MATHEMATICAL ANALYSIS OF DISPERSION OF SOLUTES IN BLOOD FLOW USING HERSCHEL-BULKLEY FLUID MODEL THROUGH AN INCLINED UNIFORM ARTERY

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DEDICATION

This dissertation is dedicated to both of my parents, who supported me unconditionally in completing my Master’s degree.

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ABSTRACT

This study aims to analyse the effect of inclination of the artery on the blood velocity and solute dispersion in an artery when it is inclined to a certain angle. Herschel-Bulkley fluid model is considered in representing the blood flow. The method of integration and perturbation are used to obtain the solution for the velocity of the steady and unsteady blood flow, respectively. The steady convection-diffusion equation is solved for the concentration of the solute using method of integration. Taylor-Aris method has been implemented to obtain the effective and relative axial diffusion. This present work focuses on the effect of arterial inclination angle on the blood flow characteristic in terms of velocity, concentration of solute, effective axial diffusion and relative axial diffusion. Other parameters' effect such as yield stress, gravitational acceleration and power-law index on the behaviour of the blood are also investigated and presented by graph representation. Observation shows that the angle of artery inclination directly influences the gravitational acceleration parameter, which correlates to the resulting blood velocity and solute concentration. $90°$ and $270°$ angles of inclination have the highest effect in increasing and decreasing the velocity, concentration of solute and diffusivity, respectively. Meanwhile, $0°$, $180°$ and $360°$ angles eliminate the gravitational acceleration effect on the blood behaviour and dispersion process. This study concludes that the angle of inclination has a strong impact on blood flow and solute dispersion behaviour.

ABSTRAK

Kajian ini adalah bertujuan untuk menganalisis kesan kecenderungan arteri pada halaju darah dan penyebaran zat terlarut dalam arteri apabila arteri tersebut cenderung pada sudut-sudut tertentu. Model bendalir Herschel-Bulkley digunakan bagi mewakili aliran darah. Kaedah pengamiran dan usikan, masing-masing digunakan untuk mendapatkan penyelesaian bagi halaju aliran darah yang stabil dan tidak stabil. Persamaan perolakan-penyebaran yang stabil diselesaikan untuk mendapatkan solusi bagi kepekatan zat terlarut menggunakan kaedah pengamiran. Kaedah Taylor-Aris telah digunapakai untuk mendapatkan kemeresapan paksi efektif dan relatif. Kajian ini memberi tumpuan kepada pengaruh sudut kecenderungan arteri pada tingkah laku aliran darah dari segi halaju, kepekatan zat terlarut, kemeresapan paksi efektif dan kemeresapan paksi relatif. Kesan parameter lain seperti tekanan hasil, pecutan graviti dan indeks hukum-kuasa pada tingkah laku bendalir juga disiasat dan dianalisa melalui graf. Pemerhatian menunjukkan bahawa sudut kecenderungan arteri secara langsung mempengaruhi parameter pecutan graviti yang berkorelasi dengan halaju darah dan kepekatan zat terlarut yang dihasilkan. Sudut kecenderungan 90° dan 270° mempunyai kesan tertinggi dalam meningkatkan dan menurunkan halaju, kepekatan zat terlarut dan penyerapan. Sementara itu, sudut kecenderungan 0°, 180° dan 360° menghilangkan kesan pecutan graviti pada tingkah laku aliran darah dan tingkah laku penyebaran zat terlarut. Kajian ini menyimpulkan bahawa sudut kecenderungan mempunyai kesan yang kuat terhadap aliran darah dan tingkah laku penyebaran zat terlarut.

