MAGNETOHYDRODYNAMICS EFFECTS ON GENERALISED POWER LAW FLUID MODEL OF BLOOD FLOW IN A STENOSED BIFURCATED ARTERY

NORLIZA BINTI MOHD ZAIN

UNIVERSITI TEKNOLOGI MALAYSIA

MAGNETOHYDRODYNAMICS EFFECTS ON GENERALISED POWER LAW FLUID MODEL OF BLOOD FLOW IN A STENOSED BIFURCATED ARTERY

NORLIZA BINTI MOHD ZAIN

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Philosophy

Faculty of Science
Universiti Teknologi Malaysia

MAY 2019

ACKNOWLEDGEMENT

First and foremost, a million thanks go to my only supervisor, Dr Zuhaila Ismail for her guidance, inspiring thoughts and advices and for always keep trusting me throughout my research study. Her positive feedbacks have contributed to my self-motivation that drive me to always give my very best in this study and never give up.

I also would like to deeply thanks my seniors, Dr Lim Yeou Jiann and Ms Huda Salmi Binti Ahmad for sharing their time and knowledge teaching me the method which are new to me. Without them, it must be tough for me to fully understand the formulation of that method and its implementation on the source code.

Special thanks to my parents for their endless prayers and encouragement that has become the main inspirations to me for all this while to keep doing well in everything I do. Thank you so much to my beloved mom and my loving husband for always being my great listeners through my hardest and joyful moments. To my siblings, nieces and nephews, thank you for always cheers me up whenever I am feeling down.

Greatest appreciation to my friends in Universiti Teknologi Malaysia for their motivation and prayers for me to fulfilled my study with excellent results. Last but not least, I would like to thanks Ministry of Higher Education Malaysia for sponsoring my study in UTM through a scholarship programme of MyBrainSc.

ABSTRACT

The production of Lorentz force arises according to the magnetohydrodynamics principle which allows fluid motion to slow down an accelerated fluid and thus resulted in a uniform, calm flow. Due to that, most of the clinical treatments of certain cardiovascular diseases utilise this principle in magnetic therapy. Unfortunately, an excessive exposure to high magnetic intensity may contribute to an irreversible change and it can be harmful towards the organ. Hence, in this research, the effects of uniform external magnetic field along the bifurcated artery that possesses an overlapping stenosis at the parent's arterial lumen are investigated for further understanding. Several assumptions are considered in this study. In particular, the streaming blood is considered steady, laminar, incompressible, fully developed and electrically conducted. In addition, the rheological behaviour of the streaming blood is assumed to be characterised by a generalised power law model corresponding to shear-thinning, Newtonian and shear-thickening nature of blood. However, in these conditions, applying the finite element technique using the classical Galerkin approach may result in spurious oscillation. In order to deal with this issue, the resulting governing equations are solved using a stabilized finite element technique called Galerkin least-squares method. This method is convenient to compute a highly viscous streaming blood and compatible to circumvent the Babuska-Brezzi stability conditions. Comparison of velocity contour, pressure drop and skin friction obtained from this present study using MATLAB and COMSOL Multiphysics softwares are found in satisfactory agreement with previous work in the literature. Thus, convincing enough to be extended by including the magnetohydrodynamics and non-Newtonian effects along a bifurcated channel. The results from pattern of streamlines show that shear-thinning fluid creates the largest recirculation area in comparison to shear-thickening and Newtonian fluids. In addition, when external magnetic field is applied in a transverse direction, the flow velocity is reduced considerably, restricting the occurrence of flow reversal, consequently generating a uniform, calm flow. However, the magnetic intensity shows little effect on the constricted region due to the smaller diameters of the vessel. Furthermore, as the severity of stenosis is increased, significant rise in wall shear stress magnitudes at the throat of an overlapping stenosis are noticed which may lead to thrombosis occurrence. Therefore, application of Galerkin least-squares method to the flow of blood in a stenosed bifurcated artery with the influence of an external magnetic field can be beneficial for magnetic therapy predictions and helps to understand the flow dynamics in a stenotic region.

