

UNSTEADY MATHEMATICAL ANALYSIS OF NON-NEWTONIAN BLOOD  
FLOW MODELS IN A DOUBLE STENOSED ARTERY

REIMA DAHER ALSEMIRY

A thesis submitted in fulfilment of the  
requirements for the award of the degree of  
Doctor of Philosophy

Faculty of Science  
Universiti Teknologi Malaysia

MARCH 2021

## **DEDICATION**

Special dedication to my parents for their love and encouragement. To my beloved husband and my children Rinad, Rama, Khalid and Faris for their patience and endurance for supporting me from the beginning to the end of my study – thank you for everything

## ACKNOWLEDGEMENT

Alhamdulillah, I am eternally grateful to Allah for graciously granting me the strength to complete this research. I am grateful to everybody that assisted, strengthened and guided me throughout this time.

First, I am grateful to my supervisor, Prof. Dr. Norsarahaida S. Amin from the Department of Mathematical Sciences, Universiti Teknologi Malaysia. With her dedication, encouragement and support, I have gained a lot of experience in handling research exercises. I have been inspired by her open-door policy, her outstanding actions, kindness and stimulating academic work. She expended much time and effort to enhance my research and scientific writing skills, and ensured that I gained the knowledge necessary to complete complex research exercises.

I also owe my deepest gratitude to my respected co-supervisor, Dr. Hamed M. S. Ali from the Department of Mathematics, Taibah University in Saudi Arabia and Ain Shams University in Egypt, for his unstinted support and proper direction in critical junctures of my research work. I have received invaluable assistance and innumerable suggestions from him.

I want to thank Prof. Dr. Prashanta K. Mandal from the Department of Mathematics, Visva-Bharati University, Santiniketan, West Bengal, India and Dr. Sarifuddin from the Department of Mathematics, Berhampore College, West Bengal, India. Together, they ensured that I continued to work diligently in my research. I am indebted to Dr. Sayantan Biswas and all the non-teaching employees at the Department of Mathematics, Visva-Bharati, West Bengal, India, for their support and cooperation during my time at their institution. I must also mention Dr. Mohamed H. Haroun from the Department of Mathematics, Ain Shams University, Egypt, who gave me much support during my first year of research.

Furthermore, I want to thank Dr. Zuhaila Binti Ismail from the Department of Mathematical Sciences, Universiti Teknologi Malaysia, and all the other faculty members, staff and specialists at the Faculty of Science, Universiti Teknologi Malaysia, for their generous support.

I am particularly grateful for the financial support offered by Taibah University, which ensured that I was able to undertake the research under the joint supervision program with Universiti Teknologi Malaysia.

Sincere thanks to my friends, who have greatly helped me while on campus. I want to mention Dr. Ghadah Aljohani, Dr. Abeer Mogadam, Dr. Hana Alluhaybi and Dr. Areej Alqarni in particular, for their kindness. I will never forget the help you gave me.

I want to thank my parents for their help, love and prayers, as well as my beloved sisters and brothers for all their moral support throughout this journey. I do not have words to express how much I have appreciated this. Finally, without the immense support of my husband and children, their patience and strength, I would not have been able to complete this work. All my love and thanks.

## ABSTRACT

The simultaneous effect of pulsatile blood flow and double stenoses with different severities, lengths, and interspacing on mass transport using the Newtonian as well as the non-Newtonian power-law models of blood flow are considered in this thesis. These models are important from a physiological perspective as their effects on certain blood flow characteristics that are clinically significant can be analysed. The effect of some essential issues like the diffusivity of mass and the rate of absorption at the lumen-tissue interface are also studied to investigate the effectiveness of solute delivery. The flow is considered two-dimensional, unsteady and axisymmetric in the cylindrical polar coordinate system, while the transport of mass is modelled as an unsteady convection-diffusion equation. A numerical technique in the form of finite-difference approximations in staggered grids, widely known as the Marker and Cell (MAC) method has been used to tackle the coupled system of non-linear partial differential equations. Simultaneous effects of pulsatile flow conditions and double stenoses show an increase in the pressure drop across the stenosis length, as well as in the transport of mass at the throat and mass flux at the artery wall. The delivery of solute is observed to be more effective in the non-Newtonian model. In this study, another concern is on the effect of catheter's eccentricity on blood flow and heat transfer characteristics using the Carreau model. The perturbation method which is an approximate analytical technique, has been applied to the catheter problem. The accuracy of results is confirmed in the limiting cases, where the existing solutions in the literature are recovered as special cases. The position of the catheter's eccentricity in Carreau fluid leads to a reduction in the number and size of the circulating bolus zone which agree with physiological observations that the risks and complications associated with catheterization are alleviated when the eccentric position of the catheter is considered. The results of the simulation could provide insights towards the detection of aggregation sites, allowing the treatment of disease to be initiated quickly before it becomes clinically significant.

## ABSTRAK

Kesan serentak aliran darah berdenyut dan stenosis berganda dengan tahap keparahan, panjang, dan jarak yang berbeza terhadap pengangkutan jisim menggunakan model aliran darah Newtonan dan tak-Newtonan hukum kuasa dipertimbangkan di dalam tesis ini. Model-model ini penting dari perspektif fisiologi kerana kesannya terhadap beberapa ciri aliran darah tertentu yang signifikan secara klinikal dapat dianalisis. Kesan daripada beberapa isu utama seperti penyebaran jisim dan kadar penyerapan di kawasan antara-muka tisu-lumen juga dikaji untuk menguji keberkesanan pengangkutan bahan terlarut. Model aliran bendalir dianggap dua dimensi, tak mantap dan simetri sepaksi dalam sistem koordinat silinder, sementara pengangkutan jisim dimodelkan sebagai persamaan perolakan-penyebaran tak mantap. Teknik berangka dalam bentuk penghampiran perbezaan terhingga dengan grid berperingkat, yang lebih dikenali sebagai kaedah Marker dan Cell (MAC) telah digunakan untuk menyelesaikan sistem persamaan pembezaan separa tak linear. Kesan gabungan antara aliran berdenyut dengan stenosis berganda menunjukkan peningkatan dalam tahap penurunan tekanan di sepanjang stenosis, serta pengangkutan jisim di kawasan kerongkong dan fluks jisim di dinding arteri. Pengangkutan bahan terlarut diperhatikan lebih berkesan bagi model tak-Newtonan. Di dalam kajian ini, satu lagi pemerhatian adalah kepada kesan eksentrik kateter terhadap aliran darah dan ciri pemindahan haba menggunakan model Carreau. Kaedah usikan yang merupakan suatu kaedah analisis penghampiran, telah diterapkan pada masalah kateter. Ketepatan hasilnya disahkan dalam kes-kes penghad, di mana hasil kajian yang terdapat di dalam literatur diperoleh sebagai kes khas. Kedudukan eksentrik kateter dalam bendalir Carreau menyebabkan bilangan dan saiz zon bolus yang beredar berkurang, dengan menepati pemerhatian fisiologi bahawa risiko dan komplikasi yang berkaitan dengan kateter dapat dikurangkan ketika kateter berada pada kedudukan eksentrik. Hasil simulasi dijangka dapat mengesan situs agregasi yang membolehkan rawatan segera di ambil sebelum sesuatu penyakit itu dikesan secara klinikal.

## TABLE OF CONTENTS

	<b>TITLE</b>	<b>PAGE</b>
	<b>DECLARATION</b>	ii
	<b>DEDICATION</b>	iii
	<b>ACKNOWLEDGEMENT</b>	iv
	<b>ABSTRACT</b>	vi
	<b>ABSTRAK</b>	vii
	<b>TABLE OF CONTENTS</b>	viii
	<b>LIST OF TABLES</b>	xi
	<b>LIST OF FIGURES</b>	xii
	<b>LIST OF ABBREVIATIONS</b>	xvii
	<b>LIST OF SYMBOLS</b>	xviii
	<b>LIST OF APPENDICES</b>	xx
<b>CHAPTER 1</b>	<b>INTRODUCTION</b>	1
1.1	Research Background	1
1.2	Problem Statement	5
1.3	Research Objectives	6
1.4	Scope of the Study	7
1.5	Significance of Research	7
1.6	Thesis Organization	9
<b>CHAPTER 2</b>	<b>LITERATURE REVIEW</b>	11
2.1	Introduction	11
2.2	Characteristics of Blood	11
2.3	Structure of Blood Vessel	13
2.4	Constitutive Equations of Blood	14
2.5	Mass Transport in Artery	18
2.6	Geometry of Stenoses and Boundary Conditions	25
2.7	Blood Flow and Heat Transfer through an Eccentric Catheterized Artery	28
2.8	Methods of Solutions	31

