

UNSTEADY MAGNETOHYDRODYNAMIC FLOW AND HEAT TRANSFER OF  
NANOFLUID AND HYBRID NANOFLUIDS OVER A VERTICAL CONE  
EMBEDDED IN A POROUS MEDIUM

HANIFA

UNIVERSITI TEKNOLOGI MALAYSIA

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## ABSTRACT

Nanotechnology has played a significant role in multi-fields of heat transfer processes and has made impressive advances in energy applications. This technology has significantly developed the science of thermal energy by improving various properties of energy transmitting fluids. This includes the development of nanofluid that can provide high heat transfer rates in a thermal energy system. A new class of nanofluid is known as hybrid nanofluid. Hybrid nanofluid has better chemical and mechanical strength, excellent thermal and electrical conductivity, lower cost, high heat transfer rates, and reliable physio-chemical properties. Bearing in mind such interesting features of nanofluid, the predominant idea of this thesis is to investigate heat transfer in the boundary layer flow of unsteady viscous nanofluids and hybrid nanofluid. Specifically, the water based nanofluids and hybrid nanofluid flow along a vertical cone enclosed in a porous medium is considered. The effects of external magnetic field and thermal radiation are additional features to the innovation of the constructed mathematical model. The system of nonlinear coupled equation supported by related initial and boundary conditions are solved numerically by using finite difference method. In the analysis, the impact of various physical parameters are scrutinized and the results are exhibited graphically. The physical quantities of wall shear stress and heat transfer coefficient versus governing constraints are evaluated and their results are summarized in the form of tables. The heat transfer performance of hybrid nanofluid is compared with the performance of nanofluid. The results show that the thermal performance of the system increases in the presence of magnetic field and thermal radiation. In addition, high heat transfer rates are observed when the flow is induced by varied heat flux as compared to varied wall temperature. Moreover, the viscosity is also responsible to enhance the heat transfer rates of the fluids. This research contributes to a better understanding on the effects of magnetohydrodynamic in mixed convection for radiative hybrid nanofluid flow.

## ABSTRAK

Nanoteknologi telah memainkan peranan yang signifikan dalam pelbagai bidang proses pemindahan haba dan telah mencapai kemajuan yang impresif dalam penggunaan tenaga. Teknologi ini telah mengembangkan sains tenaga termal dengan menambahbaik pelbagai sifat penghantaran tenaga bendalir. Ini termasuk perkembangan dalam nanobendalir yang dapat, menyediakan kadar pemindahan haba yang tinggi dalam sistem tenaga termal. Satu kelas nanobendalir yang baharu dikenali sebagai nanobendalir hibrid. Nanobendalir hibrid mempunyai kekuatan kimia dan mekanikal yang lebih baik, kekonduksian termal dan elektrik yang sangat baik, kos yang lebih rendah, kadar pemindahan haba yang tinggi, dan sifat fisiokimia yang boleh dipercayai. Dengan mengambil kira ciri menarik dalam nanobendalir, idea utama tesis ini adalah untuk menyiasat pemindahan haba dalam aliran lapisan sempadan nanobendalir likat tidak stabil dan nanobendalir hibrid. Khususnya, nanobendalir berasaskan air dan nanobendalir hibrid yang mengalir di sepanjang kon menegak yang tertutup dalam poros medium telah dipertimbangkan. Kesan medan magnet luaran dan sinaran termal adalah ciri tambahan kepada inovasi model matematik yang dibina. Sistem persamaan gandingan tak linear yang disokong oleh syarat awal dan syarat sempadan yang berkaitan telah diselesaikan secara berangka dengan menggunakan kaedah beza terhingga. Dalam analisis ini, kesan pelbagai parameter fizikal telah diteliti dan keputusan ditunjukkan secara grafik. Kuantiti fizikal bagi tegasan ricih dinding dan pekali pemindahan haba melawan kekangan tertakluk telah dinilai dan keputusannya diringkaskan dalam bentuk jadual. Prestasi pemindahan haba bagi nanobendalir hibrid telah dibandingkan dengan prestasi nanobendalir. Keputusan menunjukkan bahawa prestasi termal bagi sistem meningkat dengan kehadiran medan magnet dan sinaran termal. Tambahan lagi, kadar pemindahan haba yang tinggi telah diperhatikan ketika aliran disebabkan oleh perubahan fluks haba berbanding dengan perubahan suhu dinding. Selain itu, kelikatan juga berperanan dalam meningkatkan kadar pemindahan haba cecair. Kajian ini menyumbang kepada pemahaman yang lebih baik ke atas kesan hidromagnetik magnet dalam olakan campuran untuk aliran nanobendalir hibrid sinaran.

