

ENHANCED UNIVARIATE AND MULTIVARIATE CONTROL CHARTS VIA
OUTLIERS' SCREENING TECHNIQUE, ROBUST ESTIMATORS AND
VARYING SAMPLE SIZE SCHEMES

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DEDICATION

I dedicate this research work to my amazing parents, supportive siblings, lovely wife and adorable children, for their unending-show of love, support, patience and prayers.

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ABSTRACT

The ability to monitor processes using control charts for contaminated environments is vital. Typical control charts may not serve the purpose because of violations in the underlined assumptions or the presence of outliers in such environments. Fixed sample size (FSS) based control charts are not only less efficient as compared to varying sample size (VSS) but are sometimes more expensive to administer. Therefore, this study has developed new control charts to improve the statistical process control for contaminated processes. The goals are to design univariate and multivariate control charts that are more sensitive, efficient, and robust in the presence of outliers and violation of the model's assumptions. The study enhances the Shewhart, the exponentially weighted moving average, and exponentially and homogeneously double-weighted moving average charts, with outliers' screening techniques to improve the sensitivity of the charts in the estimation and monitoring processes. Next, robust multivariate location estimators were applied to Hotelling T² and multivariate cumulative sum (MCUSUM) charts, to retain their efficiency when underlying assumptions are violated in contaminated process environments. In addition, this research proposes a new adaptive homogeneous weighted moving average features (HWMA) chart with VSS, for location monitoring. This study also employed Monte-Carlo simulations to evaluate the effectiveness of the proposed control charts, using the run length properties to measure the performance of the control charts. The results show that the enhanced control charts for outlier detection are more sensitive and efficient than their counterparts at detecting anomalies. The efficiency of the multivariate Shewhart and CUSUM charts is improved when the robust multivariate estimators were employed in contaminated settings. The results also indicate that the adaptive VSS-HWMA charts outperform their counterparts. In conclusion, the proposed control charts incorporating an outlier detection model and employing robust estimators could be used to monitor processes adequately for contaminated environments.

ABSTRAK

Keupayaan untuk memantau proses menggunakan carta kawalan untuk persekitaran yang tercemar sangat penting. Carta kawalan yang biasa mungkin tidak dapat memenuhi tujuan ini kerana pelanggaran andaian atau pencilan yang terdapat di persekitaran tersebut. Carta kawalan berasaskan saiz sampel yang tetap (FSS) bukan hanya kurang cekap berbanding dengan saiz sampel yang berbeza-beza (VSS) malah lebih mahal menguruskannya. Oleh itu, kajian ini telah membangunkan carta kawalan baru untuk meningkatkan kawalan proses statistik. dalam proses yang tercemar. Matlamatnya adalah untuk merekabentuk carta kawalan univariat dan multivariat yang lebih sensitif, cekap, dan mantap di mana terdapat pencilan dan pelanggaran terhadap andaian model. Kajian ini menambahbaik carta kawalan Shewhart, purata bergerak berwajaran secara eksponen dan purata bergerak berwajaran berganda secara exponent dan homogen, dengan teknik penyaringan pencilan untuk meningkatkan kepekaan carta dalam proses anggaran dan pemantauan. Seterusnya, penganggar lokasi multivariat yang kukuh digunakan kepada carta Hotelling T² dan jumlah terkumpul multivariat (MCUSUM), untuk mengekalkan kecekapan carta kawalan yang melanggar andaian model dalam keadaan yang tercemar. Sebagai tambahan, penyelidikan ini mencadangkan carta kawalan adaptif berwajaran homogen dengan ciri bergerak purata (HWMA) yang baharu dengan VSS, untuk pemantauan lokasi. Kajian ini juga menggunakan simulasi Monte-Carlo untuk menilai keberkesanan carta kawalan yang dicadangkan, menggunakan sifat panjang larian untuk mengukur prestasi carta kawalan. Hasil kajian menunjukkan bahawa carta kawalan yang dipertingkatkan untuk pengesanan pencilan lebih sensitif dan cekap daripada carta kawalan lain dalam mengesan anomali. Kecekapan carta Shewhart multivariat dan CUSUM ditingkatkan apabila penganggar multivariat yang kukuh digunakan dalam keadaan yang tercemar. Keputusan juga menunjukkan bahawa carta adaptif VSS-HWMA mengatasi carta kawalan lain. Kesimpulannya, carta kawalan yang dicadangkan yang menggabungkan model pengesanan pencilan dan menggunakan penganggar yang kukuh dapat digunakan untuk memantau proses dengan secukupnya untuk persekitaran yang tercemar.