<b>CHAPTER 3</b>	<b>GOVERNING EQUATIONS</b>	35
3.1	Introduction	35
3.2	Governing Equations	35
3.2.1	Equation of Continuity	36
3.2.2	Equation for Momentum Transport	36
3.2.3	Equation for Mass Transport	39
3.2.4	Equation for Heat Transport	39
3.3	Boundary and Initial Conditions	40
3.4	Methods of Solutions	43
<b>CHAPTER 4</b>	<b>THE EFFECT OF FLOW PULSATILITY AND DOUBLE STENOSES ON THE MASS TRANSPORT OF BLOOD FLOW IN AN ELASTIC ARTERY</b>	45
4.1	Introduction	45
4.2	The Governing Equations	45
4.3	Solution Procedure	49
4.3.1	Non-dimensionalization of the Equations	49
4.3.2	Radial Coordinate Transformation	51
4.3.3	Marker and Cell Method	53
4.4	Time-Stepping Procedure for Numerical Stability	65
4.5	Numerical Algorithm	66
4.6	Numerical Results and Discussion	67
<b>CHAPTER 5</b>	<b>NON-NEWTONIAN MODEL OF PULSATILE BLOOD FLOW AND MASS TRANSPORT PAST A PAIR OF STENOSES IN AN ELASTIC ARTERY</b>	81
5.1	Introduction	81
5.2	The Governing Equations	82
5.3	Solution Procedure	83
5.4	Numerical Results and Discussion	87
<b>CHAPTER 6</b>	<b>THE EFFECT OF WALL ABSORPTION ON SOLUTE DISPERSION THROUGH AN ARTERIAL LUMEN AND ITS UPTAKE IN THE TISSUE WITH THE POWER-LAW MODEL OF BLOOD FLOW IN AN ARTERY WITH DOUBLE STENOSES</b>	101



6.1	Introduction	101
6.2	The Governing Equations	102
6.3	Solution Procedure	104
6.4	Numerical Results and Discussion	106
<b>CHAPTER 7</b>	<b>THE EFFECT OF ECCENTRIC CATHETERIZATION ON CARREAU MODEL OF BLOOD FLOW AND HEAT TRANSFER IN A STENOSED ARTERY</b>	117
7.1	Introduction	117
7.2	The Governing Equations	118
7.3	Solution Procedure	120
	7.3.1 Non-dimensionalization of the Equations	120
	7.3.2 Perturbation Solution	123
7.4	Results and Discussion	134
<b>CHAPTER 8</b>	<b>CONCLUSION</b>	151
8.1	Summary of Content	151
8.2	Summary of Results	152
8.3	Conclusion and Suggestions for Future Research	155
	<b>REFERENCES</b>	157
	<b>APPENDICES A-B</b>	173-191

## LIST OF TABLES

<b>TABLE NO.</b>	<b>TITLE</b>	<b>PAGE</b>
Table 4.1	Results of velocity for different grid sizes.	67
Table 7.1	Comparison of Nusselt number results for the eccentric parameter.	147

## LIST OF FIGURES

<b>FIGURE NO.</b>	<b>TITLE</b>	<b>PAGE</b>
Figure 1.1	Progression of atherosclerosis ( <a href="https://www.newbeginningshealthcare.net/blog/what-is-atherosclerosis">https://www.newbeginningshealthcare.net/blog/what-is-atherosclerosis</a> ).	2
Figure 1.2	Arterial catheter ( <a href="https://www.medline.com/product/Arterial-Catheters/Kits-by-Argon-Medical/Z05-PF27732">https://www.medline.com/product/Arterial-Catheters/Kits-by-Argon-Medical/Z05-PF27732</a> ).	3
Figure 2.1	Blood components ( <a href="https://www.britannica.com/science/blood-biochemistry">https://www.britannica.com/science/blood-biochemistry</a> ).	12
Figure 2.2	The percentage values for the components of blood in a normal adult ( <a href="https://wickedbiology.wordpress.com/2016/01/21/1-2-the-circulatory-system/blood-components/">https://wickedbiology.wordpress.com/2016/01/21/1-2-the-circulatory-system/blood-components/</a> ).	12
Figure 2.3	Blood vessel structure ( <a href="http://antranik.org/blood-vessels/">http://antranik.org/blood-vessels/</a> ).	13
Figure 2.4	Drug delivery ( <a href="https://pharmanotes.wordpress.com/tag/pharmacology/">https://pharmanotes.wordpress.com/tag/pharmacology/</a> ).	20
Figure 2.5	The pathway from an intravenous injection site to a tumour site (Dewhirst and Secomb, 2017).	22
Figure 2.6	The ADME processes (Sakai (2009)).	25
Figure 3.1	Schematic diagram for the catheterization arterial stenosis.	41
Figure 4.1	Schematic diagram for the double arterial stenoses.	47
Figure 4.2	Schematic of the computational domain under study.	53
Figure 4.3	A typical MAC cell.	54
Figure 4.4	Grids of MAC method to discretize the continuity equation.	54
Figure 4.5	Grids of MAC method to discretize the radial momentum equation.	55
Figure 4.6	Grids of MAC method for hybrid discretization of the convective term of the radial momentum equation.	57
Figure 4.7	Grids of MAC method to discretize the axial momentum equation.	58
Figure 4.8	Grids of MAC method for hybrid discretization of the convective term of the axial momentum equation.	60
Figure 4.9	Grids of MAC method to discretize the mass transport equation.	61

Figure 4.10	Flow chart for the proposed numerical simulation.	66
Figure 4.11	Comparison of the non-dimensional pressure drop across the stenoses.	68
Figure 4.12	Axial velocity profile for different Re at $z = 10$ ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	69
Figure 4.13	Axial velocity profile for different Re at $z = 19$ ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	69
Figure 4.14	Radial velocity profile for different Re at $z = 10$ ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	70
Figure 4.15	Radial velocity profile for different Re at $z = 19$ ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	70
Figure 4.16	Axial velocity profile for different axial positions at Re = 300, ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	71
Figure 4.17	Radial velocity profile for different axial positions at Re = 300, ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	72
Figure 4.18	Axial variation of the wall shear stress for different Re ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	73
Figure 4.19	Axial variation of the wall shear stress for various combinations of severities at Re = 300.	74
Figure 4.20	Axial variation of the wall shear stress for various lengths and distances between stenoses at Re = 300, ( $\delta_1 = 0.2, \delta_2 = 0.4$ ).	74
Figure 4.21	Pattern of streamlines for different inlet conditions ( $\delta_1 = 0.4, \delta_2 = 0.2$ ) (a) Re = 300 parabolic inlet, (b) Re = 500 parabolic inlet, (c) Re = 300 pulsatile inlet, (d) Re = 500 pulsatile inlet.	75
Figure 4.22	Variation of mass concentration profile for different axial positions at Re = 300.	76
Figure 4.23	Mass concentration profile of double and single stenosis at $t = 30$ .	77
Figure 4.24	Distribution of local Sherwood number for different Reynolds number.	78
Figure 4.25	Distribution of local Sherwood number for different severity of stenoses.	78
Figure 4.26	Distribution of local Sherwood number for double and single stenosis.	79
Figure 4.27	Iso-concentration line for (a) parabolic inlet (b) pulsatile inlet.	80
Figure 5.1	Comparison of the non-dimensional pressure drop across the stenoses.	88

Figure 5.2	Axial velocity profile corresponding to different axial positions at $Re = 300$ , ( $\delta_1 = 0.2, \delta_2 = 0.3$ ).	89
Figure 5.3	Radial velocity profile for different axial positions at $Re = 300$ , ( $\delta_1 = 0.2, \delta_2 = 0.3$ ).	89
Figure 5.4	Axial variation of the wall shear stress for different $Re$ ( $\delta_1 = 0.2, \delta_2 = 0.3$ ).	90
Figure 5.5	Axial variation of the wall shear stress at $Re = 300$ for same combinations of severity.	91
Figure 5.6	Axial distribution of the wall shear stress at $Re = 300$ for various combinations of severity.	92
Figure 5.7	Axial distribution of the wall shear stress for various combinations of the lengths of stenoses at ( $\delta_1 = 2, \delta_2 = 3$ ) with $Re = 300$ .	92
Figure 5.8	Patterns of streamlines for various inlet conditions and blood rheology for $Re = 300$ (a) parabolic inlet, (b) pulsatile inlet, (c) pulsatile inlet (Newtonian).	93
Figure 5.9	Pattern of streamlines for different different severities for pulsatile inlet at $Re = 300$ (a) $\delta_1 = \delta_2 = 0.2$ , (b) $\delta_1 = \delta_2 = 0.3$ , (c) $\delta_1 = \delta_2 = 0.4$ .	94
Figure 5.10	Trasmural variation of mass concentration for different axial positions at $Re = 300$ , ( $\delta_1 = 2, \delta_2 = 3$ ).	95
Figure 5.11	Transmural variation of mass concentration in case of a single and double stenoses at $t = 30$ .	96
Figure 5.12	Distribution of local Sherwood number for different Reynolds number at ( $\delta_1 = 2, \delta_2 = 3$ ).	97
Figure 5.13	Distribution of local Sherwood number for different severity of stenosis at $Re = 300$ .	97
Figure 5.14	Distribution of local Sherwood number for double and single stenosis at $Re = 300$ , ( $\delta_1 = 2, \delta_2 = 3$ ).	98
Figure 5.15	Variation of patterns of iso-concentration lines at $Re = 300$ for different inlet conditions (a) parabolic inlet, (b) pulsatile inlet.	99
Figure 6.1	Schematic diagram of the physical model for a coupled system comprised of arterial lumen and tissue.	103
Figure 6.2	Longitudinal distribution of the normalized wall shear stress for various $Re$ ( $\delta_1 = 0.2, \delta_2 = 0.25$ ).	107
Figure 6.3	Transluminal profile of the axial velocity for different axial positions at $Re = 300$ ( $\delta_1 = 0.2, \delta_2 = 0.25$ ).	108