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LIST OF ABBREVIATIONS

H-B	-	Herschel-Bulkley
LHS	-	Left-hand side
MHD	-	Magnetohydrodynamics
PDE	-	Partial differential equation
RBC	-	Red blood cell
RHS	-	Right-hand side
UTM	-	Universiti Teknologi Malaysia

LIST OF SYMBOLS

$\bar{a}$	-	Radius of artery
c	-	Constant of integration
$\bar{C}_1$	-	Solute concentration in the core flow region
C_1	-	Non-dimensionalized solute concentration in the core flow region
$\bar{C}_2$	-	Solute concentration in the outer flow region
C_2	-	Non-dimensionalized solute concentration in the outer flow region
$\bar{C}_c$	-	Solute concentration in the core flow region where the core region is evaluated at $\bar{r} = \bar{r}_c$
C_c	-	Non-dimensionalized solute concentration in the core flow region where the core region is evaluated at $r = r_c$
$\partial\bar{C}/\partial\bar{z}^*$	-	Concentration gradient
$\partial C/\partial z^*$	-	Non-dimensionalized concentration gradient
$\bar{D}_m$	-	Constant molecular diffusion
D_m	-	Non-dimensionalized constant molecular diffusion
$\bar{D}_{eff}$	-	Effective axial diffusion
$E(r_c)$	-	Measure of the change in Herschel-Bulkley dispersion relative to Newtonian fluid
$\mathbf{F}$	-	Body force of gravity in all directions
$\bar{g}$	-	Gravitational acceleration in the downward direction
g	-	Non-dimensionalized gravitational acceleration in the downward direction
m	-	Power-law index of Herschel-Bulkley fluid

n	-	Vector normal to the control volume
$\bar{p}$	-	Pressure
p		Non-dimensionalized pressure
Pe	-	Peclet number
Q_1, Q_2	-	Integration of the dispersion flow rate in core and outer flow regions for a circular pipe
$\bar{q}$	-	Dispersion flow rate
$\bar{r}$	-	Radius
r	-	Non-dimensionalized radius
$\bar{r}_c$	-	Radius of core flow region
r_c	-	Non-dimensionalized radius of core flow region
S	-	Surface area
$d\mathbf{S}$		Area of a fixed smooth surface of the control volume
$\bar{t}$	-	Time
t	-	Non-dimensionalized time
V	-	Control volume
$\bar{w}(\bar{z})$	-	Steady velocity at axial direction
$\bar{w}(\bar{z}, \bar{t})$	-	Unsteady velocity at axial direction
w_0	-	First term of the perturbation series of velocity
w_1	-	Second term of the perturbation series of velocity
$\bar{w}_c$	-	Velocity in the core flow region
w_c	-	Non-dimensionalized velocity in the core flow region
$\bar{w}_o$	-	Velocity in the outer flow region
w_o	-	Non-dimensionalized velocity in the outer flow region
$\hat{w}_c$	-	Relative velocity in the core flow region
$\hat{w}_o$	-	Relative velocity in the outer flow region
$\bar{w}_m$	-	Mean velocity
w	-	Vector normal to the control volume

$\bar{z}$	-	Direction of blood flow
z	-	Non-dimensionalized direction of blood flow
$\bar{z}^*$	-	Axial coordinate
z^*	-	Non-dimensionalized axial coordinate
α	-	Reynolds number
$\bar{\theta}$	-	Angle of pipe inclination
θ	-	Non-dimensionalized angle of pipe inclination
$\bar{\mu}$	-	Viscosity of the fluid
$\bar{\mu}_H$	-	Viscosity coefficient of the Herschel-Bulkley fluid
$\bar{\rho}$	-	Density of the fluid
ρ	-	Non-dimensionalized density of the fluid
σ	-	Stress and strain tensor
$\bar{\tau}$	-	Shear stress
τ	-	Non-dimensionalized shear stress
τ_0	-	First term of the perturbation series of shear stress
τ_1	-	Second term of the perturbation series of shear stress
$\bar{\tau}_y$	-	Yield stress
τ_y	-	Non-dimensionalized yield stress
$\bar{\psi}$	-	Azimuthal angle of pipe cross-sectional area normal to the direction of blood flow

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CHAPTER 1

INTRODUCTION

1.1 Introduction

This chapter discusses the background of study conducted on the problem concerning the solute dispersion, Herschel-Bulkley fluid model and inclination of the artery. The effectiveness of the solute dispersion is affected by several factors, such as the angle of artery inclination, gravitational acceleration, core flow radius and power-law index. This study focuses on blood flow and solute dispersion through an inclined narrow artery.

