2018); (Sarif *et al.*, 2013).

Mainly, mathematics can simulate any physical phenomenon. However, not all the problems in fluid dynamics can be solved by direct calculation. Under these circumstances, numerical approaches based on computer simulations must be solve the mathematical equations that govern these processes, called computational fluid dynamics (CFD). The computational codes (algorithms) can be developed in various languages: FORTRAN, C++, MATLAB, Python, and many more. Additionally, CFD is a computer-based science that predicts the fluid flows as well as it will figures how the fluids could affects the surface of the object when passed through it by running the algorithms or codes.

In light of the previous paragraphs, this study had been conducted to study the influence of suspended dust particles flow behaviour on Newtonian and non-Newtonian fluid, where the fluid flow is considered on an exponentially stretching sheet. The Jeffrey fluid model was chosen over others to represent the non-Newtonian fluid. Heat and mass transfer were included in the study considering magnetic and chemical reaction effects on mixed convection flow. The governing partial differential equations have been reduced into non-dimensional governing equations by employing non-dimensional variables via a similarity transformation. Consequently, the non-dimensional governing equations are then solved numerically by adopting the implicit finite-difference method, called Keller-box method. To obtain the numerical outcomes, the numerical algorithms from Keller-box method are programmed in FORTRAN 77 software, continued with the aid of MATLAB to plot the graphs. In this study, FORTRAN programming has been proposed as a tool for obtaining the numerical results. This tool is a handy and pleasant language that can handle large matrices for two-phase fluid flow while doing the Keller-box method.

1.3 Problem Statement

The interest in the two-phase flow problems has increased substantially over the past decade due to the circumstance of these fluids in many industrial and technological applications. This happened because the investigation of single-phase fluid flow are not relevant enough to describe the nature of fluids that exist in daily life, where the single-phase flow model cannot solve the fluids problems with the suspended dust particles. Due to this reasons, the two-phase flow model is introduced so that both properties of the fluids and dust particles are reckoned. The environmental pollution, crude oils purification and polymer manufacturing processes are the examples corresponding to those the two-phase flow problems. Therefore, this thesis will consider a Newtonian and non-Newtonian fluids type to represent the fluid phase.

In contrast, the spherical solid particles dispersed in the fluid flow is treated as dust phase. The Jeffrey fluid model is the simplest model, which is considered to be used with fluids of non-Newtonian type. Owing to this, the two-phase flow model of Newtonian and non-Newtonian fluids, together with dust particles, is formulated. Therefore, this research investigates the presence of suspended dust particles in affecting both Newtonian and non-Newtonian fluids behaviour. Specifically, a two-phase flow of dusty Newtonian and dusty Jeffrey fluid with the effects of convective heat and mass transfer is considered. The problem, then, is to analyse the behaviour of suspended dust particles on the flow motion concerning the velocity profiles, the temperature profiles, the concentration profiles, as well as the Nusselt and Sherwood numbers and the skin friction coefficient.

Moreover, the study on these fluid types past an exponentially stretching sheet with some significant effects such as MHD and chemical reaction is rarely studied. Yet, they are highly motivated to be learned. Recent research acknowledges that the heat and mass transfer rate possesses a significant impact on the final product quality. Thus, this study has been carried out to explore the following research questions:

(i) How are the mathematical models of dusty Newtonian and dusty Jeffrey fluids past an exponentially stretching sheet formulated?

(ii) How would the skin friction, Nusselt and Sherwood numbers, and the fluid flow characteristics be affected due to the presence of suspended dust particles, together with the physical parameters for dusty Newtonian fluid and dusty Jeffrey fluid model?

(iii) How do the Keller-box method for convective heat and mass transfer of dusty Newtonian and dusty Jeffrey fluids be solved?

(iv) How to develop the numerical algorithms in FORTRAN software for computation of dusty Newtonian and dusty Jeffrey fluids?

(v) How are the dusty Newtonian and dusty Jeffrey fluid models compared with the existing single-phase of Newtonian and Jeffrey fluid models in the convective heat and mass transfer problems?