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## LIST OF ABBREVIATIONS

BTCS	-	Backward-time central-space
FTCS	-	Forward-time central-space
MHD	-	Magnetohydrodynamic
MWCNTs	-	Multi wall carbon nanotubes
ODEs	-	Ordinary differential equations
PDEs	-	Partial differential equations
PHF	-	Prescribed heat flux
PWT	-	Prescribed wall temperature
SWCNTs	-	Single wall carbon nanotubes

## LIST OF SYMBOLS

### Roman Letters

$a_0$	–	Nanoparticles shape constant
Ag	–	Gold
$Al_2O_3$	–	Aluminum oxide
Au	–	Silver
$b_0$	–	Nanoparticles shape constant
$B$	–	Strength of magnetic field ( $T$ )
Be	–	Bejan number
Br	–	Brinkman number
$B_0$	–	Strength of applied magnetic field ( $T$ )
$B_1$	–	Strength of induced magnetic field ( $T$ )
$c$	–	Arbitrary constant
CdTe	–	Cadmium telluride
$C_p$	–	Heat capacity at constant pressure ( $Jkg^{-1}K^{-1}$ )
Cu	–	Copper
$F$	–	Arbitrary function
$Fe_3O_4$	–	Iron oxide
FFI	–	Fluid friction
$g$	–	Acceleration due to gravity ( $ms^{-2}$ )
$Gr$	–	Grashof number
HFI	–	Entropy generated by heat transfer
$k$	–	Thermal conductivity ( $Wm^{-1}K^{-1}$ )
$K$	–	Non-dimensional porosity parameter

$k_b$	–	Absorption coefficient
$k_0$	–	Permeability of porous medium ( $m^2$ )
$l$	–	Constant
$L$	–	Reference length of a cone ( $m$ )
$L_x$	–	Maximum value of $x$
$L_y$	–	Maximum value of $y$ compatible with $y \rightarrow \infty$
$m$	–	Shape factor of nanoparticles
$M$	–	Non-dimensional magnetic parameter
$n$	–	Index power
$Nu_x$	–	Nusselt number
$N_1$	–	Irreversibility due to heat transfer
$N_2$	–	Irreversibility due to fluid friction
$p_n$	–	Number of nodes in $x$ direction
$Pe$	–	Péclet number
$Pr$	–	Prandtl number
$q_n$	–	Number of nodes in $y$ direction
$q_r$	–	Radiative heat flux
$q_w$	–	Heat flux at the surface of the cone ( $Wm^{-2}$ )
$Q$	–	Non-dimensional heat generation/absorption parameter
$Q_0$	–	Heat generation/absorption parameter ( $Jm^{-3}K^{-1}s^{-1}$ )
$r$	–	Radius of the cone ( $m$ )
$Rd$	–	Non-dimensional thermal radiation
$Re$	–	Reynolds number
$S_{gen}$	–	Entropy generation ( $kgm^{-1}s^{-3}K^{-1}$ )
$S_{GEN}$	–	Non-dimensional entropy generation
$S_0$	–	Characteristic entropy generation ( $kgm^{-1}s^{-3}K^{-1}$ )

$t$	–	Time ( $s$ )
$t_n$	–	Maximum time
$T$	–	Temperature ( $K$ )
$T_w$	–	Temperature at the surface of the cone ( $K$ )
$T_\infty$	–	Ambient temperature ( $K$ )
$u$	–	Velocity component along $x$ axis ( $ms^{-1}$ )
$u_p$	–	Mass transfer velocity at the surface ( $ms^{-1}$ )
$u_0$	–	Free stream velocity ( $ms^{-1}$ )
$v$	–	Velocity component along $y$ axis ( $ms^{-1}$ )
$x$	–	Coordinate axis along the flow ( $m$ )
$y$	–	Coordinate axis normal to the flow ( $m$ )