Many studies have been conducted to formulate an equation describing the blood flow and its relation to the factors affecting the blood flow. The most well-known equation is the Hagen-Poiseuille equation, which considers the viscosity of the fluid, pressure gradient at the constant cross-section of the pipe, length and diameter of the pipe. The equation was derived independently by Jean Léonard Marie Poiseuille in 1838 and continued by Gotthilf Heinrich Ludwig Hagen in 1839 (Sutera & Skalak, 1993). The ability to mathematically approach the behaviour of blood flow gives many contributions to the medical field in which one of those is the theory of dispersion of solute in blood flow. Many researchers and scientists have explored the study of solute dispersion as it contributes to findings and applications in the medical field. The medical field concerning the circulatory system implements the knowledge of solute dispersion in solving problems related to the transport of solute in the blood circulation system. Since the human circulatory system diverse from one patient to another, studies on solute dispersion are still being extended to further explore problems and solutions under various other conditions. For instance, cardiovascular diseases, specifically atherosclerosis (stenosis) which claims the lives of people, make the study of blood flow through arteries significant as it is closely related to the nature of blood movement and the dynamic behaviour of blood vessel (Ratchagar & Kumar, 2019).

Thus, studies on solute dispersion benefit not only the researchers but also the society who depends on these study developments to live a better life.

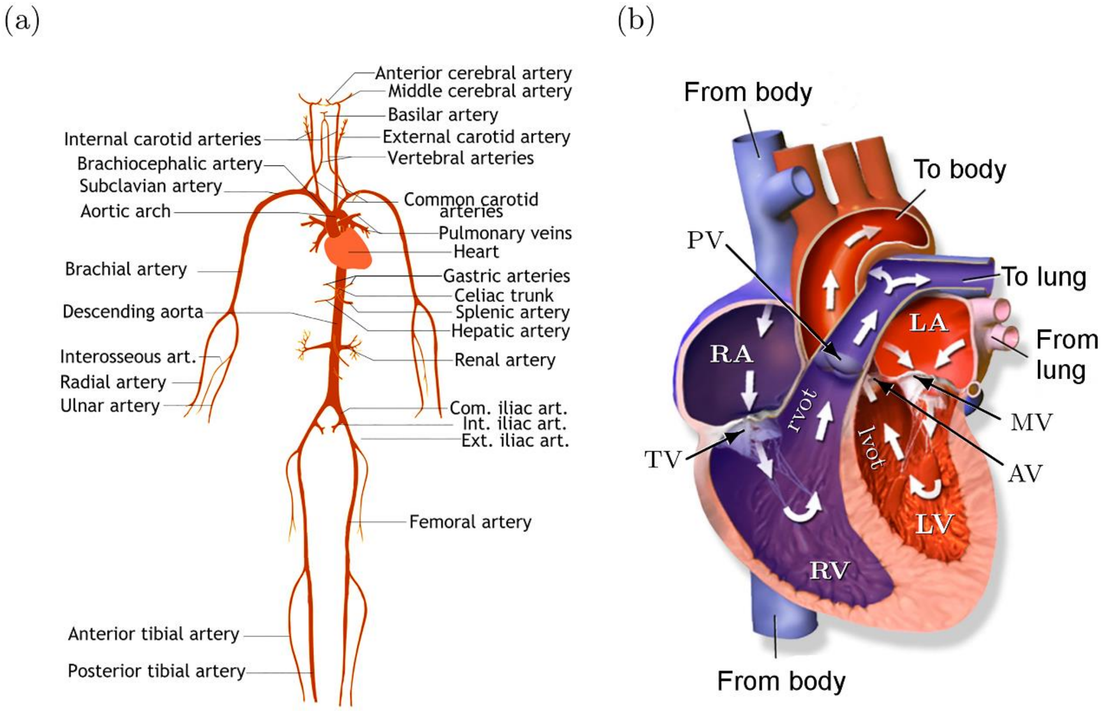

Figure 1.1 (a) Arteries in the circulatory system and (b) the direction of blood flow inside the heart (Huttunen et al., 2019).