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LIST OF ABBREVIATIONS

ARL	- Average Run Length
CL	- Centre Line
CSEWMA	- Combined Shewhart EWMA
CUSUM	- Cumulative Sum
DCUSUM	- Dual Cumulative Sum
DEWMA	- Double Exponentially Weighted Moving Average
DHWMA	- Dual Homogeneously Weighted Moving Average
DW	- Durbin-Watson
EQL	- Extra Quadratic Loss
EWMA	- Exponentially Weighted Moving Average
FSS	- Fixed Sample Size
FSS-EWMA	- Fixed Sample Size-EWMA
FSS-HWMA	- Fixed Sample Size-HWMA
HL	- Hodges-Lehman
HWMA	- Homogeneously Weighted Moving Average
IC	- In Control
KDE	- Kernel Density Estimation
LCL	- Lower Control Limit
LCNE	- Location Contaminated Normal Environment
LVCNE	- Location-Variance Contaminated Normal Environment
LWL	- Lower Warning Limit
MCD	- Minimum Covariance Determinant
MCE	- Mixed CUSUM EWMA
MCUSUM	- Multivariate Cumulative Sum
MEC	- Mixed EWMA CUSUM
MEWMA	- Multivariate Exponentially Weighted Moving Average
MGC	- Mixed Generally Weighted Moving Average Cumulative Sum
MHWMA	- Multivariate Homogeneously Weighted Moving Average
OoC	- Out of Control
RARL	- Relative Average Run Length

SDRL	- Standard Deviation Run Length
SPC	- Statistical Process Control
TM	- Trimean
UCL	- Upper Control Limit
UNE	- Uncontaminated Normal Environment
UWL	- Upper Warning Limit
VCNE	- Variance Contaminated Normal Environment
VSI	- Variable Sample Interval
VSS	- Variable Sample Size
VSS-EWMA	- Variable Sample Size-EWMA
VSS-HWMA	- Variable Sample Size-HWMA

LIST OF SYMBOLS

Y	-	Random variable of interest
μ	-	Mean
σ^2	-	Variance
μ_0	-	In control mean
σ_0^2	-	In control variance
n	-	Sample size
\bar{Y}_i	-	Sample Mean
LCL_{Ei}	-	EWMA lower control limit
UCL_{Ei}	-	EWMA upper control limit
m	-	Phase-I preliminary sample
S_l	-	Sample variance
c_4	-	Bias correction constant
$\hat{\mu}_0$	-	Estimated In-Control mean
$\hat{\sigma}_0$	-	Estimated In-Control standard deviation
δ	-	Shift
$N(\mu, \sigma^2)$	-	Normal distribution with mean and variance
$\chi_{(v)}^2$	-	Chi-square distribution with v degrees of freedom
α	-	Percentage of outliers
ω	-	Magnitude of outliers
\tilde{Y}	-	Sample median
IQR	-	Interquartile range
Q_3	-	Third quartile
Q_1	-	First quartile
p	-	Tukey's confidence factor
b	-	MAD confidence factor
W_i	-	DEWMA statistic
LCL_W	-	DEWMA lower control limit
UCL_W	-	DEWMA upper control limit
H_i	-	HWMA statistic