### **Greek Letters**

$\alpha$	–	Half angle of the cone
$\beta$	–	Thermal expansion ( $K^{-1}$ )
$\chi$	–	Any arbitrary solution
$\Delta t$	–	Time step
$\Delta T$	–	Reference temperature ( $K$ )
$\Delta x$	–	Mesh size in $x$ direction
$\Delta y$	–	Mesh size in $y$ direction
$\gamma$	–	Viscosity variation parameter
$\mu$	–	Dynamic viscosity ( $kgm^{-1}s^{-1}$ )
$\nu$	–	Kinematic viscosity ( $m^2s^{-1}$ )
$\phi_1 - \phi_{12}$	–	Nanofluid constants
$\psi_0$	–	Sphericity of nanoparticles
$\rho$	–	Fluid density ( $kgm^{-3}$ )

$\sigma$	–	Electrical conductivity ( $S m^{-1}$ )
$\sigma_b$	–	Stefan Boltzman constant ( $W m^{-2} K^{-4}$ )
$\tau_x$	–	Non-dimensional wall shear stress
$\tau_w$	–	Wall shear stress ( $Pa$ )
$\varphi$	–	Nanoparticle volume fraction

### **Subscript/Superscript**

$f$	–	Base fluid
$hnf$	–	Hybrid nanofluid
$i$	–	Regularly spaced grid points in $x$ direction
$j$	–	Regularly spaced grid points in $y$ direction
$k$	–	Time level
$nf$	–	Nanofluid
$s_1, s_2$	–	Nanoparticles
*	–	Non-dimensional



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# CHAPTER 1

## INTRODUCTION

This chapter addresses the important research area that focuses on heat transfer enhancement using nanofluid and hybrid nanofluid along with some essential and relevant physical effects. It includes an overview of the research background, problem statement, research objectives, scope and significant of the research.

### 1.1 Research Background

Throughout the latest literature and engineering curricula, heat transfer has become not just an autonomous discipline but also an essential discipline at the interface with other crucial and older disciplines. For example, fluid mechanics is able to explain heat transport and other contentment due to the great strides made in modern convective heat transfer. On the other hand, thermodynamics is prepared to teach modeling, simulation and optimization of the natural energy system due to the great advances in transfer of heat. In general, heat transfer phenomena describes the heat flow (thermal energy) due to the difference in temperature. A few common examples of heat transfer in day to day life include heating and cooling system of buildings, water boiling, hot air rising, light bulb fire and sun warming.

There are three main mechanisms of heat transfer; thermal conduction, thermal convection, and thermal radiation. The definition of conduction refers to the molecular activity of the subject material or body. Conduction is the transfer of energy from higher-energy molecules to the less-energy molecules. Such molecular energies are directly linked to the temperature, which means the transfer of heat takes place from the higher temperature side to the lower temperature side. Then, it can be assumed that thermal energy is diffused.

Convection is the second form of heat transfer. Under this mode, a fluid under motion is the medium in which the heat transfer is performed. This motion

leads to the transfer of heat in the presence of a temperature gradient [1]. As in the same conduction scenario, energy diffusion is also present. There are two distinctive broad types of convective heat transfer, namely, natural convection and forced convection. Convection is called natural convection when the density change, arising from temperature variations within the fluid, induces fluid motion. In forced convection, an external agent such as a pump or blower induces the fluid movement. Sometime in forced convection scenario, natural convection also exists due to gravitational body forces. This mode of convection is termed as mixed convection, when natural convection and forced convection work together to transfer heat. In many engineering and industrial applications the mixed convective flow contributed significantly including solar central receivers exposed to wind currents, nuclear reactors cooled during emergency shutdown, electronic devices cooled by fans and heat exchangers placed in a low velocity environment [2].

In thermal radiation, energy transfers in the form of electromagnetic waves by mean of photon. In other words, radiation causes energy transfer by the emission of electromagnetic radiation. This implies radiation is everywhere as all matters absorb and emit electromagnetic radiation. The radiative heat transfer can not be overlooked when a high temperature is needed for final product preparation. Many processes occur at high temperature in engineering environments, and knowledge of the radiative heat transfer is very important for the design of the related devices such as gas turbine, nuclear power plant and several propulsion equipment from satellites, aircraft, missiles and space vehicles [3].