ABSTRAK

Pengeluaran daya Lorentz yang wujud berdasarkan prinsip *magnetohydrodynamics* membolehkan gerakan bendalir memperlahankan cecair yang sedang memecut justeru menghasilkan aliran yang seragam dan tenang. Oleh itu, kebanyakan rawatan klinikal bagi beberapa penyakit kardiovaskular menggunakan prinsip ini dalam terapi magnetik. Malangnya, pendedahan yang berlebihan terhadap keamatan magnet yang tinggi boleh menyumbang kepada perubahan yang tidak dapat dipulihkan dan ia boleh memudaratkan organ. Oleh itu, dalam kajian ini, kesan medan magnet luar seragam terhadap saluran darah bercabang yang mempunyai stenosis bertindih di lumen cabang utama disiasat untuk pemahaman yang lebih lanjut. Beberapa andaian dianggap dalam kajian ini. Khususnya, aliran darah dianggap mantap, laminar, tidak boleh dikompresi, aliran terbentuk sepenuhnya dan boleh mengalirkan elektrik. Tambahan pula, sifat rheologi aliran darah diandaikan sebagai model hukum kuasa teritlak berdasarkan sifat penipisan ricih, Newtonian dan penebalan ricih darah. Walaubagaimanapun, dalam situasi ini, dengan menggunakan teknik elemen terhingga melalui pendekatan Galerkin klasik boleh mengakibatkan ayunan palsu. Bagi menangani masalah ini, persamaan pentadbiran yang terhasil diselesaikan dengan menggunakan kaedah satu bentuk teknik unsur terhingga yang stabil dikenali sebagai Galerkin kuasa dua-terkecil. Kaedah ini sesuai untuk mengira aliran darah yang mempunyai kelikatan yang sangat tinggi dan serasi untuk mengelakkan keadaan kestabilan *Babuska-Brezzi*. Perbandingan ke atas kontur halaju, penurunan tekanan dan geseran kulit yang diperoleh dari kajian ini menggunakan perisian MATLAB dan COMSOL Multiphysics didapati memperoleh persetujuan yang memuaskan dengan kajian sebelumnya dalam literatur. Oleh itu, kaedah ini cukup meyakinkan untuk diperluaskan dengan menambah kesan *magnetohydrodynamics* dan bukan-Newtonian pada saluran darah yang bercabang. Hasil dari corak aliran menunjukkan bahawa cecair penipisan ricih menghasilkan kawasan pengulangan semula yang lebih besar berbanding dengan cecair penebalan ricih dan cecair Newtonian. Di samping itu, apabila medan magnet luar digunakan dalam arah melintang, halaju aliran berkurangan dengan ketara, menyekat berlakunya pembalikan aliran, seterusnya menghasilkan aliran seragam, yang tenang. Walau bagaimanapun, keamatan magnet menunjukkan kesan yang tidak ketara ke atas kawasan yang sempit disebabkan oleh diameter salur darah yang lebih kecil dari vesel tersebut. Tambahan pula, apabila keterukan stenosis meningkat, peningkatan ketara dalam magnitud tekanan geseran dinding di tekak stenosis yang bertindih dikesan yang kemudiannya boleh menyebabkan terjadinya trombosis. Oleh itu, penggunaan kaedah Galerkin kuasa dua-terkecil terhadap aliran darah dalam arteri bifurkasi yang berpusat dengan pengaruh medan magnet luar boleh memberi manfaat untuk ramalan terapi magnetik dan membantu memahami dinamika aliran di rantau stenosis.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamics
FHD	-	Ferrohydrodynamics
BFD	-	Biomagnetic Fluid Dynamics
FEM	-	Finite Element Method
GLS	-	Galerkin Least-Squares

LIST OF SYMBOLS

x	-	Axial coordinate
y	-	Radial coordinate
$R_1(x)$	-	Radii of the outer wall
$R_2(x)$	-	Radii of the inner wall
a	-	Radii of the mother artery
r_1	-	Radii of the daughter artery
r_0	-	Radii of curvature for the lateral junction
$r_0^{'}$	-	Radii of curvature for the flow divider
d	-	Onset of the stenosis
l_0	-	Length of the stenosis at a distance d from the origin
x_1	-	Location of the onset and offset of the lateral junction
x_2,	-	Location of the offset of the lateral junction
x_3	-	Apex
τ_m	-	Maximum height of stenosis
β	-	Half of the bifurcation angle
ρ	-	Density of blood
u	-	Axial velocity component
v	-	Radial velocity component
w	-	Tangential velocity component
t	-	Time
f_x	-	Body force per unit mass of the fluid element
μ	-	The respective molecular viscosity coefficient
λ	-	The second viscosity coefficient
τ_{ij}	-	Stress in the $j-$ direction applied on a plane perpendicular to the $i-$ axis
Δ	-	The symmetrical “rate of deformation tensor
m	-	Fluid consistency coefficient