Figure 6.4	Luminal drug concentration for different axial positions ( $Re = 300, \eta = 1, Pe_l = 500, Pe_t = 1000$ ).	109
Figure 6.5	Tissue drug concentration for different times ( $Re = 300, \eta = 1, Pe_l = 500, Pe_t = 1000$ ).	110
Figure 6.6	Temporal variation of luminal averaged concentration for different values of $\eta$ ( $Re = 300, Pe_l = 500, Pe_t = 1000$ ).	111
Figure 6.7	Temporal change of tissue averaged concentration for various values of $\eta$ ( $Re = 300, Pe_l = 500, Pe_t = 1000$ ).	111
Figure 6.8	Temporal change of the effectiveness of delivery for various values of $\eta$ ( $Re = 300, Pe_l = 500, Pe_t = 1000$ ).	112
Figure 6.9	Temporal variation of the effectiveness of delivery in stenosed and normal artery ( $Re = 300, Pe_l = 500, Pe_t = 1000$ ).	113
Figure 6.10	Axial variation of drug concentration in lumen for different values of $\eta$ ( $Re = 300, Pe_l = 500, Pe_t = 1000$ ).	114
Figure 6.11	Axial variation of drug concentration for various values of $\eta$ ( $Re = 300, Pe_l = 500, Pe_t = 1000$ ) in tissue.	114
Figure 6.12	Temporal change of averaged concentration for various values of $Pe_l$ ( $Re = 300$ ) in lumen.	115
Figure 6.13	Temporal change of averaged concentration in tissue for various values of $Pe_t$ ( $Re = 300$ ).	116
Figure 6.14	Temporal change of the effectiveness of delivery for various values of $Pe_t$ ( $Re = 300$ ).	116
Figure 7.1	Schematic diagram of eccentric catheterized stenosed artery( Mekheimer and Kot (2012)).	119
Figure 7.2	Variation of axial velocity with radial distance for Newtonian fluid.	135
Figure 7.3	Variation of axial velocity with radial distance and $n = 0.3568$ for different values of (a) eccentricity parameter ( $\varepsilon$ ), (b) catheter radius ( $\sigma$ ).	136
Figure 7.4	The wall shear stress-dependent (a) eccentricity parameter ( $\varepsilon$ ), (b) angle of circumferential direction ( $\theta$ ), (c) catheter radius ( $\sigma$ ).	138
Figure 7.5	The wall shear stress-dependent (a) Weissenberg number ( $We$ ), (b) velocity of the catheter ( $V_0$ ).	139
Figure 7.6	The effects of (a) eccentricity parameter ( $\varepsilon$ ), (b) catheter radius ( $\sigma$ ) on the impedance.	140
Figure 7.7	The effects of (a) Weissenberg number ( $We$ ), (b) velocity of catheter ( $V_0$ ) on the impedance.	141

Figure 7.8	Pattern of streamlines for various values of eccentricity parameter ( $\varepsilon$ ).	142
Figure 7.9	Pattern of streamlines for various values of catheter radius ( $\sigma$ ).	143
Figure 7.10	Pattern of streamlines for various values of Weissenberg number ( $We$ ).	144
Figure 7.11	Profiles of the temperature for different values of (a) eccentricity parameter ( $\varepsilon$ ), (b) angle of circumferential direction ( $\theta$ ), (c) catheter radius ( $\sigma$ ).	145
Figure 7.12	Profiles of the temperature for different values of (a) Weissenberg number ( $We$ ), (b) velocity of catheter ( $V_0$ ).	145
Figure 7.13	Profiles of the temperature for different values of (a) Prandtl number ( $Pr$ ), (b) Eckert number ( $Ec$ ).	146
Figure 7.14	Nusselt number versus eccentricity parameter for various values of (a) catheter radius ( $\sigma$ ), (b) velocity of catheter ( $V_0$ ), (c) Weissenberg number ( $We$ ).	148
Figure 7.15	Nusselt number versus eccentricity parameter for various values of (a) Prandtl number ( $Pr$ ), (b) Eckert number ( $Ec$ ).	149
Figure A.1	Finite control volume fixed in space.	173
Figure A.2	Cylindrical coordinates system.	179

## LIST OF ABBREVIATIONS

CVD	-	Cardiovascular Disease
CFD	-	Computational Fluid Dynamics
ECs	-	Endothelial Cells
LDL	-	Low-Density Lipoprotein
MAC	-	Marker and Cell Method
PK	-	Pharmacokinetics
RBCs	-	Red Blood Cells
SMCs	-	Smooth Muscle Cells
SOR	-	Successive Over Relaxation Method
WSS	-	Wall Shear Stress
WBCs	-	White Blood Cells



## LIST OF SYMBOLS

$\mathbf{F}$	-	Body forces
$\mathbf{D}$	-	Rate of strain tensor
$\mathbf{V}$	-	Velocity vector
$u$	-	Velocity component in radial direction
$w$	-	Velocity component in axial direction
$v$	-	Velocity component in theta direction
$V_0$	-	Velocity of catheter
$U_0$	-	The cross-sectional average velocity
$C_i$	-	Mass concentration, $i = l$ for lumen, $i = t$ for tissue
$C_s$	-	The reference concentration at the inlet
$c_p$	-	Specific heat
$D_i$	-	Coefficient of mass diffusion, $i = l$ for lumen and $i = t$ for tissue
$T$	-	Blood temperature
$T_0$	-	The respective temperature at the inner wall
$T_1$	-	The respective temperature at the arterial wall
$K_c$	-	Reaction rate constant
$L$	-	Total length of arterial segment
$R_i$	-	Radius of the arterial segment in the stenotic region, $i = 1$ double stenoses and $i = 3$ overlapping stenosis
$R_0$	-	Radius of the annular region in the non-stenotic artery
Re	-	Reynolds number
Sc	-	Schmidt number
Pe <sub>i</sub>	-	Peclet number, $i = l$ for lumen and $i = t$ for tissue
Sh <sub>D</sub>	-	Sherwood number
We	-	Weissenberg number
Pr	-	Prandtl number
Ec	-	Eckert number

$Nu_{ave}$	-	The mean Nusselt number
$m$	-	Consistency factor
$n$	-	Flow behavior index
$p$	-	Pressure
$\alpha$	-	Womersley number
$\eta$	-	The wall absorption parameter
$\Delta t$	-	Time increment
$\Delta x$	-	Increment in the radial direction
$\Delta z$	-	Increment in the axial direction
$\rho$	-	Density of blood
$\boldsymbol{\tau}$	-	Stress tensor
$\mu_{app}$	-	Non-Newtonian viscosity
$\sigma$	-	Catheter radius
$\epsilon$	-	Eccentricity parameter
$\beta$	-	Combination factor
$\kappa$	-	Thermal conductivity
$\Gamma$	-	Time constant
$\mu$	-	Constant viscosity
$\mu_{\infty}$	-	Infinite shear rate viscosity
$\mu_0$	-	Zero shear rate viscosity
$\lambda$	-	Impedance to flow
$\omega$	-	Angular frequency
$\omega_0$	-	Over relaxation parameter
$\omega_u$	-	Under relaxation parameter
$\theta$	-	Angle of circumferential direction
$\delta$	-	Critical height of stenosis

## LIST OF APPENDICES

<b>APPENDIX</b>	<b>TITLE</b>	<b>PAGE</b>
Appendix A	DERIVATION OF GOVERNING EQUATIONS	173
Appendix B	MATHEMATICAL PROGRAMMING	191

# CHAPTER 1

## INTRODUCTION

### 1.1 Research Background

Stenosis is the narrowing or restriction of blood vessel or valve that reduces blood flow. For example, aortic stenosis is the aortic valve in the heart. This narrowing often causes a sharp increase in the resistance to flow through the vessels. Over time, stenosis can advance to a complete blockage of the artery (Young (1968)). It affects the velocity of blood flowing through the artery, affecting blood pressure and may cause the heart to collapse. The damage caused to the heart or blood vessels can cause cardiovascular disease (CVD). Examples of CVD are coronary artery and carotid artery diseases. In coronary artery disease, a plaque buildup occurs in the arteries of the heart and can cause a heart attack while carotid artery disease can cause stroke (Ougrinovskaia *et al.* (2010)).

The narrowing usually results from atherosclerosis, which refers to a build-up of plaque on the inside of the arteries. The process of build up is called arterogenesis. The artery walls are normally smooth to allow blood flow easily through the artery and for easy transportation of mass or atherogenic particles such as fat, cholesterol which exists in blood in the form of low-density lipoproteins (LDL), calcium, and other substances within the artery wall. When the plaque is brittle and ruptures the most serious harm takes place in which the tear of plaque causes blood clots which could block the arterial lumen and/or move to another part of the circulatory system, which are eventually responsible for strokes, and heart attacks, difficulty in walking and gangrene (Thubrikar (2007)). Atherosclerosis is the major cause of CVD and remains the leading factor of death. Ross (1999) wrote that atherosclerosis is an

inflammatory disease, in which high concentrations of cholesterol in the blood is one of the principal risk factors.