As a consequence of the global energy crisis, which is one of the most critical issues due to the significant and persistent rise in demand, the growing lack of energy resources and the high cost. Many researchers have accomplished to increase the capacity of thermal systems and to reduce the size and thus energy consumption levels. Enhancement of heat transfer corresponds to the application of basic principles of heat transfer techniques to increase the rate of heat removal or deposition on a surface. The heat transfer enhancement methods are used to improve the heat transfer without adversely impacting the overall understanding of the systems. It includes a broad variety of fields in which heat exchangers are used for such functions as refrigeration,

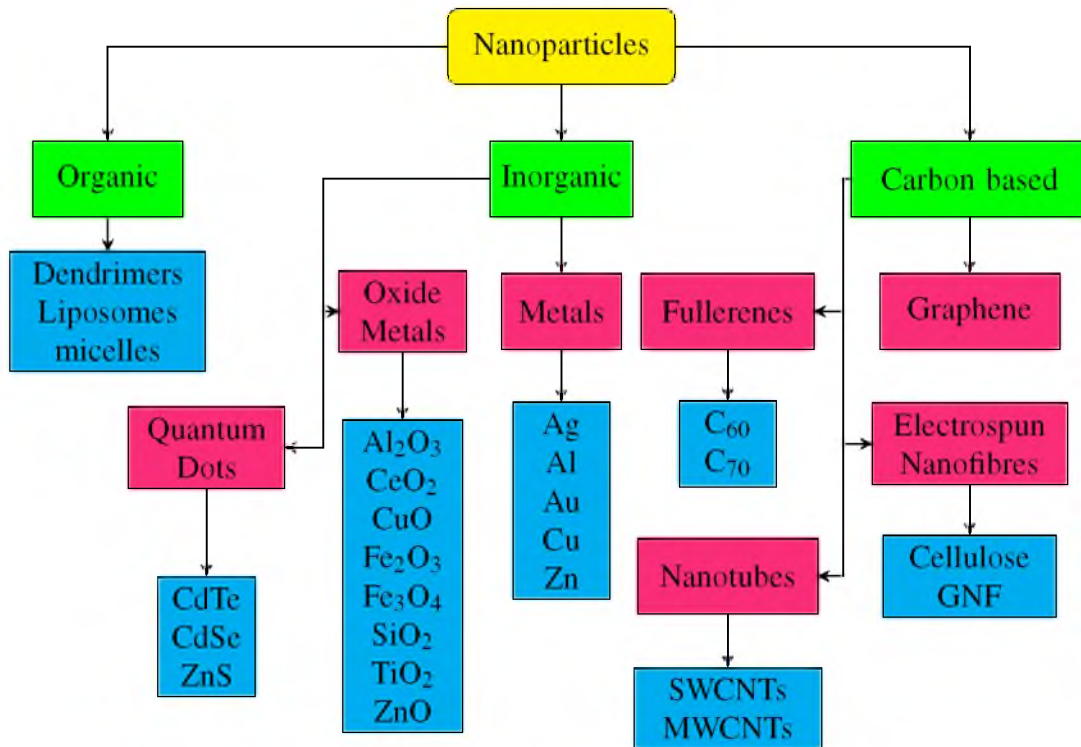


Figure 1.1: Classification of nanomaterials

air-conditioning, heating systems, automotive cooling equipment and other chemical industry applications.

There are many approaches in place to increase the efficiency of heat transfer. Some approaches include using extended surfaces, applying vibration to the heat transfer surfaces and using micro channels. Increasing the thermal conductivity of the working fluid can also increase heat transfer efficiency. In contrast to solid, widely used heat transfer fluids such as water, kerosene, ethylene glycol and engine oil have comparatively low thermal conductivity. Scientist and engineers have therefore recommended a new class of fluid termed as nanofluid, which are produced by adding small size nanoparticles/nanotubes in traditional single-phase liquids including water, kerosene, ethanol, ethylene glycol, etc. The presence of nano-scaled materials, even with a low volume concentration, plays a notable role in heat transfer (cooling and heating) processes. This beautiful discovery resulted from the experimental work of Choi and Eastman [4]. They declared that a small amount of nanoparticles can boost the efficiency of thermal system dramatically. Many researchers have been carried out on their footsteps which are significant for numerous industrial, environmental and clinical applications. The revolutionary features of nanofluids can be implemented practically in

solar energy field [5], cancer treatment [6], antibacterial and anti cancer activities [7], as a coolant in electrical and mechanical devices [8, 9], heat enhancement in transformer [10], nano-refrigerant boiling heat transfer enhancement[11], CO<sub>2</sub> absorption [12], plate heat exchangers [13]. Based on the morphology, size, shape, composition, physio-chemical properties, nanomaterials can be categorized into different groups, depicted in Figure 1.1.