n	-	Flow behaviour index
$\mathbf{F_m}$	-	Electromagnetic force
$\mathbf{J}$	-	Current density
$\mathbf{B}$	-	Magnetic flux intensity
$\mathbf{E}$	-	Electric field intensity
σ	-	Electrical conductivity
$\mathbf{B}_0$	-	External magnetic field
$\mathbf{B}_1$	-	Induced magnetic field
B_0	-	Magnitude of $\mathbf{B}_0$
Ω	-	Domain
$\mathbf{n}$	-	Unit vector outward normal to the outflow boundary
p	-	Pressure
Γ_h	-	Boundary corresponding to Dirichlet condition
Γ_g	-	Boundary corresponding to Neumann condition
h	-	Length of the inlet
u_r	-	Averaged mean inflow velocity
Re	-	Reynolds number
M	-	Hartmann number
C_h	-	Finite element partition
K	-	Triangular element
$Q_m(x, y)$	-	Weighting function for continuity equation
$N_l(x, y)$	-	Weighting function for momentum equations
B_{cx}	-	Natural boundary conditions of $x-$momentum equations
B_{cy}	-	Natural boundary conditions of $y-$momentum equations
(η_i, ξ_i)	-	Element natural coordinates
T_{st}	-	Standard triangle
$N_j^{e_i}$	-	Nodal shape functions j in element e_i for velocity components
$Q_j^{e_i}$	-	Nodal shape functions j in element e_i for pressure components

(x_j, y_j)	-	Vertices of triangular element K
$R(\eta,\xi)$	-	Coordinates transformation operator for global coordinate x
$S(\eta,\xi)$	-	Coordinates transformation operator for global coordinate y
A_K	-	Area of triangular element K
e_i	-	Triangular element
u^{e_i}, v^{e_i}	-	Spatial variations of velocity components within an element e_i
p^{e_i}	-	Spatial variations of pressure component within an element e_i
u_j, v_j	-	Velocities at corner nodes j
p_j	-	Pressure at corner nodes j
R_1	-	Polynomial spaces of degree 1 for both velocities and pressure components
$L^2(\Omega)$	-	Space of square integrable functions
$L_0^2(\Omega)$	-	Space functions in $L^2(\Omega)$
$H^1(\Omega), H_0^1(\Omega)$	-	(Section 4.2.2) Sobolev spaces
n_q	-	Number of quadrature points (η_i, ξ_i)
w_i	-	Weights that are normalized with respect to triangle area
K	-	Stiffness matrix
U	-	Vector of degree of freedom
F	-	Force vector
$\mathcal{J}$	-	Jacobian matrix
τ	-	Convergence tolerance
$I(u,v,p)$	-	Least squares functional
$\tau(\mathrm{Re}_K)$	-	Stabilization parameter
h_K	-	Size of triangular element K
$\eta(\dot{\gamma})$	-	Viscosity function

δu	-	Small correction on u
δv	-	Small correction on v
δp	-	Small correction on p
b	-	Iteration
N_e	-	Number of domain element
τ_w	-	Wall shear stress
ΔP	-	Pressure drop
P_a	-	Pressure at the starting point of stenosis
P_b	-	Pressure at the end point of stenosis

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Research Background

Cardiovascular diseases are recorded as the most popular cause of deaths among humans worldwide. This fact has been verified by World Health Organization (WHO 2017) which has confirmed that about 17 million people were estimated to have died per year due to these diseases (World Health Organization, 2017; Sankar, Gunakala and Comissiong, 2013). This number is expected to grow by year 2030 to more than 23.6 million deaths. Among these huge number of mortality, two main sources of deaths are caused by cerebrovascular disease and coronary heart disease due to the blockage of blood vessels to the brain and heart muscle, respectively.

The genesis, diagnosis and prognosis of this vascular diseases are still being investigated by researchers all around the world and have become a favourite subject of scientific research to be explored. Their effects on biological fluid dynamics are the main concern to be discovered and understood. The connection between these two helps greater in the prediction of disease progression and how it influences the blood flow dynamics. By considering other external forces influence such as body acceleration, gravity and external magnetic field on blood flow, these will generate more knowledge in this area which in turn may benefit others especially an expert in the field of medicine for improving the design of medical devices.

Blood is a two-phase fluid, consisting of about 55% continuous Newtonian phase fluid (plasma) with a number of suspended formed elements make up over 45% volume of blood which composes of red blood cells (erythrocytes), white blood cells (leukocytes) and platelets (Kowalewski, 2005). Figure 1.1 summarizes the human blood composition as it is being centrifuged.

