MATHEMATICAL MODELLING AND PARAMETER STUDIES OF TORSIONAL SURFACE WAVE

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DEDICATION

This thesis is dedicated to my father, Teng Yin Chee, who taught me that no pain, no gain, and success belongs to those who make their best efforts. It is also dedicated to my mother, Ng Poh Lian, who taught me that a small effort a day may bring great rewards in the furure.

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ABSTRACT

In seismological studies, torsional surface wave is important in predicting earthquake. Torsional surface wave is restricted to propagate in the upper crustal earth, and it can only exist in the solid materials. The studies of torsional surface waves are limited in the public domain as compared to Rayleigh, Love, and Stoneley waves, although the research of torsional surface wave in the elastic medium is very impactful in seismology. The focus of this thesis is to determine the effects of sandy level, gravity field, homogeneous and initial stress parameters on the torsional surface waves propagation in stress-free and rigid surfaces. The derivative of the phase velocity of torsional surface waves is derived by using dynamical equations of motion in a cylindrical coordinate system with displacement characteristic, where $u = 0\,; w = 0\,; v = v(r, z, t)$. The solution is found through the separation of variable method in the form of expanded linear Whittaker function. Hence, the phase velocity of torsional surface wave can be obtained. The graphical analysis of the solution indicates the relationship between phase velocity of torsional surface wave with sandy level, gravity field, initial stress, homogeneous and rigidity parameters respectively. The results show that in the presence of the gravity field, torsional surface waves can propagate in elastic and sandy media. However, in the absence of gravity field, the torsional surface waves cannot propagate in the sandy medium but can propagate in elastic medium. In addition, homogenous, initial stress and rigidity parameters also play important roles in influencing the phase velocity of the torsional surface waves.

ABSTRAK

Dalam kajian seismologi, gelombang permukaan kilasan adalah penting dalam meramalkan gempa bumi. Gelombang permukaan kilasan terhad untuk penyebaran di bahagian atas kerak bumi, dan ia hanya boleh wujud dalam bahan pepejal. Kajian gelombang permukaan kilasan adalah terhad dalam domain awam berbanding gelombang Rayleigh, Love dan Stoneley, walaupun kajian gelombang permukaan kilasan dalam media elastik sangat memberi kesan dalam kajian seismologi. Fokus tesis ini adalah untuk menentukan kesan parameter aras berpasir, medan graviti, homogen dan tekanan awalan bagi penyebaran gelombang permukaan kilasan pada permukaan bebas tekanan dan permukaan tegar. Derivatif halaju fasa gelombang permukaan kilasan diterbitkan dengan menggunakan persamaan gerakan dinamik dalam satu sistem koordinat silinder dengan sifat sesaran, $u = 0\,; w = 0\,; v = v(r, z, t)$. Penyelesaiannya diperolehi melalui kaedah pemisahan pembolehubah dalam bentuk pengembangan linear fungsi Whittaker. Oleh itu, halaju fasa gelombang permukaan kilasan dapat dihasilkan. Analisis grafik penyelesaian menunjukkan hubungan antara halaju fasa gelombang permukaan kilasan masing-masing dengan parameter aras berpasir, medan graviti, tekanan awalan, homogen dan ketegaran. Hasil kajian menunjukkan bahawa dengan kewujudan medan graviti membolehkan gelombang permukaan kilasan tersebar dalam medium elastik dan berpasir. Walau bagaimanapun, ketiadaan medan graviti, gelombang permukaan kilasan tidak boleh tersebar dalam medium berpasir tetapi boleh tersebar dalam medium elastik. Tambahan lagi, parameter homogen, tekanan awalan dan ketegaran memainkan peranan penting dalam mempengaruhi kelajuan fasa gelombang permukaan kilasan.

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LIST OF ABBREVIATIONS

P wave	-	Primary Wave (Compressive Wave)
S wave	-	Secondary Wave (Shear Wave)
SH wave	-	Horizontal Shear Wave
SV wave	-	Vertical Shear Wave

LIST OF SYMBOLS

C_{ijkl}	-	Elastic Constants
C_{ij}, C_{ji}	-	Elastic Constants with Symmetry Stress and Strain
σ_{ij}	-	Stress Components
e_{kl}	-	Strain Components
λ, μ	-	Lame's Parameter
u_i, u_j	-	Wave Displacement Vector
F_i	-	Body Force
F_r, F_θ, F_z	-	Body Force in r, θ, z direction
δ_{ij}	-	Kronecker Delta
e_{kk}, Ω	-	Volume Dilatation
μ	-	Shear Modulus (Rigidity)
∇	-	Gradient
$\nabla \cdot$	-	Divergence
∇^2	-	Laplacian
ω	-	Angular Velocity
t	-	Time
i	-	Imaginary Number
$M_{\lambda,\mu}(z)$	-	Whittaker Function of the First Kind
$W_{\lambda,\mu}(z)$	-	Whittaker Function of the Second Kind
r, θ, z	-	Direction of Wave Propagation
c	-	Phase Velocity
$\sigma_{rr}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{r\theta}, \sigma_{\theta\theta}, \sigma_{\theta z}, \sigma_{zz}$	-	Stress Components due to respective direction
g	-	Acceleration due to Gravity
k	-	Wave Numbers
$e^{i\omega t}$	-	Plane Wave Equation