Caro *et al.* (1971) reported that atherosclerosis (see Figure (1.1)) may occur based on shear-dependent mass transfer mechanism between blood cholesterol and the arterial wall. Caro and Nerem (1973) noted that the correlations between shear stress and atherosclerosis may be due to the alteration in convection mass transfer since the mass transfer due to convection depends on the velocity gradient, which is related to shear stress. This means the balance between convection and diffusion in the bloodstream and arterial wall determines which molecules enter, exit and remain entrapped (Fry (1987)). In order to make an appropriate assessment regarding the possible relationship between the spots of atherosclerotic lesions and the mass transfer patterns, an accurate characterization of mass transport behavior is very important. Moreover, a clear knowledge of mass transfer in a stenosed artery is of considerable medical interest in the formation and development of atherosclerosis.

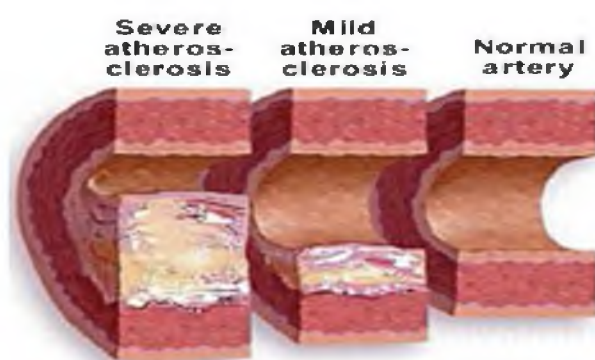


Figure 1.1: Progression of atherosclerosis (<https://www.newbeginningshealthcare.net/blog/what-is-atherosclerosis>).

The study of blood rheology and the dynamical characteristics of its flow is a very important step towards the comprehension, prediction, diagnosis, and therapy of CVD. Blood is a complex and exhibits various types of rheology behavior depending on the size of the vessel in a specific location. From a biofluid mechanics point of view, blood would not be expected to obey the very simple, one parameter, and linearized law of viscosity as developed by Newton (Mustapha and Amin (2008)).

However, CVD usually develops over a long time, thus it can be prevented or delayed by effectively managing and modifiable risk factors. Corrective therapies involve drug regimens and various forms of surgical intervention. The delivery of a solute from the bloodstream to the site of drug action primarily depends on blood flow but blood flow to different organs of the body is not equal. The effect of some essential issues like the diffusivity of mass, the rate of its absorption at the lumen-tissue interface with the flowing blood are important to analyze.

In some instances, surgery may be necessary to treat clogged arteries and prevent arterial plaque accumulation. The most common surgical intervention is artery catheterization and it is a quick procedure involving minimal risk while in more severe cases coronary bypass is used. Artery catheterization is the insertion of small plastic tubes (catheters) into arteries and veins (see Figure (1.2)) to the heart to obtain x-ray pictures (angiography) of coronary arteries, to determine whether blood vessels supplying the heart muscle are obstructed and to measure pressures and flow velocity or flow rate in the heart (hemodynamics). The transportation of drug or its delivery also involves a catheter being inserted into the artery.



Figure 1.2: Arterial catheter (<https://www.medline.com/product/Arterial-Catheters/Kits-by-Argon-Medical/Z05-PF27732>).

The procedure involving catheter brings about potential complications which

may include reaction to sedation, infection, and bleeding. When a catheter is inserted into the artery, it creates an annular region between the walls of the catheter and artery. Insertion of catheter further increases impedance frictional resistance to flow that would change the flow characteristics such as velocity, pressure, and streamlines (Dash *et al.* (1996)).

Blood does not only transport metabolites, oxygen and other dissolved substances to and from the tissues, but it also alters heat transfer or the transport of heat within the body. This is to meet the changing demands of the organism whose cardiovascular system for example, is sensitive to changes in the environment such as temperature change. In biological systems, it is very important to consider the variation of temperature because a slight change in temperature, for example, if the temperature of human blood rises above  $1^{\circ}C$ , irreversible harm occurs in the blood proteins (Quast and Kimberger (2014)).

Over the years, many mathematical models have been developed to study blood flow. These mathematical models are being continuously improved and upgraded to take into account more realistic physical conditions. The solution to these problems involve complex mathematical equations which are difficult to solve and most solutions obtained involved numerical methods. Analytical methods can only be approached in special cases when the equations are very much simplified. In this thesis, four different situations based on the research background above are analysed to study the effect of boundary conditions, the type of fluid, the effect of certain parameter on mass transport at the lumen-artery interface and the effect of catheter insertion on heat transfer, with specific problem statements and research questions as outlined in Section 1.2.

## 1.2 Problem Statement

Most mathematical models to investigate mass transfer in stenosed artery assume the artery to have a single mild stenosis, the artery wall is rigid, the flow is time independent and the inlet velocity is parabolic. In real situation, the patient is found to have multiple stenoses in the same arterial segment, the arterial blood flow is unsteady and the effect of the unsteadiness on pressure drop is important and flow from the heart comes from a large pressure reservoir into successively smaller tubes resulting in pulsatile velocity at the inlet region. The first research question is how are the blood flow and mass transfer characteristics altered when flow pulsatility and double stenoses are considered simultaneously.

The next problem is concerned with the nature of blood itself. Experimental investigations have revealed that blood exhibits non-Newtonian properties at low shear rate. The non-Newtonian behavior of blood is most evident in small vessels or at very low shear rates (Tu and Deville (1996)). Since the shear rate is low in the downstream of the stenosis, a correct analysis of the flow pattern should include the non-Newtonian factor. The question then is how the non-Newtonian model of blood flow affects the characteristics of flow and mass transport in an artery with double stenoses and pulsatile inlet condition.

Where mass transfer is concerned, the majority of works on mass or solute transport were performed by considering tubular geometrical model with thin boundary and the analysis was restricted to the solute dispersion in the fluid phase of the tube only. In reality, the arterial wall is thick and mass is transported via a number of realistic lumen-tissue interface conditions, thus the unsteady coupled problem interconnecting the dispersion of mass from lumen into tissue must be taken into consideration. The interphase mass transfer due to the absorption at the tube wall plays an important role on the dispersion process of solute. This phenomenon has lots of applications in hemodynamics for permeable blood vessels, tubular flow reactors with heterogeneous catalysis, bioengineering (haemodialysers and oxygenators) and



physiology. Applications in which absorptive wall is also of major importance include tubular flow reactors and soluble gas uptake by the walls of pulmonary airways, and is of particular significance in the mixing and the transportation of drugs or toxins in physiological systems. Since the therapeutic domain for atherosclerotic disease is the stenosed arterial wall and for the better treatment of the patient, study the uptake of an injected drug under various luminal conditions at the absorptive lumen-tissue interface and its subsequent dispersion into the arterial wall is an important consideration because it closely resembles the physiological situation concerning intravenous drug delivery. The research question here is how the absorptive wall affects the diffusivity of mass within lumen and the arterial tissue.

In the course of drug transport or delivery, where the insertion of a catheter is concerned, the catheter is usually placed in an eccentric position, one reason being to reduce pain in the patient. However, most analysis involving catheterized flow considered that the catheter to be in concentric position. But a more realistic condition is that the catheter is usually placed in an eccentric position, one reason being to reduce pain. Another condition to consider is a mathematical model that takes into account temperature change as it is common knowledge that a temperature rise of merely  $1^{\circ}C$  or  $2^{\circ}C$  for example would cause a fever. A more realistic flow model should include the energy equation. The research question here is how the eccentric catheter affects blood flow characteristics when temperature is taken into account.

### **1.3 Research Objectives**

The main objective of this research is to carry out a mathematical analysis using both analytical and numerical means to investigate the effects of different boundary conditions and flow domains on the flow, mass transport and heat transfer characteristics of blood in a stenosed artery.

Specific objectives are:

- (a) To determine the simultaneous effect of pulsatile blood flow and double stenoses on mass transport.
- (b) To analyze the effects of power-law model of blood flow and pulsatile inlet on mass transfer in an artery with double stenoses.
- (c) To calculate the distribution of mass from lumen to tissue through a stenosed artery in the presence of absorption at the tube wall.
- (d) To determine the effects of eccentric catheterization on Carreau model of blood flow with heat transfer in an overlapping stenosed artery.

#### **1.4 Scope of the Study**

This research takes into consideration blood flow through a double stenosed artery with pulsatile boundary condition. The geometry of stenosis considered is the double and overlapping one. The fluid is assumed to be incompressible where only the Newtonian and non-Newtonian models of blood characterized by the power-law model and Carreau model are considered. The flow is assumed laminar, two-dimensional axisymmetric in the cylindrical coordinate system. For the solution procedure, the analytical technique is the perturbation method with mild condition approximation while the numerical technique is the finite difference the Marker and Cell (MAC) method. Simulations are carried out using published data in the literature.

#### **1.5 Significance of Research**

Mathematical models of blood flow in the cardiovascular system provide insight into normal and diseased conditions in blood vessels and have applications in areas such as surgical planning and designs of medical devices. A mathematical analysis on the flow characteristics of blood with the consideration of the real situation could make it possible to predict whether the problems faced by patients need medical

intervention in the form of invasive surgery or otherwise. For example in CVD, the cardiologist needs to determine whether the patient need a bypass or not based mostly on his experience. This is very risky for the patient if the doctor is inexperienced.