Even though nanofluids help the engineers and scientists in improving the performance of thermal system but a better form of a fluid is still in quest until today. Since the availability and low cost of the nanoparticles are the most crucial preconditions for the practical applications of nanofluid in thermal energy systems. But, unfortunately, the availability of nanoparticles with high thermal conductivity like copper (Cu), silver (Ag) and gold (Au) is limited due to their high cost. Also, unmodified Cu, Ag and Au particles might be the source to exhibit toxicity risks [14]. On the other hand, oxide nanoparticles are mostly available nanoparticles due to low cost, but their thermal conductivity is lower than other nanomaterials and high volume concentration of oxide nanoparticles (> 5%) need to achieve the desired improvement in thermal conductivity. At low concentration, high performance with low cost remains the most important challenge in the field of nanofluid technology. In dealing with this, a new class of nanofluid, ' hybrid nanofluid ' has been introduced. These are the fluids containing two or more nanoparticles, and could be the most potential in terms of cost [15]. Moreover, hybrid nanofluids are capable to ameliorate the thermal performance of energy system because of synergistic effects [16]. The desired heat transfer efficiency can be attained even with small volume fraction of nanoparticles by hybridizing a suitable combination of nanoparticles. The current research therefore focuses primarily on the significant applications of hybrid nanofluids, which is authors' ultimate goal of improving heat transfer performance of nanofluid.

Besides on the characteristics of nanofluids, another important factor affecting the behavior of fluid is an applied magnetic field. The mutual interaction between magnetic field and moving fluid is dealt in magnetohydrodynamic (MHD), which is the physical-mathematical framework dealing with the dynamics of magnetic field in electrically conductive fluids. Hannes Alfvén pioneered the field of MHD in 1942, for

which he was awarded the Nobel Prize in physics in 1970. In the past few years, MHD has played an incredibly important role in the development and progress of industrial science and technology, for instance, in astrophysics processes, cooling of nuclear reactors, MHD power generators, MHD pumps, accelerators [17, 18]. In addition, MHD boundary layer flow with heat transfer of nanofluids is becoming a major topic of modern-day interest over the last few decades [19–21].

Moreover, porous media are ideal candidates for transport phenomena and heat-intensive applications including pollution control of soil, groundwater flow, production of crude oil, solar receivers, porous burners, catalytic chemical reactors, chemical reformers and heat exchangers [22–24]. It is well understood that the complicated pore structure is the unique property of a porous medium which distinguishes it from solid bodies. In addition to that, MHD flow inside porous media has provided a new interdisciplinary concept that can enhance and amend convective heat transfer. It has extensive applications in agricultural/mechanical engineering and petroleum industries science such as underground water resources, energy extractions, geothermal energy recovery, oil exploration, hydromagnetic generators [25, 26].

For physical applications, the geometry of a problem plays an important role. In particular, fluid flow over a cone has high demands in several real-life situations including aeronautical engineering, health care systems, energy conservation, astrophysics, space engineering and technology. In addition, canonical surfaces are often used in grinding, pumping, drilling, and degassing machines and so on in the chemical industry. Therefore, it is important not only to investigate these parameters, but also to get an idea of the fluid flow behavior when it especially flows along this type of geometry. Besides, the nature of flow and the participation of nanoparticles in heat transfer applications in diverse industrial chemical processes is also crucial to understand and predict. In reference to the aforementioned discussion, the heat transfer in the fluid, containing solo and hybrid nanoparticles, flow along a vertical cone is considered in current thesis. It is assumed that the cone is encapsulated in a porous media and a uniform magnetic field is induced normal to the surface of the cone. The effects of thermal radiation using Rosseland heat flux approximation and heat generation/absorption are the additional feature of the current

study. Moreover, convective heat transfer is considered in two different modes, natural and mixed convection, subject to different boundary conditions including constant wall temperature, variable wall temperature, constant heat flux and variable heat flux. To be able to fully understand the physics of the flow in easy way, the governing equations of proposed problems are nondimensionalized using appropriate set of non-dimensional variables. Thereafter, the non-dimensional, coupled, nonlinear partial differential equations (PDEs) are discretized by exploiting an implicit finite difference method, specifically, Crank Nicolson method [27]. This method was proposed by John Crank and Phyllis Nicolson in the mid 20th century for solving the heat type parabolic PDEs. It is one of the most reliable, convergent, second order accurate in space and time and unconditionally stable scheme [28–30]. After discretization, the algebraic difference equations are evaluated using Thomas algorithm with the aid of MATLAB software.