$\frac{P}{2\mu_1}$	-	Initial Stress (Compressive Initial Stress in negative)
$\frac{a}{k}$	-	Heterogeneity Parameter
η	-	Sandy Parameter
$\frac{C}{C_1}$	-	Phase Velocity of Torsional Surface Wave
G	-	Biot's Gravity Parameter
ρ_1, μ_1, P_1	-	Parameter Constants
ρ	-	Density
$e_{rr}, e_{\theta\theta}$,	-	Strain Components due to respective direction
e_{zz} , $e_{r\theta}$		
e_{rz} , $e_{\theta z}$		
u, v, w	-	Displacement Components due to r,θ, z direction
a	-	Modulus of rigidity
$V(z)$	-	Displacement along z direction
$J_1(kr)$	-	First order Bessel Function (Jacobian Equation)

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

Research on seismic wave is significant because it can help us understand the structure of the earth and predict earthquake and tsunami (Kanao, 2011). A seismic wave is a wave generated when an earthquake is happening, and it can be recorded by a seismograph (William et al., 2002). Seismic wave can be divided into two phrases, which is the body wave with higher velocity and the surface wave with lower velocity (Stein and Wysession, 2009). Body wave can be divided into Primary wave (also known as Compressive wave or P wave) and Secondary wave (also known as Shear wave or S wave), which can be further divided into horizontally polarized shear wave (SH wave) and vertically polarized shear wave (SV wave). Likewise, surface wave, a seismic wave that travels along or parallel to the earth's surface, can be divided into Rayleigh wave, Love wave, Stoneley wave, and Torsional surface wave (Figure 1.1).

Among the surface waves, a torsional surface wave is a wave that twists at the medium. Since it is a surface wave, it travels much slower than body wave, but has a higher amplitude and longer period, leading to major damage (Ghorai and Tiwary, 2014). Francisco, Reboredo & Torrisi (2004) stated that severe damage due to the earthquake that happened in Mexico City in 1985, resulting to the collapse of Mexico City general hospital (Figure 1.2) is due to torsion effects carried by torsional surface wave, which can influence internal structure of a building, causing the building to collapse. Therefore, the experts emphasized that torsional effects must be considered when designing seismic-resistant building so that the building can withstand earthquake attacks. In order to investigate seismic wave, the geophysical study which involved mathematical physics and mathematical modelling are commonly used in various research (Ewing, Jardetzky & Frank, 1957**;** Novotny, 1999; William et al., 2002; Pujol, 2003) to study motion behaviour of seismic wave in various media and it is extensively involved in multiple research fields, such as modelling of internal structure of the earth, exploring underground sources, constructing earthquake resistant building, etc. (Ito, Kuroki & Yoshida, 2001).

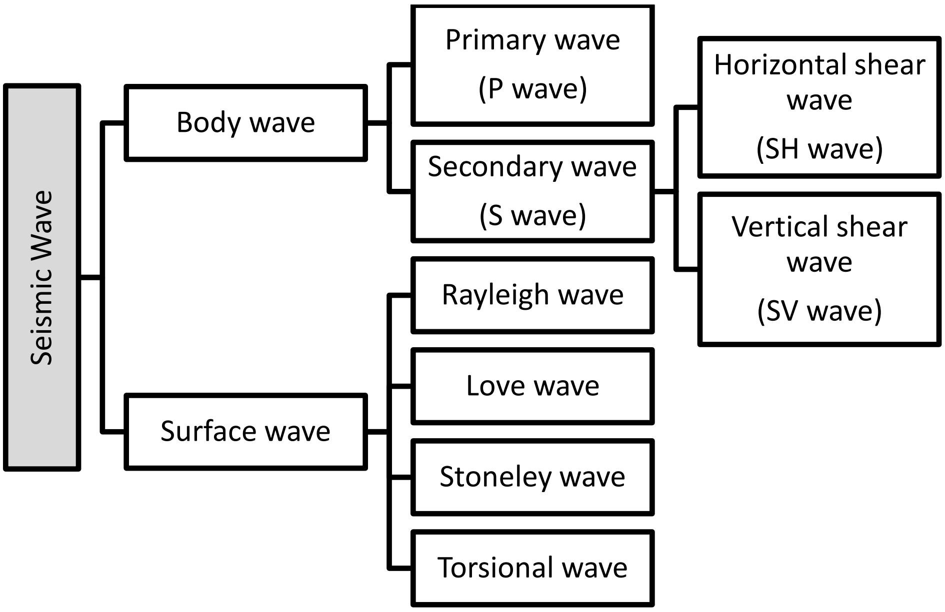

Figure 1.1 Classification of Seismic Wave

Figure 1.2 Earthquake Happened in Mexico City in 1985 (Francisco et al., 2004)