DH_i	-	DHWMA statistic
LCL_{DH_i}	-	DHWMA lower control limit
UCL_{DH_i}	-	DHWMA upper control limit
ARL_0	-	In control average run length
$SDRL_0$	-	In control standard deviation run length
ARL_1	-	Out of control average run length
$SDRL_1$	-	Out of control standard deviation run length
Y	-	A vector of variable of interest
μ'	-	Mean vector
Σ	-	Variance-covariance matrix
T_i^2	-	Hotelling statistic
$F_{p,n-p}$	-	F-distribution with p and $n - p$ degrees of freedom
UCL	-	Upper control limit
MC_i	-	MCUSUM statistic
\underline{Y}	-	Sample mean vector
\tilde{Y}	-	Sample median vector
HL	-	Hodges-Lehman vector
H_i^z	-	VSS-HWMA statistic
LWL_i	-	HWMA lower warning limit
UWL_i	-	HWMA upper warning limit

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CHAPTER 1

INTRODUCTION

1.1 Background of the Study

In statistical process control (SPC), the control chart is the most powerful and widely used tool of quality control. Its applicability in almost all sectors and different types of processes distinguishes it from its other six counterparts namely: scatter diagram, check sheets, Pareto diagram, cause and effect diagram, flowcharts, and histograms. Control chart monitors and controls processes (in different sectors: industrial, health, production, refineries, manufacturing, etc.) from any unwanted changes that might affect the process targets and specifications. The underlying assumption of control charts is the normality of the process under study. The assumptions dictate the process be moderately normal, with little deviation from the process location and dispersion. When these assumptions are not met, or there is severe deviation from the process targets, the efficiency and performance of the charts are brought to questioning. Presence of outliers in the preliminary samples employed for estimating the process parameters also affect the chart's performance, negatively.

Processes are expected to change time in and time out. These changes (causes of variation) are of two types: natural changes (random causes of variation) and unnatural changes (assignable causes of variation). The former, as the name suggests, occurs naturally, is not harmful and is inevitable in any process. While the latter displaces the process from its target, harmful, and should be corrected. Control charts monitor and control these assignable causes of variation in processes. Depending on the magnitude of the variations they monitor, control charts are classified into two (2) categories: the memory-less charts and the memory charts. The memory-less charts are suitable and appropriate for monitoring large magnitudes of variations in

processes. They are called memory-less because they use only the present value of the process in their structures. An example of the memory-less chart is the Shewhart control chart (Shewhart, 1931) and all other charts emerging from modifying the Shewhart chart. On the other hand, the memory charts, as the name implies, keep and use the past information of the process in addition to its present value in constructing the charts statistics. Such charts are efficient for monitoring small and moderate variations. The major examples of the memory charts are exponentially weighted moving average (EWMA) (Roberts, 1958), and cumulative sum (CUSUM) (Page, 1954), and the modifications and improvements of the EWMA and CUSUM charts..

Depending on the number of variables they monitor, control charts are further classified into univariate and multivariate charts. The univariate charts monitor only one variable of interest while the multivariate charts monitor three or more variables of interest with a single charting structure. Examples of the former are Shewhart, EWMA, and CUSUM. While their extensions to multivariate are the Hotelling T^2 (Hotelling, 1947), multivariate EWMA (MEWMA) (Sharad S Prabhu and Runger, 1997), multivariate CUSUM (MCUSUM) (Crosier, 1988) charts, respectively. Furthermore, some monitored variable of interest comes in individual observation while others are with subgroups (i.e. the sample size is more than 1). Examples include Shewhart chart with rational subgroups by (Nelson, 1988), EWMA chart with estimated parameters by (Jones, 2018) and Robust CUSUM control charting (Zafar Nazir *et al.*, 2013). In the case of subgroups, the chart uses an appropriate estimate from the sample size in place of the individual observation.