1.4 Research Objectives

This research numerically examines the mixed convection flow of two-phase dusty Newtonian fluid and dusty Jeffrey fluid past an exponentially stretching sheet. This investigation mainly reports the flow behaviour of dust particles suspended in the fluid flows. Specifically, the objectives of this research are:

(i) to derive the mathematical models including the continuity, momentum, energy and mass equations of two-phase flow and construct the numerical algorithms for computation;

(ii) to obtain the numerical results of the velocity, temperature and concentration profiles, and compute the variation of skin friction, Nusselt and Sherwood numbers, and

(iii) to analyse the heat and mass transfer characteristics influenced by magnetic field, mixed convection, mass concentration of dust particles parameter, local fluid-particle interaction parameter for velocity/temperature/concentration, Deborah number, and Jeffrey fluid parameter for both fluid and dust phase.

1.5 Scope of Research

This study focuses on the boundary layer analysis of two-phase dusty Newtonian and dusty non-Newtonian fluids. The flow is established over an exponentially stretching sheet. The study related to the two-phase flow (fluid-particle) assumes the common fluid to represent the Newtonian fluid model in nature. In contrast, Jeffrey fluid has been selected to represent the non-Newtonian fluid model. Thus, in this study, two types of fluid containing suspended dust particles, namely dusty Newtonian and dusty Jeffrey fluids, are considered. According to Singleton (1964), solid sphere particles have been treated as a continuum since the moving particles do no interact with the fluid flows.

As a consequence, there will be no collisions between particles. Specifically, this research concerns the presence of dust particles in affecting the behaviour of Newtonian and Jeffrey fluid models. The following four main problems are addressed as follows:

(i) Chapter 4: the influence of magnetic parameter on mixed convection flow of dusty Newtonian past an exponentially stretching sheet induced by heat transfer,

(ii) Chapter 5: the influence of magnetic and chemical reaction parameters on mixed convection flow of dusty Newtonian past an exponentially stretching sheet induced by the combined effects of heat and mass transfer,

(iii) Chapter 6: the influence of magnetic parameter on mixed convection flow of dusty Jeffrey fluid past an exponentially stretching sheet induced by heat transfer, and

(iv) Chapter 7: the influence of magnetic and chemical reaction parameters on mixed convection flow of dusty Jeffrey fluid past an exponentially stretching sheet induced by the combined effects of heat and mass transfer.

The effects of Prandtl number, fluid particle interaction for velocity, fluid-particle interaction for temperature, fluid-particle interaction for concentration, mixed convection parameter, Deborah number, a ratio of relaxation to retardation times, and Schmidt number on velocity, temperature, and concentration profiles are evaluated based on their graphical behaviours in all four problems. Consequently, the equations that govern under the Newtonian and Jeffrey model, together with dust particles, are transformed to a system of non-linear ordinary differential equations by applying the appropriate similarity transformation. Also, Boussinesq and boundary layer approximations have been considered to simplify the governing equations. The Keller-Box method is employed to achieve the numerical solutions. The numerical results are achieved via the FORTRAN 77 algorithm, and the acquired data will then be presented visually via MATLAB software. In a nutshell, Figure 1.1 depicts the research framework for this study.