The structure of the earth consists of the earth's crust, mantle, and core. The crust, together with the upper mantle, consists of solid rock, known as the lithosphere (Figure 1.3), which is about 200 km thick (Chenicoff and Whitney, 2007). In the lithosphere, numerous rock blocks are in contact with each other, and the edge of the contacted blocks are called a fault. When the contacted blocks attempt to move away from each other, the friction in the fault makes it incapable of moving, but the stress between the blocks increases and is stored in the fault. When the stored stress overcomes the frictional force, the lithosphere blocks slide over each other, the accumulated stress stored in the fault is released in the form of energy, generating seismic wave that radiates outward from the focus of the fault in all directions (Figure 1.4) (Ohnaka, 2013). When the seismic wave propagates through the lithosphere, it vibrates the surface of the ground and everything on it, causing buildings to collapse, nuclear leaks, tsunami, and often leading to serious casualties. An earthquake causes an average of 27,000 lives lost a year since year 1990 (Robin, Emily & Charles, 2011).

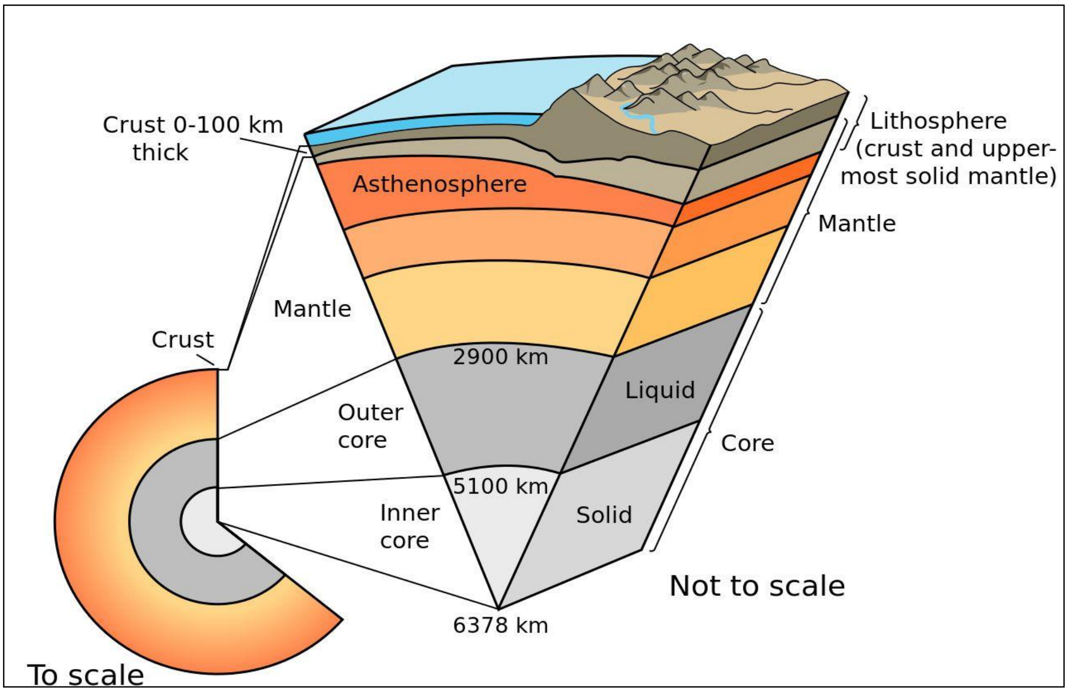

Figure 1.3 Structure of the Earth (U.S. Geological Survey, 1999)