## **1.2 Problem Statement**

Heat transfer enhancement is a technique to improve the rate of heat removal or deposition on the surface [31]. It is a topic of interest to researchers as it contributes in both energy and cost savings. Nowadays, adding nanoparticles to traditional fluids is among the most efficient ways to improve heat transfer [32]. The presence of nano-scaled materials, even with a low volume concentration, plays a notable role in heat transfer (cooling and heating) processes. Besides nanomaterials, MHD is another important factor which plays a key role in intensifying the heat transfer capabilities of a thermal system. Nonetheless, placing porous materials in path of the fluid is one of the passive ways to improve heat transfer of thermal system. In the light of the above rationale, present research shows that the thermal performance of poor convectational fluid can be improved impressively with the help of solo and hybrid nanoparticles under existent magnetic field. In particular, this research will envision MHD boundary layer flow and convective heat transfer of fluids, suspended with solo/hybrid nanoparticles, past a vertical cone inside a porous medium. The mathematical heat transfer and fluid flow models are derived assuming single-phase flow. The proposed models are comprised of non linear coupled PDEs which ensures that the solutions do not exist in closed integral. Therefore, an implicit finite difference method, namely, Crank Nicolson method is proposed in order to find the numerical solutions.

The research might be incomplete without responding to the following questions: How do the existing steady, natural convective flow models can be modified to analyze unsteady, natural as well as mixed convection in nanofluid and hybrid nanofluid over a vertical cone through porous medium? How do mixed convective flow behave together with variable wall temperature, constant heat flux and variable heat flux boundary conditions? How do the presence of MHD together with porous medium affect the heat transfer characteristics of nanofluid and hybrid nanofluid? How do the Rosseland heat flux together with heat generation/absorption affect the heat transfer characteristics of nanofluid and hybrid nanofluid? How to develop a programming in MATLAB software to find the numerical solutions of the problems?

In this study, the proposed problems are as follow:

- (i) MHD flow and heat transfer of nanofluids with constant heating.
- (ii) MHD flow and heat transfer of nanofluids with variable heating.
- (iii) MHD flow and heat transfer of hybrid nanofluid with variable heating.
- (iv) MHD flow and heat transfer of hybrid nanofluid with variable heat flux.
- (v) MHD flow and heat transfer of hybrid nanofluid with variable viscosity and heat flux.

### **1.3 Research Objectives**

This research investigates unsteady, convectonal heat transfer in nanofluids and hybrid nanofluid past a vertical cone in a porous medium. The external magnetic field, thermal radiation and heat generation/absorption effects are also taken into account. The nonlinear coupled PDEs are discretized by Crank Nicolson method. Further, Thomas algorithm is implemented to get the numerical results with the help of MATLAB software. In addition, for certain special cases, computational results are compared with the results provided in previous published studies for validation purpose. Following are the objectives of present study:

- (i) To derive the mathematical models representing the unsteady two dimensional MHD flow of nanofluid and hybrid nanofluid over a vertical cone in presence of porous medium.



- (ii) To solve the proposed problems numerically using Crank Nicolson method combined with Thomas algorithm.
- (iii) To develop a programming in MATLAB software in order to obtain the solutions for proposed problems and also verify the results with the tolerance rate of  $10^{-5}$ .
- (iv) To investigate the effects of physical parameters including nanoparticle volume fraction, porosity, magnetic, radiation and heat generation/absorption on virtual flow profiles, i.e., velocity, temperature, wall shear stress and Nusselt number.

#### **1.4 Scope of Research**

Researchers have been paying attention to the utilization of nanofluid and hybrid nanofluid since they have disclosed their potentiality as a magnificent working fluid in thermal energy systems. Holding such interesting characteristics of nanofluid in mind, the main concern in the present research is to scrutinize the heat transfer characteristics of nanofluids along a vertical cone embedded in a porous medium. The governing equations, continuity, momentum and energy equations, are simplified with the aid of boundary layer and Boussinesq approximations. Thereafter, appropriate non-dimensional parameters are used to obtain the non-dimensional models. Moreover, solo (CdTe, Cu,  $\text{Fe}_3\text{O}_4$ ) and hybrid nanoparticles (Cu- $\text{Fe}_3\text{O}_4$ ) are used to analyze the heat transfer enhancement in convectional fluids. To the best of the authors' knowledge, CdTe nanoparticles are not used to enhance the heat transfer rates of electrically conducting fluids. Therefore, this research analyzes the heat transfer enhancement due to CdTe nanoparticles. On the other hand, magnetite ( $\text{Fe}_3\text{O}_4$ ) nanoparticles are capable to increase the heat transfer rates of thermal system in presence of magnetic field. However, the thermal conductivity of  $\text{Fe}_3\text{O}_4$  is low as compared to other metal nanoparticles. Therefore, highly conducting metal specifically, Copper (Cu) has been used to increase the thermal conductivity of  $\text{Fe}_3\text{O}_4$ -water nanofluid. The spherical shaped nanoparticles are suspended in water based nanofluid, except where specified. In addition, the behavior of magnetic field, porosity, thermal radiation and heat generation/absorption on fluid features, velocity, temperature, wall shear stress and Nusselt number, are also analyzed.