On the other hand, with mathematical analysis possible complications could be determined non-invasively and cheaply without unnecessary complications. In the present work, if the geometry of stenosis of a cardiac patient could be obtained through angiograms, the flow characteristics namely the wall shear stress, the pressure drop and the streamlines could be calculated and a suitable range of values that cause complications could be established.

Specifically in this project, the critical length of stenoses , severity etc could determine whether they affect mass transport, whether recirculations have occurred and so on. Further, it provides information on how absorption affects mass transport at the lumen, how catheter position, size and velocity of insertion affects the artery, thus enabling medical practitioners to choose a suitable catheter for insertion, and by how much it should expand an artery. If a catheter is not expanded enough it will not be doing its job, but if it is expanded too much it can risk damaging the artery. Most importantly, with sufficient statistical data, mathematical analysis has the ability to predict various flow characteristics that could be used for validation purposes with experimental and clinical results.

Mathematical analysis could significantly reduce the cost of diagnosing and treating a disease. It could be expected to minimize physiological through non-invasive procedure psychological stress and cost of treatment on patients. For manufacturers of products and regulators, mathematical analysis can be the mechanism that supports efficient review of novel products and approaches without compromising on safety and effectiveness.

## 1.6 Thesis Organization

This thesis is comprising of eight chapters as given below: Chapter 1 starts with the research background which outlines the general introduction followed by problem statement, research objectives, scope and significance of the study.

In Chapter 2, a literature review with respect to the issues sketched out within the problems outlined in the objectives is displayed and discussed in detail.

In Chapter 3, the differential form of the equations and the mathematical model that governed the flow namely Newtonian, power-law and Carreau models are presented. This chapter starts with the conservation of mass, momentum, mass and energy equations. It then follows with the discussion about the appropriate boundary conditions.

In Chapter 4, the first problem which looked at the numerical investigation to determine the effect of severity, the distance between double narrowings and flow pulsatility on the transport of blood and mass. This chapter is divided into six main sections including the introduction, governing equations, solution procedure, stability and accuracy, numerical algorithm and numerical results and discussion.

Chapter 5, accounts for shear-thinning model of blood flow, known as power-law model through a double stenosed artery to improve the previous chapter by considering the non-Newtonian nature of blood.

Chapter 6, addresses the distribution of mass in a stenosed arterial segment as well as in the tissue with the streaming blood represented by the power-law model. The coupled system of non-linear mass and momentum transport along with appropriate boundary conditions is solved numerically using the finite difference scheme MAC method.

In Chapter 7, describes the effect of catheter insertion on blood flow and heat transfer characteristics of a Carreau fluid model by using the perturbation method involving two appropriate small parameters. Finally, in chapter 8 concludes the thesis together with useful suggestions and recommendations for pursuing future research.

## REFERENCES

- Abd El Hakeem, A. E. N. and El Misiery, A. (2002). Effects of an Endoscope and Generalized Newtonian Fluid on Peristaltic Motion. *Applied Mathematics and Computation*. 128: 19–35.
- Ahmed, S. A. and Don, P. G. (1981). Pulsatile Poststenotic Flow Studies with Laser Doppler Anemometry. *Journal of Biomechanics*. 7(9): 965–705.
- Akbar, N. S. (2016). Non-Newtonian Model Study for Blood Flow through a Tapered Artery with a Stenosis. *Alexandria Engineering Journal*. 55(1): 321–329.
- Akbar, N. S. and Nadeem, S. (2014). Carreau Fluid Model for Blood Flow through a Tapered Artery with a Stenosis. *Ain Shams Engineering Journal*. 5(4): 1307–1316.
- Anderson, J. D. (1995). *Computational Fluid Dynamics: The Basics with Applications*. New York: McGraw-Hill.
- Astarita, G. and Marrucci, G. (1974). *Principles of Non-Newtonian Fluid Mechanics*. McGraw-Hill Companies.
- Azahari, A., Ismail, Z. and Abdullah, N. (2018). 3D Model of Generalized Power Law Blood Flow in a Stenosed Bifurcated Artery. *Matematika*. 34(1): 87–102.
- Back, L. (1994). Estimated Mean Flow Resistance Increase During Coronary Artery Catheterization. *Journal of Biomechanics*. 27(2): 169–175.
- Back, L., Kwack, E. and Back, M. (1996). Flow Rate-Pressure Drop Relation in Coronary Angioplasty: Catheter Obstruction Effect. *Journal of Biomechanical Engineering*. 118: 84–89.
- Bakheet, A., Alnussairy, E. A., Ismail, Z. and Amin, N. (2018). Generalized Power-Law Model of Magnetohydrodynamic Blood Flow with Heat Transfer. *Indian Journal of Public Health Research & Development*. 9(12): 794–800.

- Bakheet, A., Alnussaiby, E. A., Ismail, Z. and Amin, N. (2016). Blood Flow through an Inclined Stenosed Artery. *Applied Mathematical Sciences*. 10(5): 235–254.
- Bakhti, H. and Azrar, L. (2016). Steady Flow of Couple-Stress Fluid in Constricted Tapered Artery: Effects of Transverse Magnetic Field, Moving Catheter, and Slip Velocity. *Journal of Applied Mathematics*. 2016: 1–11.
- Bird, R. B., Stewart, W. E. and Lightfoot, E. N. (2002). *Transport Phenomena Second Edition*. Wiley.
- Bourhan, T. and Magableh, A. (2008). Magnetic Field Effect on Heat Transfer and Fluid Flow Characteristics of Blood Flow in Multi-Stenosis Arteries. *Heat and Mass Transfer*. 44(3): 297–304.
- Buchanan, J. R., Kleinstreuer, C. and Comer, J. K. (2000). Rheological Effects on Pulsatile Hemodynamics in a Stenosed Tube. *Computers and Fluids*. 29(6): 695–724.
- Calo, V. M., Brasher, N. F., Bazilevs, Y. and Hughes, T. J. (2008). Multiphysics Model for Blood Flow and Drug Transport with Application to Patient-Specific Coronary Artery Flow. *Computational Mechanics*. 43(1): 161–177.
- Caro, C. G., Fitz-Gerald, J. M. and Schroter, R. C. (1971). Atheroma and Arterial Wall Shear Observation, Correlation and Proposal of a Shear Dependent Mass Transfer Mechanism for Atherogenesis. *Proceedings of the Royal Society of London. Series B. Biological sciences*. 177(1046): 109–133.
- Caro, C. G. and Nerem, R. M. (1973). Transport of <sup>14</sup>C-4-Cholesterol between Serum and Wall in the Perfused Dog Common Carotid Artery. *Circulation Research*. 32(2): 187–205.
- Carreau, P. J. (1968). *Ph.D. Thesis*. University of Wisconsin, Madison: Ph. D. Thesis.
- Chakravarty, S. (1987). Effects of Stenosis on the Flow-Behaviour of Blood in an Artery. *International Journal of Engineering Science*. 25(8): 1003–1016.
- Chakravarty, S. and Chowdhury, A. G. (1988). Response of Blood Flow through an Artery under Stenotic Conditions. *Rheologica Acta*. 27(4): 418–427.

- Chakravarty, S., Datta, A. and Mandal, P. (1995). Analysis of Nonlinear Blood Flow in a Stenosed Flexible Artery. *International Journal of Engineering Science*. 33(12): 1821–1837.
- Chakravarty, S. and Mandal, P. (1994). Mathematical Modelling of Blood Flow through an Overlapping Arterial Stenosis. *Mathematical and Computer Modelling*. 19(1): 59–70.
- Chakravarty, S. and Mandal, P. K. (1996). A Nonlinear Two-Dimensional Model of Blood Flow in an Overlapping Arterial Stenosis Subjected to Body Acceleration. *Mathematical and Computer Modelling*. 24(1): 43–58.
- Chakravarty, S. and Mandal, P. K. (2000). Two-Dimensional Blood Flow through Tapered Arteries under Stenotic Conditions. *International Journal of Non-Linear Mechanics*. 35(5): 779–793.
- Chakravarty, S. and Sen, S. (2005). Dynamic Response of Heat and Mass Transfer in Blood Flow through Stenosed Bifurcated Arteries. *Korea Australia Rheology Journal*. 17(2): 47–62.
- Cho, Y. I. and Kensey, K. R. (1991). Effects of the Non-Newtonian Viscosity of Blood on Flows in a Diseased Arterial Vessel. Part 1: Steady Flows. *Biorheology*. 28(3-4): 241–262.
- Darby, R. and Chhabra, R. P. (2016). *Chemical Engineering Fluid Mechanics Third Edition*. CRC Press.
- Daripa, P. and Dash, R. K. (2002). A Numerical Study of Pulsatile Blood Flow in an Eccentric Catheterized Artery Using a Fast Algorithm. *Journal of Engineering Mathematics*. 42(1): 1–22.
- Dash, R. K., Jayaraman, G. and Mehta, K. N. (1996). Estimation of Increased Flow Resistance in a Narrow Catheterized Artery - a Theoretical Model. *Journal of Biomechanics*. 29(7): 917–930.
- Dash, R. K., Jayaraman, G. and Mehta, K. N. (2000). Shear Augmented Dispersion of a Solute in a Casson Fluid Flowing in a Conduit. *Annals of Biomedical Engineering*. 28(4): 373–385.