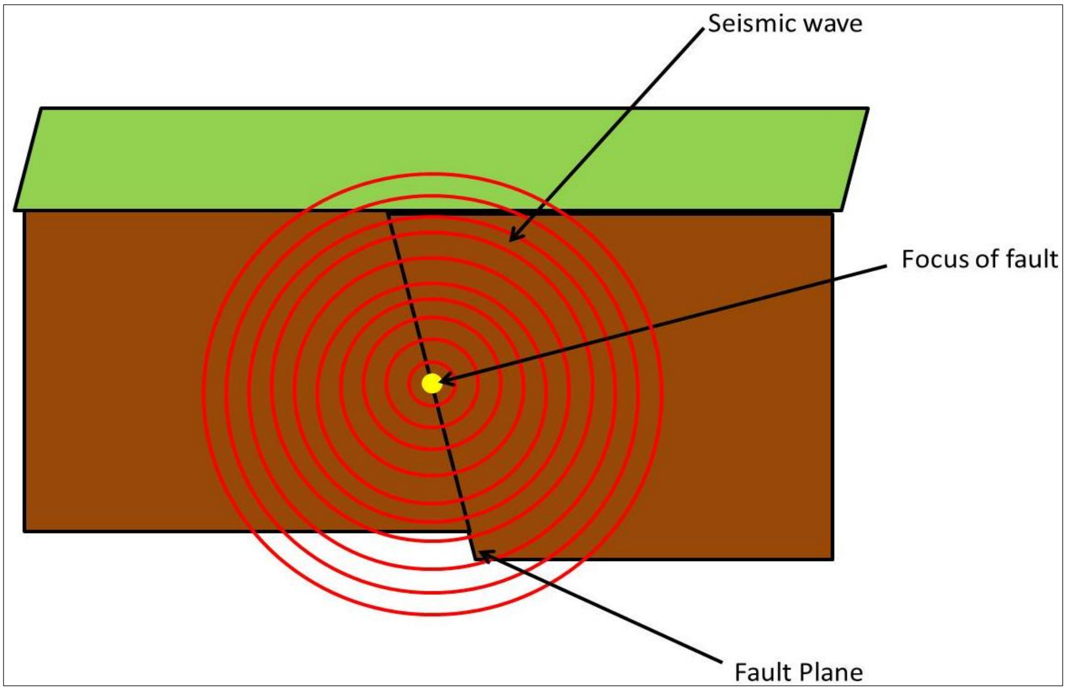

Figure 1.4 Radiation of Seismic Wave

Seismology is a study that links seismic waves to geophysical activities in the solid earth. It revealed the structure of the earth and how an earthquake occurs (Kanao, 2011). In seismological research, there are two main types of seismic waves, body wave, and surface wave (Peter, 2009). Body wave is a type of wave that propagates through the inner layers of the earth in three dimensions, with a frequency higher than surface wave, and it can be further divided into P wave and S wave (Figure 1.5). P wave is a compressive wave that is essentially longitudinal and can travel in solid or liquid materials. In addition, it is the fastest wave propagating at the speed of sound and arrives earlier than any other wave on the seismograph, hence why scientists named it as primary wave or pressure wave. S wave, also known as shear wave or secondary wave, is essentially transverse. It can propagate either side to side (horizontally polarised – SH wave) or up and down (vertically polarised - SV wave). S wave arrived after the fastest P-wave – it has 60% of P wave velocity and can propagate through solid materials. Although the S wave is slower than P wave, it usually caused more damage because it has a larger amplitude (Stein and Wysession, 2009).

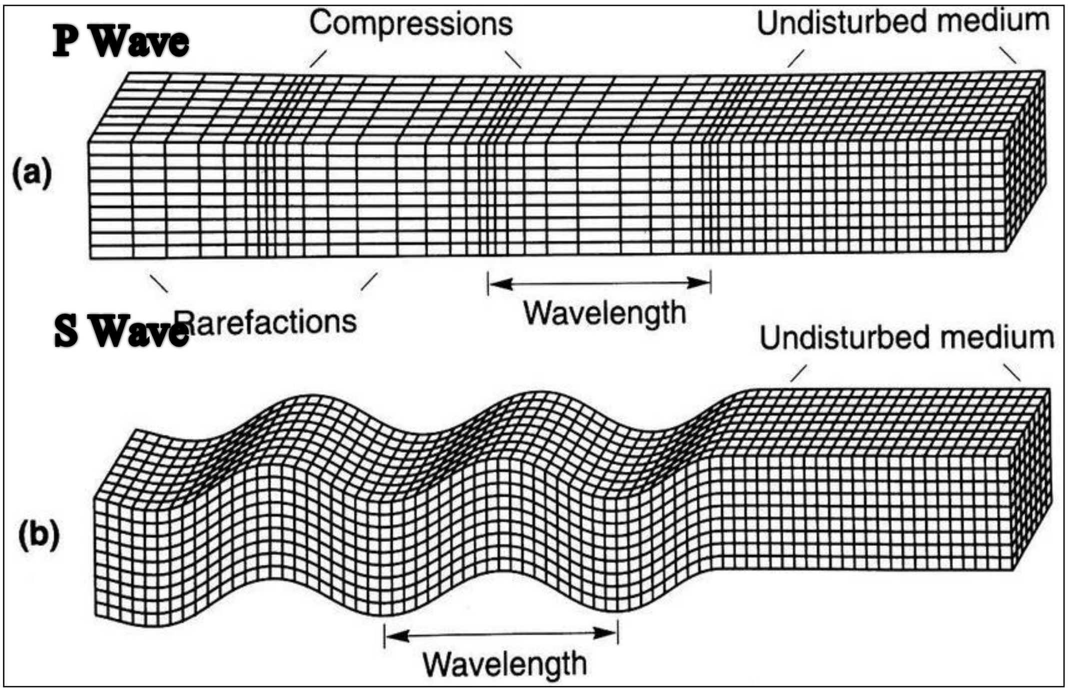

Figure 1.5 Propagation of Body Wave (Bolt, 1993)