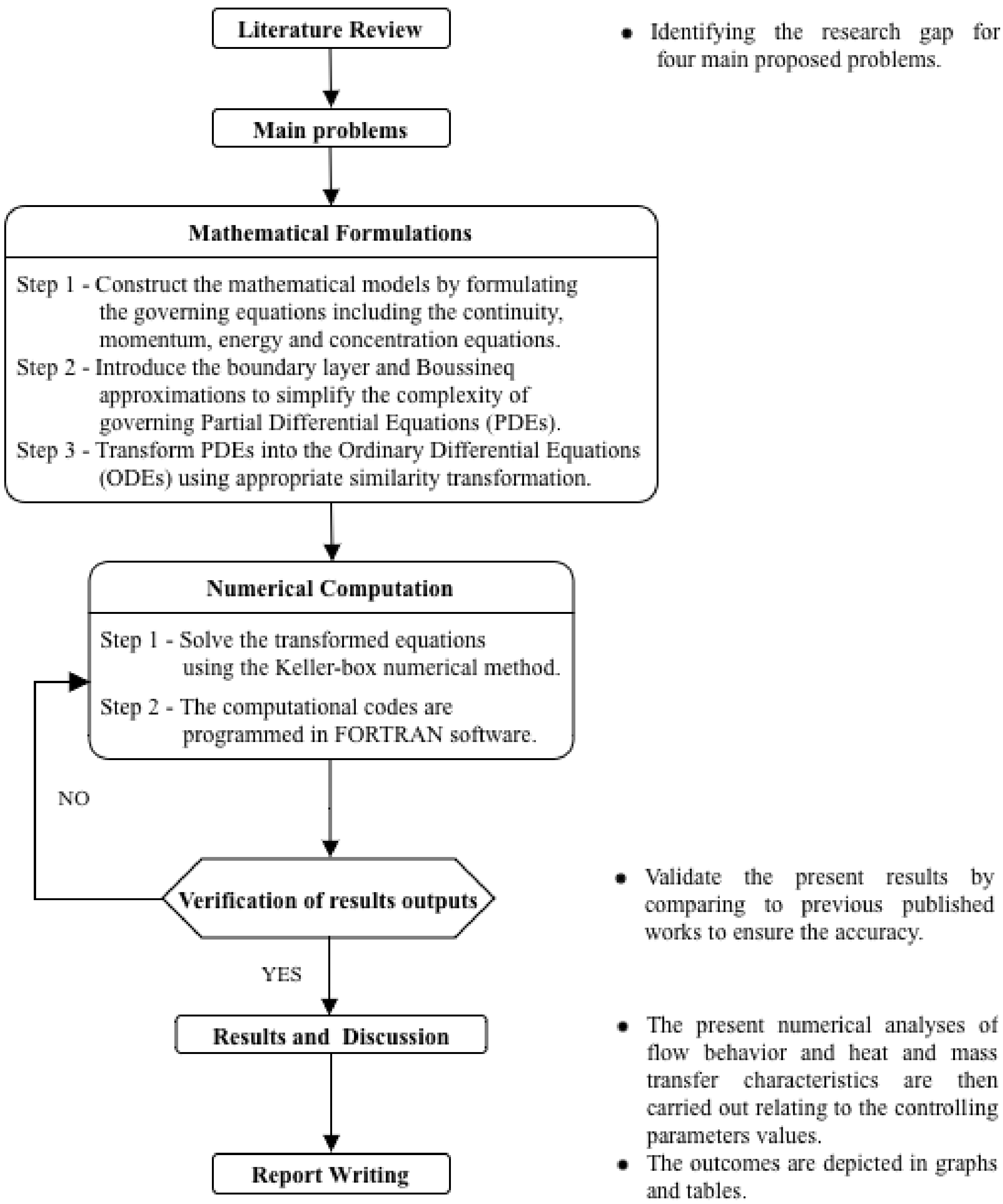

Figure 1.1 Research Framework

1.6 Significance of Research

The significance of the study is stated as follows

(i) The development of mathematical models for two-phase flow has been introduced, where the interaction between dust particles and fluid medium is analysed carefully. A modification on an existing single-phase flow model into a two-phase flow model (consider both Newtonian and non-Newtonian fluids model) has been constructed successfully. This contributes to the new mathematical knowledge of the two-phase flow theory and, consequently, could help future studies.

(ii) This study intends to have a deeper understanding of the rheological flow behaviour of suspended dust particles in the fluid flows. It aims to understand the emerging role of dust particles in varying the behaviour of Newtonian and non-Newtonian fluids. This study can be used as a good reference for other research and publications as well as the industrial applications.

(iii) The knowledge of heat and mass transfer on convective boundary layer flow of dusty Newtonian and dusty non-Newtonian fluids problems is important due to its basic phenomena in our everyday life and engineering applications. It is needed to determine how the heat and mass transfer has a significant effect when dealing with convective fluid flow. Learning the heat and mass transfer would explain why some processes like crystallisation and distillation are performed at specific density and temperature for a positive outcome. Thus, by considering the following knowledge, it may provide some insight on how to enhance the flow rates, which could improve the flow as well as cut the operation cost.

(iv) One of the numerical methods of the Keller-box method is implemented to solve and analyse the two-phase fluid flow problems. Then, the numerical computation is developed in the FORTRAN software to obtain the numerical results. All of the proposed problems are solved numerically with the help of computer simulations. The achievable results of this present study can provide a better understanding of basic engineering knowledge and the development of computational tools. Moreover, the mathematical modelling of dusty

Newtonian and dusty non-Newtonian gives an excellent knowledge to the actual global application controlling the flow characteristics.

1.7 Thesis Outline

This thesis comprises eight chapters. Chapter 1 introduces the research background, statement of the problem, objectives of the research, scope of the research, the significance of research, research methodology, and the thesis outline. The literature review on topics of concern has been thoroughly reviewed and discussed in Chapter 2.