Another dimension to classifying control charts is the type of sample size the chart adopts. Traditional control charts are designed with a fixed sample size (FSS) such that fixed sample size is chosen for all observations in the process being monitored (Shewhart, 1931; Page, 1954; Roberts, 1958). On the other hand, adaptive charting schemes suggest varying sample size (VSS) in the charts structures to improve the capability of such charts (Costa, 1994, 1999; Reynolds and Arnold, 2001; Amiri *et al.*, 2014; Muhammad *et al.*, 2018; Chong *et al.*, 2019). Unlike the FSS charts, the VSS charts change the sample size of the next observation in the process based on conditions set for the current chart statistics. In addition to the usual

control limits, VSS charts include warning limits to their design. The VSS charts start with a smaller sample size for monitoring, if the statistics plot beyond the warning limits, larger sample size is sought to get more insight into the process. This process continues until the statistics plot is within the warning limit, and then a smaller sample is sought for the next observation. Furthermore, control charts are classified by the parameters they monitor in processes. On this basis, there are two major classifications. The location process monitoring and the dispersion process monitoring. Like this thesis, many charts are designed to monitor the location parameter, some monitor the dispersion parameter, and some charts monitor both the location and dispersion parameters simultaneously in a single chart.

All the categories earlier mentioned; memory and memory-less, univariate and multivariate, FSS and VSS charts are constructed in two stages; the retrospective stage (phase-I) and the prospective stage (phase-II). In phase-I, the process specifications are set. These specifications are the control limits. The control limits; the lower control limit (LCL), center line (CL), and the upper control limit (UCL), are structured based on the process parameters. A chart is regarded to be in-control (IC) in as much it plots within the lower and upper control limits. The moment the chart plots beyond the LCL and UCL, it is declared out-of-control (OoC). In the absence of these parameters, they are estimated from some preliminary samples in phase-I, afterward, the control limits are constructed. Having set the controls limits, and the process is running with the specified limits, then comes the function of the prospective stage. In phase-II, the charts monitor and report any assignable cause of variation that might distort the process to practitioners for immediate intervention.

1.2 Problem Statement

Control charts should not only identify the unnatural causes of variation in a process but also potential outliers present in the preliminary samples employed for estimating the process parameters. Furthermore, control charts should be robust enough to retain their efficiencies and sensitivities in the presence of contamination and violation of the underlying assumptions. In reality, the normality assumptions of

control charts are often violated, the parameter estimators are not sensitive and robust to detect outliers in the preliminary samples, leading to the poor performance of such control charts. The problems of this research are: i) screening outliers off the preliminary samples employed for estimating the unknown parameters in univariate control charts, ii) monitoring multivariate processes under contaminated environments, iii) employing adaptive control charting schemes for location monitoring.

1.3 Research Questions

Motivated by the problem statement, the following questions will be addressed in this research work.

- (a) How to develop outliers' detection-based univariate charts for process monitoring, both memory and memory-less?
- (b) How to design robust multivariate control charts for location monitoring in contaminated environments, for both memory and memory-less charts?
- (c) How to design an adaptive control charting scheme for location monitoring location with a homogenously weighted moving average (HWMA) chart?
- (d) How can the proposed outliers' detection-based, robust multivariate and adaptive control charts be applied in real life?

1.4 Research Objectives

This research aims at achieving the following objectives:

- (a) Develop outliers' detection based univariate (single and doubled) control charts, for both memory and memory-less types, that are more sensitive and efficient
- (b) Develop robust multivariate Shewhart and cumulative sum control charts for monitoring contaminated environments.
- (c) Design adaptive homogeneously weighted moving average chart with varying sample size for location monitoring.
- (d) Apply all proposed charting structures to real-life data set extracted from the glass, semiconductor, and carbon fibre manufacturing industries.

1.5 Significance of the Study

Developing outliers' detection-based control charts is very essential in SPC. This is because outliers have a major influence in any statistical analysis, as they increase the error variance, reduce the power of statistical tests and cause bias estimates, leading to wrong inferences and conclusions. Hence, the proposed outliers' detection-based charting schemes will enhance the charts' efficiency and sensitivity. In addendum, the proposed robust multivariate control charts for contaminated processes will restore the efficiency and performance of the charts as much as when the processes were not contaminated. This is significant as many of the real-life scenarios do not comply with the underlying assumptions of the charts. Furthermore, the proposed adaptive charting schemes will enhance the sensitivity of the HWMA chart for location monitoring. With varying sample sizes, practitioners from different sectors can flexibly monitor the process with either small or large samples. This feature will give a better performance and economical gain over the traditional HWMA chart with a fixed sample size. Finally, the application of all proposed charts to real-life data set extracted from different sectors plays a vital role for SPC practitioners.