Unlike body wave, surface wave is a type of wave which travels along the surface of the earth (Stein and Wysession, 2009). There are several types of surface waves, which are Rayleigh wave, Love wave, Stoneley wave and Torsional surface wave. Rayleigh wave, also known as ground roll, travels as ripples near the ground's surface. It combines both longitudinal and transverse vibrations (the combination of P and SV-wave) with exponentially decreasing amplitude as it gets farther away from the ground. A love wave, also known as a horizontally polarized shear wave, is an SH wave trapped at the earth's surface with horizontal motion perpendicular to the direction of wave motion. Unlike the Rayleigh wave, the Love wave has no vertical displacement (Figure 1.6). Love waves can be defined as the S wave without vertical displacement and it can only travel in the semi-infinite layer with finite thickness upper layer (Stein and Wysession, 2009). Stoneley wave is a boundary wave that propagates within two solid boundaries, or between solid and fluid boundaries (Scholte wave, it is sometimes known as tube wave, because its movement is guided by surfaces of interface of the solid (see Figure 1.7)). This wave is generated by a point source in a fluid-filled borehole and its amplitude is maximum when it is at the boundary of the media, but decays exponentially when it is away from the interface (Stoneley, 1924). Last but not least, torsional wave is a surface wave that is polarized horizontally, but it gives a twist to the medium that it propagates (Gupta et al., 2011).

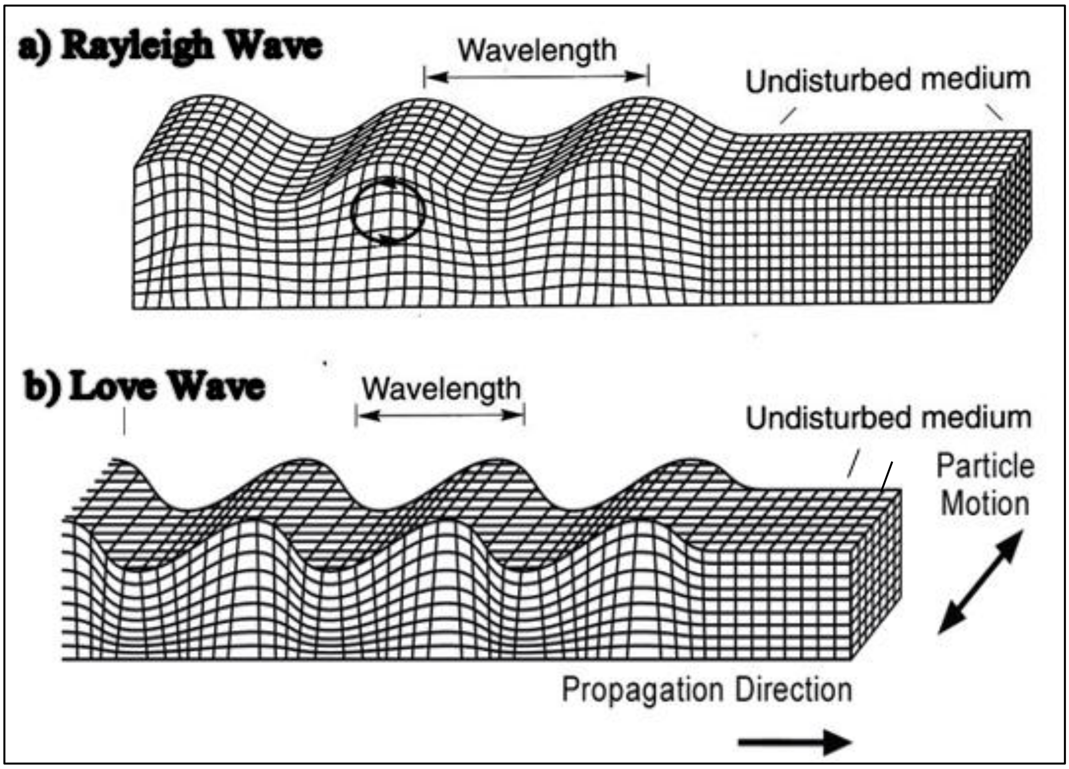

Figure 1.6 Propagation of Rayleigh and Love Waves (Bolt, 1993)