- Dejam, M., Hassanzadeh, H. and Chen, Z. (2016). Shear Dispersion in a Capillary Tube with a Porous Wall. *Journal of Contaminant Hydrology*. 185-186: 87–104.
- Deshpande, M. D., Giddens, D. P. and Mabon, R. F. (1976). Steady Laminar Flow through Modelled Vascular Stenoses. *Journal of Biomechanics*. 9(4): 165–174.
- Dewhirst, M. W. and Secomb, T. W. (2017). Transport of Drugs from Blood Vessels to Tumour Tissue. *Nature Reviews Cancer*. 17(12): 738–750.
- El Kot, M. A. and Abbas, W. (2017). Numerical Technique of Blood Flow through Catheterized Arteries with Overlapping Stenosis. *Computer Methods in Biomechanics and Biomedical Engineering*. 20(1): 45–58.
- Elblbesy, M. A. and Hereba, A. T. (2016). Computation of the Coefficients of the Power Law Model for Whole Blood and Their Correlation with Blood Parameters. *Applied Physics Research*. 8(2): 1–9.
- Ellahi, R., Rahman, S. U. and Nadeem, S. (2014). Blood Flow of Jeffrey Fluid in a Catheterized Tapered Artery with the Suspension of Nanoparticles. *Physics Letters A*. 378(40): 2973–2980.
- Ethier, C. R. (2002). Computational Modeling of Mass Transfer and Links to Atherosclerosis. *Annals of Biomedical Engineering*. 30: 461–471.
- Floyd, P. A., Mimms, S. E. and Yelding, C. (2007). *Personal Health: Perspectives and Lifestyles*. Nelson Education.
- Forrester, J. H. and Young, D. F. (1970). Flow through a Converging-Diverging Tube and its Implications in Occlusive Vascular Disease—I: Theoretical Development. *Journal of Biomechanics*. 3(3): 297–305.
- Fournier, R. L. (2017). *Basic Transport Phenomena in Biomedical Engineering*. New York: Philadelphia: Taylor and Francis.
- Fry, D. (1987). Mass Transport, Atherogenesis, and Risk. *Arteriosclerosis*. 7(1): 88–100.

- Galdi, G. P., Rannacher, R., Robertson, A. M. and Turek, S. (2007). *Hemodynamical Flows: Modeling, Analysis and Simulation (Oberwolfach Seminars)*. Birkhäuser Basel.
- Ganga, H. V. and Lundbye, J. B. (2012). Intravascular Temperature Management (IVTM) Techniques in Therapeutic Hypothermia. *EP Lab Digest*. 12(10): 1–9.
- Griffiths, I. M., Howell, P. D. and Shipley, R. J. (2013). Control and Optimization of Solute Transport in a Thin Porous Tube. *Physics of Fluids*. 25(3): 033101.
- Harlow, F. H. and Welch, J. E. (1965). Numerical Calculation of Time-Dependent Viscous Incompressible Flow of Fluid with Free Surface. *Physics of Fluids*. 8(12): 2182–2189.
- Hasan, A. B. M. T. and Das, D. K. (2008). Numerical Simulation of Sinusoidal Fluctuated Pulsatile Laminar Flow through Stenotic Artery. *Journal of Applied Fluid Mechanics*. 1(2): 25–35.
- Hirt, C. W. (1968). Heuristic Stability Theory for Finite-Difference Equations. *Journal of Computational Physics*. 2(4): 339–355.
- Hisham, M. D., Awan, A. U., Shah, N. A. and Tlili, I. (2020). Unsteady Two-Dimensional Flow of Pseudo-Blood Fluid in an Arterial Duct Carrying Stenosis. *Physica A: Statistical Mechanics and its Applications*. 124126.
- Hogan, H. A. and Henriksen, M. (1989). An Evaluation of a Micropolar Model for Blood Flow through an Idealized Stenosis. *Journal of Biomechanics*. 22(3): 211–218.
- Hossain, S. S., Hossainy, S. F., Bazilevs, Y., Calo, V. M. and Hughes, T. J. (2012). Mathematical Modeling of Coupled Drug and Drug-Encapsulated Nanoparticle Transport in Patient-Specific Coronary Artery Walls. *Computational Mechanics*. 49(2): 213–242.
- Hussain, M. A., Kar, S. and Puniyani, R. R. (1999). Relationship between Power Law Coefficients and Major Blood Constituents Affecting the Whole Blood Viscosity. *Journal of Biosciences*. 24(3): 329–337.

- Ikbal, M. A., Chakravarty, S. and Mandal, P. K. (2008). An Unsteady Peristaltic Transport Phenomenon of Non-Newtonian Fluid - a Generalised Approach. *Applied Mathematics and Computation*. 201(1-2): 16–34.
- Ikbal, M. A., Chakravarty, S., Sarifuddin and Mandal, P. K. (2010). Numerical Simulation of Mass Transfer to Micropolar Fluid Flow Past a Stenosed Artery. *International Journal for Numerical Methods in Fluids*. 67(11): 1655–1676.
- Ikbal, M. A., Chakravarty, S., Wong, K. K., Mazumdar, J. and Mandal, P. K. (2009). Unsteady Response of Non-Newtonian Blood Flow through a Stenosed Artery in Magnetic Field. *Journal of Computational and Applied Mathematics*. 230(1): 243–259.
- Ismail, Z., Abdullah, I., Mustapha, N. and Amin, N. (2008). A Power-Law Model of Blood Flow through a Tapered Overlapping Stenosed Artery. *Applied Mathematics and Computation*. 195(2): 669–680.
- Jagadeesha, S. and Rao, I. R. (2012). Solute Transfer in A Power-law Fluid Flow through Permeable Tube. *Advanced in Theoretical and Applied Mechanics*. 5(7): 309–322.
- Jamil, D. F., Roslan, R., Abdulhameed, M. and Hashim, I. (2018). Controlling the Blood Flow in the Stenosed Porous Artery with Magnetic Field. *Sains Malaysiana*. 47(10): 2581–2587.
- Jayaraman, G. and Dash, R. K. (2001). Numerical Study of Flow in a Constricted Curved Annulus: An Application to Flow in a Catheterised Artery. *Journal of Engineering Mathematics*. 40(4): 355–376.
- Jayaraman, G. and Tewari, K. (1995). Flow in Catheterised Curved Artery. *Medical & Biological Engineering & Computing*. 33(5): 720–724.
- John, P. S. (2017). *Cerebral Blood Flow and Metabolism: A Quantitative Approach*. World Scientific.
- Johnston, B. M., Johnston, P. R., Corney, S. and Kilpatrick, D. (2004). Non-Newtonian Blood Flow in Human Right Coronary Arteries: Steady State Simulations. *Journal of Biomechanics*. 37(5): 709–720.

- Johnston, P. R. and Kilpatrick, D. (1990). Mathematical Modelling of Paired Arterial Stenoses. *In [1990] Proceedings Computers in Cardiology.* 229–232.
- Kaazempur-Mofrad, M. R., Wada, S., Myers, J. G. and Ethier, C. R. (2005). Mass Transport and Fluid Flow in Stenotic Arteries : Axisymmetric and Asymmetric Models. *International Journal of Heat and Mass Transfer.* 48: 4510–4517.
- Kanai, H., Iizuka, M. and Sakamoto, K. (1970). One of the Problems in the Measurement of Blood Pressure by Catheter-Insertion: Wave Reflection at the Tip of the Catheter. *Medical and Biological Engineering.* 8(5): 483–496.
- Kenjereš, S., Van Der Krieken, J. and Li, C. (2019). Endothelium Resolving Simulations of Wall Shear-Stress Dependent Mass Transfer of LDL in Diseased Coronary Arteries. *Computers in Biology and Medicine.* 114: 103453.
- Kilpatrick, D., Webber, S. D. and Colle, J. P. (1990). The Vascular Resistance of Arterial Stenoses in Series. *Angiology.* 41(4): 278–285.
- Ku, D. N. (1997). Blood Flow in Arteries. *Annual Review of Fluid Mechanics.* 29(1): 399–434.
- Labadin, J., Ping, Y. and Walton, A. (2008). Investigating Axial Flow between Eccentric Cylinders. *Applied Computer and Applied Computational Science.* 488–491.
- Labadin, J. and Walton, A. G. (2006). Modeling of Axial Flow between Eccentric Cylinders. *Proceedings of the 2nd IMT-GT Regional Conference on Mathematics, Statistics and Applications, Universiti Sains Malaysia.* 1–7.
- Lee, T. S. (1990). Numerical Studies of Fluid Flow through Tubes with Double Constrictions. *International Journal for Numerical Methods in Fluids.* 11(8): 1113–1126.
- Li, T.-F., Wu, J., Luo, K. and Yi, H.-L. (2018). Lattice Boltzmann Simulation of Electro-hydro-dynamic (EHD) Natural Convection Heat Transfer in Horizontal Cylindrical Annuli. *International Communications in Heat and Mass Transfer.* 98(0735-1933): 106–115.