1.6 Scope of the Study

The proposed control charts in this research focus mainly on monitoring location parameters in processes. The scope of the proposed charts is limited to monitoring the location parameter in univariate and multivariate set up, memory- and memory-less charts. As regards the theoretical aspect, the necessary mathematical theories, and derivation leading to the design of the proposed control charts are presented, in addition to the performance measures of the charts. While the computational aspects entail the Monte-Carlo simulation procedures to compute the performance measures described in the theoretical aspect. These are achieved by developing some algorithms in R programming language. The computational aspects also include the presentation of results, comparison of the proposed chart with the existing ones in the literature, and graphical depiction of the results. However, the practical aspects involve applying the procedures of the proposed charts on real-life data set extracted from different sectors such as health, semiconductor manufacturing, and petrochemical refineries.

Although, this research studies some contaminated environments, the study is limited to monitoring the location parameters under normal environments. While switching from univariate to multivariate, memory and memory-less charts, some of these are not covered in the study, as they are already available in the literature.

1.7 Structure of the Thesis

The thesis consists of six chapters, A synopsis of each chapter is set out below:

Chapter 1 contains the general introduction of the thesis. This entails the background of the study, problem statement, research questions, research objectives, significance, scope and limitation of the study, and the thesis structures.

Chapter 2 gives an extensive literature review of SPC relating to the scope of the study. It reviews the pioneering and modified works on univariate control charting schemes, the Shewhart, EWMA, and CUSUM. Chapter 2 also reviews the multivariate control chart schemes for process monitoring, both the memory and memory-less charts.

Chapter 3 focuses on the methodologies of the existing and the proposed control charts as related to the realization of each of the thesis's objectives. This entails the univariate Shewhart and EWMA charts, double EWMA and HWMA charts, multivariate Shewhart and CUSUM charts, and the HWMA charts for monitoring the location and coefficient of variation parameters. Chapter 3 also explains the design algorithm of the proposed charts adopted to measure the charts' performances.

The results and findings of the proposed control charts are depicted in chapter 4. Chapter 4 discusses the implication of these results and compares the proposed charts' performance with their counterparts in the literature, numerically and graphically.

Chapter 5 demonstrates the application of each of the proposed charts on real-life data set extracted from the glass manufacturing industry, carbon fibre industry, and semiconductor manufacturing industry (photolithography).

Finally, chapter 6 concludes the study by summarizing the main contributions of the thesis and suggests future research directions and recommendations.

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LIST OF PUBLICATIONS

Journal Publications (ISI):

- Raji, I. A., Lee, M. H., Riaz, M., Abujiya, M. A. R., & Abbas, N. (2020). Outliers detection models in Shewhart control charts; An application in photolithography: A semiconductor manufacturing industry. *Mathematics*, 8(5), 857. ISSN: 2227-7390 2020 JCR Impact Factor: 2.258, Quartile: Q1
- Raji, I. A., Lee, M. H., Riaz, M., Abujiya, M. A. R., & Abbas, N. (2021). A robust multivariate Shewhart chart for contaminated normal environments. *Quality and Reliability Engineering International*, 37(6), 2665 – 2684. ISSN: 0748 – 8017. 2021 JCR Impact Factor: 2.885, Quartile: Q2
- Raji, I. A., Riaz, M., Abujiya, M. A. R., Abbas, N., & Lee, M. H. (2021) Outliers' detection with a sensitive exponentially weighted moving average control chart. *Quality and Reliability Engineering International*. JCR Impact Factor: 2.885, Quartile: Q2