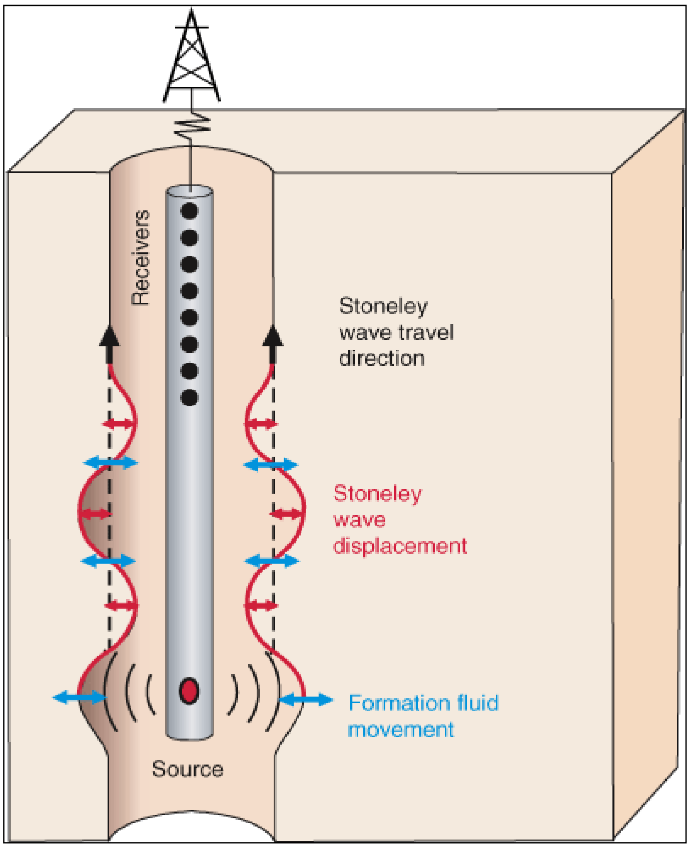

Figure 1.7 Propagation of Stoneley Wave (Qobi et al., 2001)

Torsional wave is the third type of wave other than longitudinal and transverse waves, which is also known as twisting wave. It is a wave movement that rotates periodically around the direction of wave propagation (Figure 1.8). Imaging when the end of a slinky (a toy, also known as Rainbow Spring) is twisting, it forms a wave pulse that travels along the slinky, the wave pulse is a torsional wave. Torsional waves only can propagate in solid material. Hence it is restricted to propagate in the upper crustal layer, and its amplitude decay exponentially with displacement from the surface of the earth (Chaunsali, Li & Yang, 2016). The study of torsional wave is very important to seismologists because it is useful in non-destructive testing (NDT), which can evaluate the properties of a material without causing damage to it (Cartz, 1995). Since the earth is made up of various materials (Klein and Philpotts, 2017), the study on behaviour of torsional surface wave propagating through different materials is essential to seismologists in order to investigate the main culprit that leads to the collapse of buildings due to earthquake. Besides, the torsional surface wave's effect must be considered in future architecture when carrying out construction work in the seismic zone (Francisco et al., 2004).

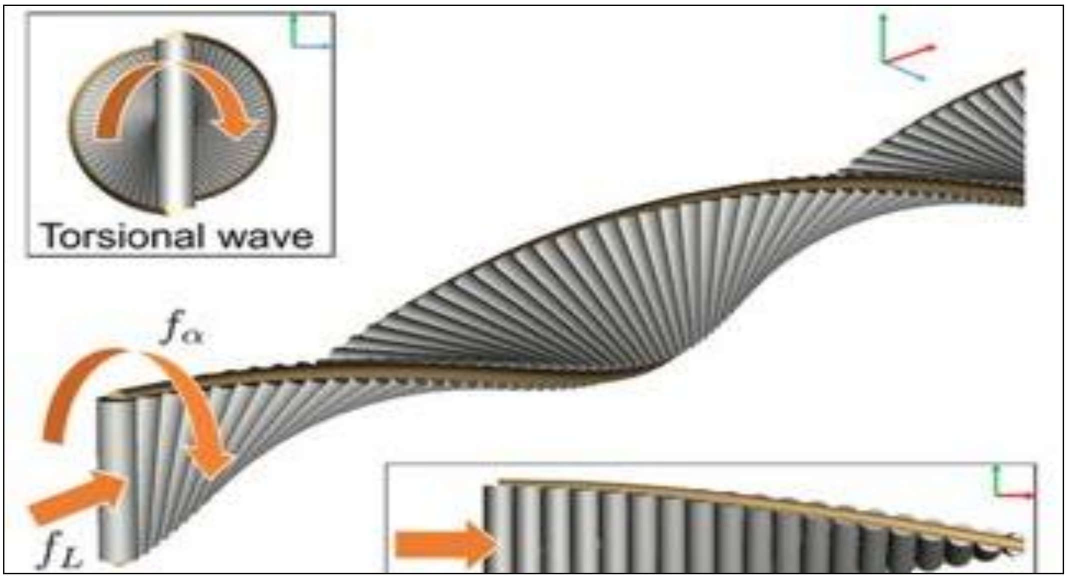

Figure 1.8 Propagation of Torsional Wave (Chaunsali, Li & Yang, 2016)