- Liu, B. and Tang, D. (2019). Influence of Distal Stenosis on Blood Flow through Coronary Serial Stenoses: a Numerical Study. *International Journal of Computational Methods*. 16(3): 1–11.
- Liu, B., Zheng, J., Bach, R. and Tang, D. (2017). Influences of Flow Parameters on Pressure Drop in a Patient Specific Right Coronary Artery with Two Stenoses. *International Conference on Computational Science and Its Applications*. 10404(0302-9743): 56–70.
- Liu, G. T., Wang, X. J., Ai, B. Q. and Liu, L. G. (2004). Numerical Study of Pulsating Flow through a Tapered Artery with Stenosis. *Chinese Journal of Physics*. 42(4 I): 401–409.
- MacDonald, D. A. (1979). On Steady Flow through Modelled Vascular Stenoses. *Journal of Biomechanics*. 12(1): 13–20.
- MacDonald, D. A. (1982). Fully Developed Incompressible Flow between Non-Coaxial Circular Cylinders. *Zeitschrift für Angewandte Mathematik und Physik*. 33(6): 737–751.
- Malik, M. Y., Hussain, A. and Nadeem, S. (2011). Flow of a Jeffery-Six Constant Fluid between Coaxial Cylinders with Heat Transfer Analysis. *Communications in Theoretical Physics*. 56(2): 345–351.
- Mandal, P. K. (2005). An Unsteady Analysis of Non-Newtonian Blood Flow through Tapered Arteries with a Stenosis. *International Journal of Non-Linear Mechanics*. 40(1): 151–164.
- Mandal, P. K., Chakravarty, S. and Mandal, A. (2007a). Numerical Study of the Unsteady Flow of Non-Newtonian Fluid through Differently Shaped Arterial Stenoses. *International Journal of Computer Mathematics*. 84(7): 1059–1077.
- Mandal, P. K., Chakravarty, S., Mandal, A. and Amin, N. (2007b). Effect of Body Acceleration on Unsteady Pulsatile Flow of Non-Newtonian Fluid through a Stenosed Artery. *Applied Mathematics and Computation*. 189(1): 766–779.

- Manglik, R. M. and Fang, P. P. (1995). Effect of Eccentricity and Thermal Boundary Conditions on Laminar Fully Developed Flow in Annular Ducts. *International Journal of Heat and Fluid Flow*. 16(4): 298–306.
- Manica, R. and De Bortoli, A. (2003). Simulation of Incompressible Non-Newtonian Flows through Channels with Sudden Expansion Using the Power-Law Model. *Trends in Applied and Computational Mathematics*. 4(3): 333–340.
- Markham, G. and Proctor, M. (1983). Modifications to the Two-Dimensional Incompressible Fluid Flow Code ZUNI to Provide Enhanced Performance. *C.E.G.B. Report TPRD*. 82.
- Mates, R. E., Gupta, R. L., Bell, A. C. and Klocke, F. J. (1978). Fluid Dynamics of Coronary Artery Stenosis. *Circulation Research*. 42(1): 152–162.
- Mazumder, B. S. and Mondal, K. K. (2005). On Solute Transport in Oscillatory Flow through an Annular Pipe with a Reactive Wall and its Application to a Catheterized Artery. *Quarterly Journal of Mechanics and Applied Mathematics*. 58(3): 349–365.
- Mekheimer, K. S. and Abd elmaboud, Y. (2008). The Influence of Heat Transfer and Magnetic Field on Peristaltic Transport of a Newtonian Fluid in a Vertical Annulus: Application of an Endoscope. *Physics Letters A*. 372(10): 1657–1665.
- Mekheimer, K. S. and El Kot, M. A. (2012). Mathematical Modeling of Axial Flow between Two Eccentric Cylinders: Application on the Injection of Eccentric Catheter through Stenotic Arteries. *International Journal of Non-Linear Mechanics*. 47(8): 927–937.
- Midya, C., Layek, G. C., Gupta, A. S. and Mahapatra, T. R. (2003). Magnetohydrodynamic Viscous Flow Separation in a Channel with Constrictions. *Journal of Fluids Engineering, Transactions of the ASME*. 125(6): 952–962.
- Misra, J. C., Patra, M. K. and Misra, S. C. (1993). A Non-Newtonian Fluid Model for Blood Flow through Arteries under Stenotic Conditions. *Journal of Biomechanics*. 26(9): 1129–1141.

- Misra, J. C., Sinha, A. and Shit, G. C. (2011). Mathematical Modeling of Blood Flow in a Porous Vessel Having Double Stenoses in the Presence of an External Magnetic Field. *International Journal of Biomathematics*. 4(2): 207–225.
- Molla, M. M., Wang, B. C. and Kuhn, D. C. (2012). Numerical Study of Pulsatile Channel Flows Undergoing Transition Triggered by a Modelled Stenosis. *Physics of Fluids*. 24(12): 1–25.
- Morgan, B. E. and Young, D. F. (1974). An Integral Method for the Analysis of Flow in Arterial Stenoses. *Bulletin of Mathematical Biology*. 36(1): 39–53.
- Mustapha, N. and Amin, N. (2008). The Unsteady Power Law Blood Flow through a Multi-Irregular Stenosed Artery. *Matematika*. 24(2): 187–198.
- Mustapha, N., Amin, N., Chakravarty, S. and Mandal, P. K. (2009). Unsteady Magnetohydrodynamic Blood Flow through Irregular Multi-Stenosed Arteries. *Computers in Biology and Medicine*. 39(10): 896–906.
- Mustapha, N., Mandal, P. K., Abdullah, I., Amin, N. and Hayat, T. (2010a). Numerical Simulation of Generalized Newtonian Blood Flow Past a Couple of Irregular Arterial. *Numerical Methods for Partial Differential Equations*. 27(4): 960–981.
- Mustapha, N., Mandal, P. K., Johnston, P. R. and Amin, N. (2010b). A numerical Simulation of Unsteady Blood Flow through Multi-Irregular Arterial Stenoses. *Applied Mathematical Modelling*. 34(6): 1559–1573.
- Nadeem, S. and Akbar, N. S. (2010). Simulation of the Second Grade Fluid Model for Blood Flow through a Tapered Artery with a Stenosis. *Chinese Physics Letters*. 27(6): 1–4.
- Nadeem, S., Akbar, N. S., Hayat, T. and Hendi, A. A. (2012). Influence of Heat and Mass Transfer on Newtonian Biomagnetic Fluid of Blood Flow through a Tapered Porous Arteries with a Stenosis. *Transport in Porous Media*. 91(1): 81–100.
- Nandakumar, N. and Anand, M. (2016). Pulsatile Flow of Blood through a 2D Double-Stenosed Channel: Effect of Stenosis and Pulsatility on Wall Shear Stress. *International Journal of Advances in Engineering Sciences and Applied Mathematics*. 8(1): 61–69.

- Nichols, W., O'Rourke, M. and Vlachopoulos, C. (1998). *McDonald's Blood Flow in Arteries, Sixth Edition: Theoretical, Experimental and Clinical Principles*. CRC Press.
- Noreen, S., Kausar, T., Tripathi, D., Ain, Q. U. and Lu, D. (2020). Heat Transfer Analysis on Creeping flow Carreau Fluid Driven by Peristaltic Pumping in an Inclined Asymmetric Channel. *Thermal Science and Engineering Progress*. 17: 100486.
- Ougrinovskaia, A., Thompson, R. S. and Myerscough, M. R. (2010). An ODE Model of Early Stages of Atherosclerosis: Mechanisms of the Inflammatory Response. *Bulletin of Mathematical Biology*. 72(6): 1534–1561.
- Papanastasiou, T., Georgiou, G. and Alexandrou, A. N. (1999). *Viscous Fluid Flow*. CRC press.
- Patankar, S. (1980). *Numerical Heat Transfer and Fluid Flow*. New York: CRC Press.
- Paul, M. C. and Molla, M. M. (2012). Investigation of Physiological Pulsatile Flow in a Model Arterial Stenosis Using Large-Eddy and Direct Numerical Simulations. *Applied Mathematical Modelling*. 36(9): 4393–4413.
- Paul, S. (2011). Effect of Wall Oscillation on Dispersion in Axi-symmetric Flows between Two Coaxial Cylinders. *Zeitschrift für Angewandte Mathematik und Mechanik*. 91(1): 23–37.
- Quast, S. and Kimberger, O. (2014). *The Significance of Core Temperature—Pathophysiology and Measurement Methods*. Dräger Medical GmbH: Lübeck, Germany.
- Rabby, M. G., Shupti, S. P. and Molla, M. M. (2014). Pulsatile Non-Newtonian Laminar Blood Flows through Arterial Double Stenoses. *Journal of Fluids*. 2014: 1–13.
- Ramana, B., Sarojamma, G., Vishali, B. and Nagarani, P. (2012). Dispersion of a Solute in a Herschel – Bulkley Fluid Flowing in a Conduit. *Journal of Experimental Sciences*. 3(2): 14–23.