The solution of non-dimensional, nonlinear, coupled PDEs are obtained numerically by using Crank Nicolson method assisted by Thomas algorithm. The

MATLAB is used for the computation of numerical results as well as for plotting the graphs for visual display. The computational method is validated by comparing the results for certain cases with the numerical results of previously published papers. The theoretical framework for this thesis can be seen in Figure 1.2.

### **1.5 Significance of Research**

Energy is one of the vital resource in all over the world and global demand for energy is steadily rising. This increasing demand is partly due to population growth and economic development. Scientists are actively looking for new technologies and equipment to cope with this problem, which at the same time are highly thermally efficient. Many engineering systems including optical devices, high precision microelectronics, transport system, high power engine, synthetic chemical process, etc., experience a rise in thermal load. Traditional heat reduction methods seem insufficient to tackle this problem. Scientist and engineers have therefore recommended to use nanofluids instead of traditional fluids. The nanofluids are the traditional fluids suspended by nanomaterials. The presence of nano-scaled materials, even with a low volume concentration, plays a notable role in intensifying the heat transfer (cooling and heating) processes. Therefore, the current study is conducted to examine heat transfer enhancement in convectional nanofluid in presence of magnetic field and porous medium.

In addition, hybrid nanoparticles have capability to improve the thermal performance of convectional nanofluid at low cost. Bearing this in mind, hybrid nanoparticles are used to enhance the heat transfer in unsteady MHD fluid flow over a vertical cone in porous medium. Moreover, the effects of thermal radiation and heat generation/absorption are also taken into account. By doing this research, it is hoped that the outputs or results will contribute on better understanding on MHD effect in natural and mixed convection for radiative hybrid nanofluid flow. Furthermore, this study gives a clear vision that how hybrid nanofluid can enhance the thermal conductivity and heat transfer rates in transient time with various boundary conditions.

A numerical method is chosen to solve all the models, hence this thesis contributes in the development of numerical programming which can be helpful to