Mathematical modelling is a translation of a real object's behaviour into the language of mathematics, and solving it with mathematical theory. Mathematics provides exact solution that can help make predictions about real object behaviour without running an experiment. Besides that, complex mathematics formula can also be easily solved by using numerical method with the help of mathematics software such as MATLAB and behaviour of the objects can be observed by plotting graphs

(Lawson and Marion, 2008). Nowadays, seismologists widely use mathematical modeling to investigate the behaviour of seismic wave to understand the formation of earthquake and hence reduce casualties caused by it (Bath, 1968). In this thesis, theory of elasticity, equation of motion, plane wave, Hooke's law and Whittaker solution will be used to derive the velocity equation to show the behaviour of torsional surface wave propagating through different media.

1.2 Background of the Problem

Over the last several years, the investigation on characteristics of torsional surface wave propagation in elastic geo-media with layered structure has been of great concern (Vishwakarma, Kaur & Panigrahi, 2019). Physical properties of torsional surface wave refracted across the earth's boundaries, such as phase velocity, energy loss, and particle motion, made physical properties of this wave no longer the same in each layer but depend on heterogeneity. In addition, it is the most destructive wave in an earthquake and can propagate very long distances without much loss of energy (Gourgiotis and Georgiadis, 2015). Besides that, it cannot travel in an isotropic homogeneous medium and its amplitude decays exponentially with the depth of the free surface, which make its nature entirely different from other waves, such as Love, Rayleigh, and Stoneley waves (Chattaraj, Samal & Debasis, 2016). Francisco, Reboredo & Torrisi (2004) claimed that major cause to buildings damage observed in past earthquakes may be due to torsional vibration induced by the ground motion. The severe damage that occurred during the 1985 Mexico Earthquake showed the importance of torsional effects and pointed out the need to understand this issue and improve building design requirements. Besides that, the study of torsional surface wave propagation is also very useful for understanding the globe's geodynamics, earthquake engineering, and geophysics.

1.3 Research Gaps on Torsional Surface Wave and Parameter Study

The investigation of the propagation of torsional wave in heterogeneous elastic materials has been the subject of many studies (R. Kakar and S. Kakar, 2016, Chattaraj, Samal & Debasis, 2016, Gourgiotis and Georgiadis, 2015). Related topics on this area of interest include study on the effects of sandy level, gravity field, initial stress, heterogeneous parameter and rigidity of surface on the torsional surface waves'. These five parameters were chosen in this thesis because they have significant effects on the propagation of torsional surface wave. First, dry sandy materials have internal frictional forces that affect wave propagation, so it is necessary to consider the sandy level of the composition (Vishwakarma and Gupta, 2013). Second, the earth is a gravitating medium, hence Biot's gravity parameter is considered in this study (Abhishek et al. 2019). Next, since the earth is an initial stressed medium due to the presence of some physical factors involving overburdened layer, atmospheric pressure, variation in temperature, slow process of creep and gravitational field, large amount of initial stress is present in the medium, hence the initial stress cannot be ignored (Gupta, Kundu & Vishwakarma, 2013). Since the composition of the earth is heterogeneous, very hard layers, medium heterogeneity and rigid interfaces play a significant role in the propagation of seismic waves (Gupta et al., 2010). According to current study, these five parameters is independent but they could be interdependent of each other. It could be seen from Anup et al. (2015) that the behavior of torsional surface wave due to gravitational field on a rigid surface is different (tendency to increase or decrease) from that on a traction free surface (Figure 2.5 – Figure 2.7). Therefore, we can boldly assume there is a correlation between these parameters and it will affect behavior of torsional surface wave. This thesis's focus is finding out the correction between the parameters and discussing their influence on the behavior of torsional surface wave.

Since torsion effect has to be considered in designing seismic resistant building, the torsional surface wave studies demand immediate attention. Seismologists have achieved outstanding progress on the effects of five parameters, namely initial stress, Biot's gravity parameter, heterogeneous parameter, sandy parameter and rigidity of surface on the propagation of torsional surface wave. Current studies reported that these five parameters play an important role in propagation of torsional surface wave, either increasing or reducing the phase velocity of torsional surface wave. Anup et al. (2015) found that when the gravitational field increases for rigid surface, the phase velocity of torsional surface wave increases linearly, but it increases exponentially in a traction free surface. In addition, a study (Dey, Gupta S & Gupta A , 1996a) found that when the initial stress in the medium is low, the phase velocity in the elastic medium is higher than in the sandy medium, and when the initial stress in the medium is high, then the phase velocity in the sandy medium has a higher value. It could be seen that there is a correlation between these parameters, but researchers did not discuss the relationship between them, and only assumed them as independent of one another. Therefore, in this study, the correlation between these parameters will be studied using a velocity equation to help study the relationship between these parameters and determine torsional surface wave behaviour on different media.