- Ramesh, K., Gnaneswara Reddy, M. and Devakar, M. (2018). Biomechanical Study of Magnetohydrodynamic Prandtl Nanofluid in a Physiological Vessel with Thermal Radiation and Chemical Reaction. *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanomaterials, Nanoengineering and Nanosystems*. 232(4): 95–108.
- Rana, J. and Murthy, P. V. (2016a). Solute Dispersion in Pulsatile Casson Fluid Flow in a Tube with Wall Absorption. *Journal of Fluid Mechanics*. 793: 877–914.
- Rana, J. and Murthy, P. V. (2016b). Unsteady Solute Dispersion in Non-Newtonian Fluid Flow in a Tube with Wall Absorption. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 472(2193): 20160294.
- Ravi Kumar, S. (2016). Analysis of Heat Transfer on MHD Peristaltic Blood Flow with Porous Medium through Coaxial Vertical Tapered Asymmetric Channel with Radiation – Blood Flow Study. *International Journal of Bio-Science and Bio-Technology*. 8(2): 395–408.
- Riyi, L., Xiaoqian, W., Weidong, X., Xinfeng, J. and Zhiying, J. (2019). Experimental and Numerical Study on Forced Convection Heat Transport in Eccentric Annular Channels. *International Journal of Thermal Sciences*. 136: 60–69.
- Ross, R. (1999). Atherosclerosis—an Inflammatory Disease. *New England Journal of Medicine*. 340(2): 115–126.
- Sabbah, H. N. and Stein, P. D. (1982). Hemodynamics of Multiple Versus Single 50 Percent Coronary Arterial Stenoses. *The American Journal of Cardiology*. 50(2): 276–280.
- Sakai, J. B. (2009). Pharmacokinetics: The Absorption, Distribution, and Excretion of Drugs. *Practical Pharmacology for the Pharmacy Technician*. 27–40.
- Saleem, A., Akhtar, S., Nadeem, S., Issakhov, A. and Ghalambaz, M. (2020). Blood Flow through a Catheterized Artery Having a Mild Stenosis at the Wall with a Blood Clot at the Centre. *Computer Modeling in Engineering and Sciences*. 125(2): 565–577.

- Saltzman, W. M. (2001). *Drug Delivery: Engineering Principles for Drug Therapy*. Oxford University Press.
- Sankar, D. S. (2010). Perturbation Analysis for Blood Flow in Stenosed Arteries under Body Acceleration. *International Journal of Nonlinear Sciences and Numerical Simulation*. 11(8): 631–653.
- Sankar, D. S. (2016). Perturbation Analysis for Pulsatile Flow of Carreau Fluid through Tapered Stenotic Arteries. *International Journal of Biomathematics*. 9(4): 1–25.
- Sankar, D. S. and Hemalatha, K. (2007). A Non-Newtonian Fluid Flow Model for Blood Flow through a Catheterized Artery-Steady Flow. *Applied Mathematical Modelling*. 31(9): 1847–1864.
- Sankar, D. S., Lee, U., Nagar, A. K. and Morsidi, M. (2018). Mathematical Analysis of Carreau Fluid Model for Blood Flow in Tapered Constricted Arteries. *AIP Conference Proceedings*. 2016: 1–28.
- Sankarasubramanian, R. and Gill, W. N. (1973). Unsteady Convective Diffusion with Interphase Mass Transfer. *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences*. 333(1952): 115–132.
- Sarifuddin, Chakravarty, S. and Mandal, P. K. (2009a). Effect of Asymmetry and Roughness of Stenosis on Non-Newtonian Flow Past an Arterial Segment. *International Journal of Computational Methods*. 6(3): 361–388.
- Sarifuddin, Chakravarty, S. and Mandal, P. K. (2009b). Effect of Heat and Mass Transfer on Non-Newtonian Flow-Links to Atherosclerosis. *International Journal of Heat and Mass Transfer*. 52(25-26): 5719–5730.
- Sarifuddin, Chakravarty, S., Mandal, P. K. and Andersson, H. I. (2009c). Mass Transfer to Blood Flowing through Arterial Stenosis. *Zeitschrift fur Angewandte Mathematik und Physik*. 60(2): 299–323.
- Sarifuddin, Chakravarty, S., Mandal, P. K. and Layek, G. C. (2008). Numerical Simulation of Unsteady Generalized Newtonian Blood Flow through Differently Shaped Distensible Arterial Stenoses. *Journal of Medical Engineering and Technology*. 32(5): 385–399.

- Sarkar, A. and Jayaraman, G. (2004). The Effect of Wall Absorption on Dispersion in Oscillatory Flow in an Annulus: Application to a Catheterized Artery. *Acta Mechanica*. 172(3-4): 151–167.
- Sathyamurthy, P., Karki, K. C. and Patankar, S. V. (1992). Laminar, Fully Developed Mixed Convection in a Vertical Eccentric Annulus. *Numerical Heat Transfer; Part A: Applications*. 22(1): 71–85.
- Schneiderman, G., Ellis, C. G. and Goldstick, T. K. (1979). Mass Transport to Walls of Stenosed Arteries: Variation with Reynolds Number and Blood Flow Separation. *Journal of Biomechanics*. 12(11): 869–877.
- Shibeshi, S. S. and William, E. C. (2005). The Rheology of Blood Flow in a Branched Arterial System. *Applied Rheology*. 15(6): 398–405.
- Smits, A. J. (2000). *A Physical Introduction to Fluid Mechanics*. Wiley.
- Sonesson, B., Länne, T., Hansen, F. and Sandgren, T. (1994). Infrarenal Aortic Diameter in the Healthy Person. *European Journal of Vascular Surgery*. 8(1): 89–95.
- Tabakova, S., Kutev, N. and Radev, S. (2015). Application of the Carreau Viscosity Model to the Oscillatory Flow in Blood Vessels. *AIP Conference Proceedings*. 1690: 040019.
- Tabakova, S., Nikolova, E. and Radev, S. (2014). Carreau Model for Oscillatory Blood Flow in a Tube. *AIP Conference Proceedings*. 1629: 336–343.
- Talukder, N., Karayannacos, P. E., Nerem, R. M. and Vasko, J. S. (1977). An Experimental Study of the Fluid Dynamics of Multiple Noncritical Stenoses. *Journal of Biomechanical Engineering*. 99(2): 74–82.
- Tekleab, Y. and Harris, W. (2012). A Two-Dimensional Model of Blood Plasma Flow with Oxygen Transport and Blood Cell Membrane Deformation. *Proceedings of the Seventh International Conference on Computational Fluid Dynamics*. ICCFD7–3205.
- Thubrikar, M. J. (2007). *Vascular Mechanics and Pathology*. New York: Springer.

- Tu, C. and Deville, M. (1996). Pulsatile Flow of Non-Newtonian Fluids through Arterial Stenoses. *Journal of Biomechanics*. 29(7): 899–908.
- Tu, C., Deville, M., Dheur, L. and Vanderschuren, L. (1992). Finite Element Simulation of Pulsatile Flow through Arterial Stenosis. *Journal of Biomechanics*. 25(10): 1141–1152.
- Varshney, G., Katiyar, V. K. and Kumar, S. (2010a). Effect of Magnetic Field on the Blood Flow in Artery Having Multiple Stenosis: a Numerical Study. *International Journal of Engineering, Science and Technology*. 2(2): 67–82.
- Varshney, G., Katiyar, V. K. and Kumar, S. (2010b). Numerical Modeling of Pulsatile Flow of Blood through a Stenosed Tapered Artery under Periodic Body Acceleration. *Journal of Mechanics in Medicine and Biology*. 10(2): 251–272.
- Verma, V. K. and Saraswat, P. (2013). Effect of Multiple Stenosis on Blood Flow through a Tube. *International Journal of Biomedical and Biological Engineering*. 7(11): 753–756.
- Welch, J. E., Harlow, F. and Shannon, J.P. and Daly, B. (1966). *The MAC Method*. Los Alamos Scientific Lab. Report LA-3425, Los Alamos, New Mexico.
- Wriggers, P. and Lenarz, T. (2017). *Biomedical Technology: Modeling, Experiments and Simulation*. Springer.
- Young, D. F. (1968). Effect of a Time-Dependent Stenosis on Flow through a Tube. *Journal of Manufacturing Science and Engineering, Transactions of the ASME*. 90(2): 248–254.
- Young, D. F. and Frank, Y. T. (1973). Flow Characteristics in Models of Arterial Stenoses. I. Steady Flow. *Journal of Biomechanics*. 6(4): 395–410.
- Zaman, A., Ali, N. and Anwar Bég, O. (2016a). Numerical Simulation of Unsteady Micropolar Hemodynamics in a Tapered Catheterized Artery with a Combination of Stenosis and Aneurysm. *Medical and Biological Engineering and Computing*. 54(9): 1423–1436.

Zaman, A., Ali, N. and Sajid, M. (2016b). Slip Effects on Unsteady Non-Newtonian Blood Flow through an Inclined Catheterized Overlapping Stenotic Artery. *AIP Advances*. 6(1): 1–11.

Zendehbudi, G. R. and Moayeri, M. S. (1999). Comparison of Physiological and Simple Pulsatile Flows through Stenosed Arteries. *Journal of Biomechanics*. 32(9): 959–965.