There are many types of arteries inside the human body, as shown in Figure 1.1. Figure 1.1 illustrates the various inclination of the artery as it branches out from the heart to carry the blood to their specific target body and the anatomy of the heart that generates the pressure to pump blood through the artery. Each artery has its own orientation according to their position in the circulatory system. The velocity of the blood flow and solute dispersion inside those arteries are affected by the angle of artery inclination. Furthermore, an inclined artery coupled with a disease such as atherosclerosis leads to the narrowing of the artery that affects drug delivery through the artery. Figure 1.2 shows a normal artery and an artery with atherosclerosis. The artery with atherosclerosis has a narrowed opening caused by the cholesterol deposit that accumulates at the wall of the artery known as stenoses. The blood in a narrowed artery requires a fluid model that can represent its behaviour flowing through a small radius opening. Therefore, studies on the behaviour of blood flow and solute dispersion inside an inclined artery that is narrowed by stenoses are deemed to be significant.

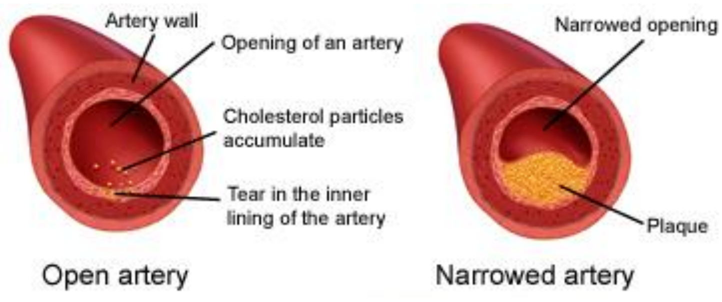

Figure 1.2 Normal artery and narrowed artery (Hui, n.d.).

Drugs are commonly administrated through the vein via intravenous injection. However, the practice of administrating drugs through the artery known as the intra-arterial injection is sometimes performed for a specific treatment and situation. Administrating drugs intravenously requires knowledge of the dispersion rate of the drug solutes in the blood since many drugs are therapeutic at low concentrations and harmful at high concentrations (Rana & Murthy, 2016). This knowledge also applies to drug administration through the artery, but more caution is needed as the risk of developing complications is higher. Increased risk of complications is due to the nature of the artery having a higher blood pressure compared to the veins and could potentially damage the tissue surrounding it as the drug is injected into the artery. Not to mention, the high blood pressure inside the artery could lead to heavy bleeding when the artery is punctured by the needle. The method of administrating drugs through the artery has a high risk of developing clinical trauma such as paraesthesia, severe pain, motor dysfunction, compartment syndrome, gangrene and limb loss (Sen et al., 2005). Nevertheless, in the event of intravenous cannulation (injection) is impossible and intraosseous access is considered too invasive, cannulation through the artery might be an alternative (Fikkers et al., 2006). Therefore, studies on behaviour of blood flow and solute dispersion inside an inclined artery are vital to reduce the risk of complications from an intra-arterial injection. An extensive study on solute dispersion inside an artery helps doctors and pharmacists in deciding the dose and distribution rate of medication to patients with less risk of causing toxicity and bleeding.

In representing the physiology of blood flowing inside an inclined artery, a circular pipe with various inclination is used to represent the inclined artery and non-Newtonian fluid is used to represent the blood. The primary concern of this study is the dispersion of solute in blood flow, especially in an inclined pipe as many ducts in a physiological system have some inclination rather than being horizontal (Prasad & Radhakrishnamacharya, 2008). Thus, adding an inclination to the study gives insights on the blood behaviour when gravity is considered.

A vast number of researches have been conducted to study the blood flow and solute dispersion behaviour due to the various type of fluid models being used to represent the blood properties known as Newtonian and non-Newtonian fluids. The viscosity of a Newtonian fluid is independent of the shear rate, while the viscosity of a non-Newtonian fluid is dependent on the shear rate. Blood has a viscosity that decreases with shear stress, and this shear-thinning property is closely related to the dynamics and mutual interactions of red blood cells (Lanotte et al., 2016). Since blood exhibits a shear-thinning property, non-Newtonian fluids are more suitable to represent the blood in studies involving haemodynamics. Non-Newtonian fluid such as Casson, Carreau, Carreau-Yasuda and Herschel-Bulkley has been proven to be useful in haemodynamics and to be applied in the study of the solute dispersion process in blood flow (Rana & Liao, 2019).