Chapter 3 shows the derivation of governing equations of dusty Jeffrey Fluid and basic dusty Newtonian fluid in two-dimensional for mixed convection flow past an exponentially stretching sheet. The derivation of continuity, momentum, energy and mass equations is presented specifically for fluid and dust phases. The expression of skin friction coefficient, Nusselt and Sherwood numbers are also discussed in detail.

Chapter 4 presents the numerical solution for MHD mixed convection of dusty fluid with heat transfer past an exponentially stretching sheet. The dimensional governing equations are reduced to non-dimensional form using certain significant non-dimensional variables and then solved numerically via the Keller-box method. The expressions of velocity and temperature profiles are acquired and plotted by utilising the MATLAB software. The influence of physical parameters on the velocity, temperature profiles for both phases are reviewed numerically and displayed in detail on plotted graphs. The numerical results for skin friction and the Nusselt number are evaluated and displayed in tabular forms.

Chapter 5 concerns the detailed analysis of suspended dust particles flow behaviour on heat and mass transfer phenomenon. This chapter is an extension of Chapter 4. The effect of chemical reaction parameter along with the parameters considered in Chapter 4 are discussed well. The new velocity and concentration profiles are generated in this chapter using similar steps as in Chapter 4.

Chapter 6 presents the study on dusty non-Newtonian fluid by taking Jeffrey fluid as a base fluid for suspended dust particles into account. The dusty Jeffrey fluid flow is investigated for both phases. Likewise, the pertinent parameters are kept similar to Chapter 4. Meanwhile, Chapter 7 is the continuation of Chapter 6. The heat and mass transfer is investigated for the dusty Jeffrey fluid together with the effect of a chemical reaction. The obtained numerical results for each chapter consisting of velocity, temperature and concentration profiles, and skin friction coefficient, Nusselt and Sherwood numbers are discussed and presented in various forms.

Finally, the summary and conclusions of this research are presented in Chapter 8. Some recommendations for future research in the present problem are also described in this chapter. Towards the end, the references used for all chapters are indicated. Next, all the published articles and conferences are also listed. The complete derivation of the governing equation of the single-phase flow is provided in Appendix A, whereas the reviews of the Keller-box method is given in Appendix B.

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LIST OF PUBLICATIONS

Indexed Journal

1. **Siti Nur Haseela Izani** and Anati Ali, 2016. Hydromagnetic mixed convection flow over an exponentially stretching sheet with fluid-particle suspension. In *AIP Conference Proceedings*. 1750: 1-030043. Presented at 23rd Malaysian National Symposium of Mathematical Sciences (SKSM23 2015), Pulai Spring Resort, Johor Bahru, 24 - 25 November. (**SCOPUS INDEXED**)
2. **Siti Nur Haseela Izani** and Anati Ali, 2016. Mixed convective boundary layer flow of a dusty Jeffrey fluid over an exponentially stretching sheet. In *AIP Conference Proceedings*. 1775: 1-030057. Presented at International Conference on Mathematics, Engineering and Industrial Applications (ICoMEIA 2016), BP Samila Beach Hotel and Resort, Songkhla Thailand, 10-12 August. (**SCOPUS INDEXED**)
3. **Siti Nur Haseela Izani** and Anati Ali, 2017. Heat and mass transfer of steady magnetohydrodynamics mixed convection of dusty fluid flow with chemical reaction past an exponentially stretching sheet. *Malaysian Journal of Fundamental and Applied Sciences*. Apr 1; 13(2) : 60-7. Presented at 5th International Science Postgraduate Conference (ISPC 2017), Pulai Spring Resort, Johor Bahru, 7 - 8 March. (**ISI INDEXED**)
4. **Siti Nur Haseela Izani** and Anati Ali, 2019. Thermal Radiation Effects on Heat and Mass Transfer of Magnetohydrodynamics Dusty Jeffrey Fluid Past an Exponentially Stretching Sheet. *MATEMATIKA: Malaysian Journal of Industrial and Applied Mathematics*. 35(2). 187-200. Presented at International Seminar on Mathematics in Industry International Conference on Theoritical and Applied Statistic (ISMI -ICTAS 2018) organized by UTM Centre for Industrial and Applied Mathematics (UTM-CIAM) and UTM Department of Mathematical Sciences, ITS Department of Statistics , ITS Department of Mathematics, Oxford Centre for Industrial and Applied Mathematics (OCIAM) and Asia Pacific Consortium of Mathematics for Industry (APCMfI) held on 4-6 September 2018 at Universiti Teknologi Malaysia, Kuala Lumpur. (**SCOPUS INDEXED**)