1.4 Problem Statement

The objectives of the research are:

1. To obtain the velocity equation of torsional surface wave, which links up all the parameters: phase velocity, initial stress, Biot's gravity parameter, heterogeneous parameter, sandy parameter, and rigidity of surface.
2. To model the torsional surface wave's behaviours when it propagates in rigid surface by using the newly developed velocity equation.
3. To model the behaviours of torsional surface wave when it propagates in traction free surface by using the new developed velocity equation.

1.5 Scope and Limitation

Since the globe is a finite dimensional sphere, the elastic wave generated on the earth is affected by the globe's boundary. Due to this, the study on propagation of the surface wave is confined to some surface during their propagation, which usually deals with an idealized earth model (Gupta et al., 2012). The initial stress of the earth is generated in the earth's layers due to several factors (Sultana and Gupta, 2017). However, in this study, the factors that generated initial stress are not going to be discussed. Still, we only focus on the effect of initial stress, similar to other parameters used in this study.

Mathematical modeling is a technique that transforms problems from the field of applications into easy-to-handle mathematical formulas whose theoretical and numerical analysis can provide useful insights, answers, and guidance to the original application (Neumaier, 2004). In this paper, the equation of motion will be used with the initial stress (Kundu, 2015) and gravitational term (Anup, 2016), and finally solved by converting it into the Whittaker function and obtaining the general Whittaker solution up to linear terms only. During the calculation, the linear variation occurs in rigidity only and other terms and external forces that are undeclared in this study are ignored. By using the newly developed velocity equation, the graphs are drawn to illustrate the effects of initial stress, sandy parameter, Biot's gravity, heterogeneous parameter and rigidity of surface on the phase velocity of the torsional surface waves. The correlation between these parameters and their influence on the phase velocity of the torsional surface wave is shown using mathematical modelling. The results are verified via comparison with outputs obtained by other researchers without conducting any laboratory experiments.

1.6 Significance of Study

In order to get a more accurate calculation on the phase velocity of torsional surface wave, a newly developed velocity equation with multi parameters, namely initial stress, Biot's gravity parameter, heterogeneous parameter, sandy parameter and rigidity of surface has been developed. This equation can comprehensively explain the connection between these parameters and the torsional surface wave's phase velocity and makes it possible to determine which parameter has the dominant effect on the phase velocity of the torsional surface wave. Besides that, this study also fills up a research gap by discussing the correlation between these parameters on the torsional surface wave. The developed velocity equation will help to determine the seismic behaviour of torsional surface wave when it propagates in various materials. Last but not least, the output velocity equation is important for civil engineers to find out the most suitable materials to construct seismic resistant buiding with anti-torsion effects (Francisco et al., 2004).

1.7 Thesis Outline

This thesis is separated into seven chapters. In this chapter, an introduction is given followed by the research background, problem statements, research objectives, scope and limitation, the significance of study, and thesis outline.

In Chapter 2, the literature review looks into the formation of an earthquake followed by the theory of elasticity. After that, Hooke's law and other mathematical equations such as equation of motion, plane wave equation, and Whittaker function are shown. Last but not least, the literature review on the relationship between initial stress, Biot's gravity parameter, heterogeneous parameter, sandy parameter, and rigidity of surface are provided.

Chapter 3 presents the derivation of the dynamic equation of torsional surface waves for gravity-sandy Gibson half-space under initial stress (Gibson half-space: modulus of rigidity varies linearly with depth whereas the density and compressive initial stress remain constant). Firstly, the formation of the problem will be given and then derivation equation in the combination of Whittaker function, first-order Bessel equation and plane wave equation will be shown by using the equation of motion. Besides that, the boundary conditions will be given in this chapter.

Chapter 4 deals with the derivation of torsional surface waves in the gravity-sandy Gibson half-space under initial stress for the rigid surface. Discussion on the relationship between phase velocity of the torsional surface wave to initial stress, Biot's gravity parameter, heterogeneous parameter, sandy parameter, and rigidity of surface for the rigid surface will be given.

Chapter 5 presents the derivation of torsional surface waves in the initial stress induced gravity-sandy Gibson half-space for the traction free surface. Discussion on the relationship between initial stress, Biot's gravity parameter, heterogeneous parameter, sandy parameter and rigidity of surface for the traction free surface will be given.

Chapter 6 will assess the performance of the model by comparing the solution of velocity equation for rigid surface and traction free surface obtained in Chapter 4 and Chapter 5.

Chapter 7 will present the summary, conclusion, and future research for this study. The chapter starts with a summary of Chapter 3 until Chapter 6 to highlight the main contributions of this research on torsional surface waves. Subsequently, the conclusion will be drawn, and some suggestions for future research will be given.

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