Nevertheless, for a low shear rate of blood flow with high yield stress in a very narrow artery, the Herschel–Bulkley fluid model is more suitable (Sankar & Hemalatha, 2007). An appropriate choice of a parameter for the power-law index of the Herschel-Bulkley fluid model can reduce it to other non-Newtonian fluid models such as power law and Bingham model. Therefore, Herschel-Bulkley fluid model is more fitting to describe the blood property flowing in a narrow artery.

In previous studies, non-Newtonian Herschel-Bulkley fluid is widely used in representing blood when it comes to solving problems related to solute dispersion in blood flow, mainly in small circular channel since blood vessels are relatively small. However, the study on solute dispersion using Herschel-Bulkley fluid in an inclined pipe has not yet been explored. Therefore, this present study focuses on the effect of pipe inclination on the behaviour of the Herschel-Bulkley fluid model and dispersion of solute to extend the study of previous researches and obtain results under different conditions. In this study, the application of fluid mechanics knowledge helps to measure the velocity profile, the concentration of solute, the rate of dispersion and the transport coefficients for the Herchel-Bulkley model in an inclined pipe.

1.2 Problem Statement

Solute dispersion occurs when solutes are injected into a circular pipe containing fluid that is flowing. This can be seen in the medical field when a doctor injects the drug into patients. However, the concentration of the drug being injected must be carefully calculated to avoid overdosing or damage to the blood vessel. Factors such as the velocity of the blood, the radius of the blood vessel and the rate of dispersion of the drugs should be considered in determining the dosage of medicine to be injected. Thus, the study of solute dispersion in a pipe contributes to understanding the dispersion process. Many researchers used the Newtonian fluid in representing the blood in studying its flow in a large diameter artery. However, in a real-life problem, certain artery has a narrow diameter due to underlying medical conditions. Thus, it is significant to study the blood flow in a narrow artery at a low shear rate to give a life-like description of blood flow. A fluid model of Herschel-Bulkley is used to represent the blood in a narrow diameter artery.

Not to mention, studies that used a horizontal pipe can deviate from a close representation of the real-life situation as blood flowing in the artery is not always or never in a perfect horizontal state. Therefore, the inclination of the pipe has been considered to study the blood flow in the real-life situation.

1.3 Research Objectives

The objectives of this present study are:

(a) To formulate the fluid flow model of the Hershel-Bulkley through an inclined uniform artery.

(b) To solve the momentum and constitutive equations for finding the velocity using the method of integration for steady blood flow and perturbation method for unsteady blood flow.

(c) To solve the steady dispersion analytically for finding effective and relative axial diffusion using the Taylor-Aris method.

(d) To analyse the graphical data of velocity profile, solute concentration, effective axial diffusion and relative axial diffusion at various angles of inclination and under the influence of different values of power-law index, gravitational acceleration and core flow radius.

1.4 Scope of Study

The scope of the study is limited to solving for the fluid velocity, solute concentration, effective axial diffusion and relative axial diffusion of steady dispersion of solute in a steady and unsteady, laminar, fully-developed Herschel-Bulkley flow in an inclined pipe. The governing equations are solved analytically for both momentum and convection-diffusion equations. The data of the velocity of Herschel-Bulkley fluid flow, the concentration of solute, the effective axial diffusion and relative axial diffusion are obtained using *Mathematica* software.

1.5 Significance of Study

Research on solute dispersion in blood flow through an inclined artery has many benefits in the science field such as medical, pharmaceutical and bioengineering fields. In the medical field, the study on solute dispersion helps doctors in deciding the suitable dosage of medicine to be given to patients. The findings of this present study can also help to depict a realistic description of solute dispersion through an inclined artery for future researchers in extending studies related to circulatory system or cardiovascular diseases. Therefore, the significance of this study are:

(a) In-depth insight on the flow and dispersion characteristics of the Herschel-Bulkley model in an inclined pipe may help future studies in extending research revolving the Herschel-Bulkley fluid model.

(b) The result of this research can help doctors in understanding the behaviour of the dispersion process of solute in treating diseases that involves injecting drugs into the artery by observing the behaviour of blood flow and solute dispersion under the influence of artery inclination.

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