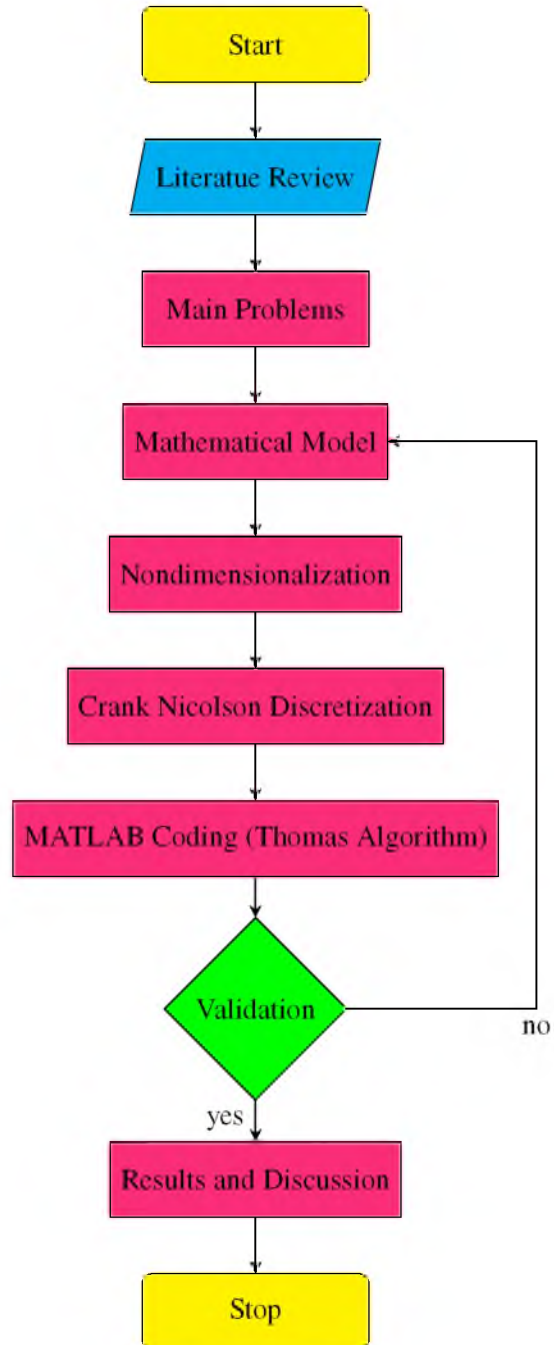


Figure 1.2: Research framework

solve the complex unsteady MHD fluid flow models. In addition, this study provides a clear picture of the simulations and analysis to other researchers.

## **1.6 Thesis Organization**

This thesis is divided into eight chapters dealing with heat transfer enhancement in electrically conducting fluid flow over a vertical cone inside a porous medium with solo/hybrid nanoparticles. Chapter 1 provides a brief overview of the research background together with problem statement, objectives, scope and significant of the research. In Chapter 2, a systematic overview of literature relating to proposed problems, as illustrated in problem statement, is discussed.

Chapter 3 is designed to examine the unsteady MHD flow and heat transfer of nanofluids over a vertical cone inside porous medium in presence of thermal radiation. This chapter is more focused on heat transfer behavior of nanofluid due to CdTe nanoparticles. However, the fluid flow features of CdTe-nanofluid are also compared with nanofluids containing copper, magnetite, gold and silver nanoparticles. The computational results for some limited cases are compared with peer reviewed literature, and an excellent agreement is found between the results. The unsteady MHD flow and heat transfer of nanofluids suspended with non-spherical CdTe nanoparticles past a vertical cone with variable wall temperature is considered in Chapter 4. The flow simulation is executed in presence of thermal radiation and heat generation/absorption. The Hamilton Crosser model of thermal conductivity is used for different shape of nanoparticles. In addition, for certain cases, the computational results are compared with the provided results of previous published studies for validation purpose. In this regard, a perfect correlation exist between peer reviewed and the current results.

In light of amazing features of hybrid nanofluid, Chapter 5 explores unsteady, MHD flow and mixed convective heat transfer of hybrid nanofluid over an inverted cone. The cone is surrounded by a porous medium and the flow influenced by an external magnetic field and variable wall temperature. In addition, the fluid is moving in upward direction with uniform velocity. The hybrid nanoparticles Cu-Fe<sub>3</sub>O<sub>4</sub> are utilized to increase the heat transfer rate in water based hybrid nanofluid. Furthermore, the entropy generation in hybrid nanofluid is also scrutinized. Chapter 6 is structured

to discuss the unsteady MHD flow and heat transfer of hybrid nanofluid flow over a vertical cone with variable wall temperature and heat flux. The effects of external magnetic field and thermal radiation in a porous medium are the additional features to the innovation of the constructed mathematical model. Moreover, the heat transfer enhancement by hybrid nanoparticles is also compared with that of solo nanoparticles. Next, Chapter 7 reveals the effects of variable viscosity on unsteady MHD flow and heat transfer of hybrid nanofluid over a vertical cone in presence of porous medium. The numerical analysis is accomplished in presence of radiative heat flux and heat generation/absorption. Finally, an overview of the thesis is presented in Chapter 8. This chapter also discusses the research contribution together with future findings. In Table 1.1, the summary of all considered problems along with considered effects are outlined.

Table 1.1: Summary of all problems with considered effects

Specifications	Problems				
	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5
Nanofluid	✓	✓	✓	✓	✓
Hybrid nanofluid	✗	✗	✓	✓	✓
Natural convection	✓	✓	✓	✗	✗
Mixed convection	✗	✗	✓	✓	✓
MHD	✓	✓	✓	✓	✓
Porosity	✓	✓	✓	✓	✓
Thermal radiation	✓	✓	✗	✓	✓
Heat generation/absorption	✗	✓	✗	✗	✓
Variable viscosity	✗	✗	✗	✗	✓
Variable wall temperature	✗	✓	✓	✓	✗
Heat flux	✗	✗	✗	✓	✓

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## LIST OF PUBLICATIONS

### Journal with Impact Factor

1. Hanifa Hanif, Ilyas Khan, and Sharidan Shafie. MHD natural convection in cadmium telluride nanofluid over a vertical cone embedded in a porous medium. *Physica Scripta* 94.12 (2019): 125208.
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