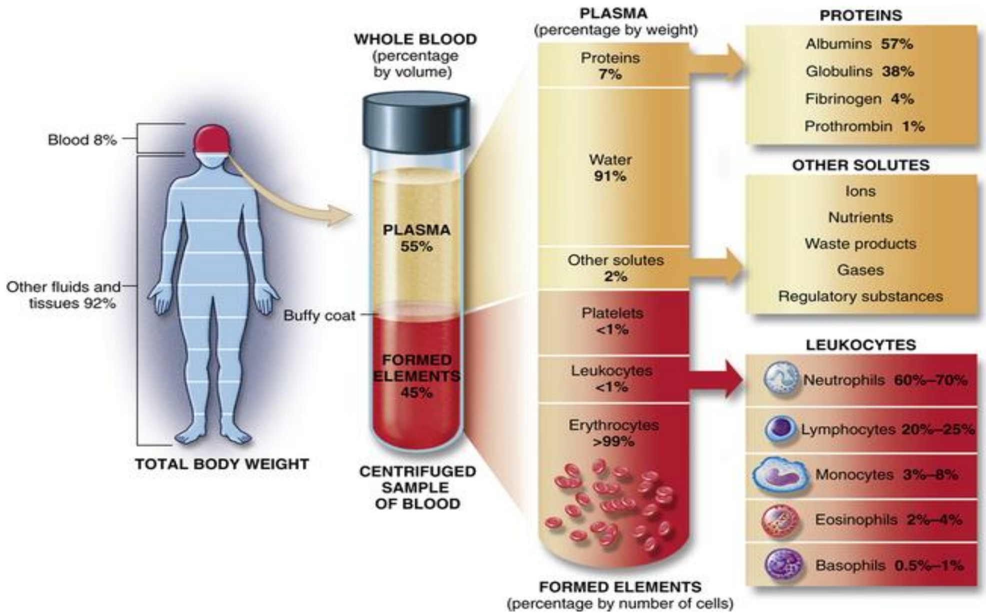

Figure 1.1 Approximate values for the components of human blood (Basicmedical Key, 2016)

Plasma is made up of 91% water, 7% protein and the rest of it is small amounts of organic and inorganic molecules together with the dissolved gasses (Kowalewski, 2005). Its concentration and volume is essential for the blood pressure control (Rabby, Shupti and Molla, 2014). Prominently, red blood cells comprise of mostly the formed elements approximately about 99% (Sherwood, Kaliviotis, Dusting and Balabani, 2014). Red blood cell is a micro polar fluid (small semisolid particles) which majorly influences the rheological behaviour of blood due to its dominated viscous effect that is predominantly being influenced by the hematocrit distribution; ratio between volume of red blood cells to the total volume of blood (Rabby et al, 2014; Sherwood et al, 2012; Srivastava and Rastogi, 2010). Meanwhile, there are five types of leukocytes which can be classified according to the absence or presence of granules in the cytoplasm of the cell as categorized in Figure 1.1, which are especially vital for the production of antibodies in order to fight infections by destroying any foreign cells (Pincombe, 1999). The smallest formed elements (platelets) functioning in blood clotting to stop bleeding of the damaged vessels is thought to be important in the development of atherosclerotic lesions (Pincombe, 1999).

Zaman, Ali, Sajid and Hayat (2015) stated that in larger arteries, blood is assumed as a Newtonian fluid for values of shear rate greater than $100\,s^{-1}$ and behaves differently as a non-Newtonian fluid in narrow arteries (of diameter $\leq 2400\mu m$) (Srivastava et al, 2010). It is clearly described by Rabby et al (2014) that Newtonian fluid viscosity is always constant with the shear rate while the non-Newtonian fluid viscosity is continuously changing with the shear rate. This characteristic of Newtonian fluid would then later reveal a high potential to the individual being attacked by any of the cardiovascular diseases compared to non-Newtonian fluid (Rabby et al, 2014).

Blood vessels are liable for the transportation of blood throughout the human body system. Arteries, capillaries and veins are the three main types of blood vessels, each with different roles to be carried out in the human circulatory system. For instance, arteries take control of carrying blood away from the heart (Yahya, 2010). The main artery is aorta that bifurcates into other major arteries which are the carotid arteries, brachial arteries, thoracic arteries, coronary arteries and iliac arteries. The major arteries then branch into minor arteries that will further diverge into smaller vessels (arterioles) to reach far down into the muscles and organs (Yahya, 2010). Arterioles then diverge into the capillary beds which consist of about 10 to 100 capillaries that deviate further to cells and tissues of the body to enable the blood supply to reach every tiny part of the human body (Yahya, 2010). Exchange of oxygen, nutrients and waste with tissues at the cellular level takes part at these capillary beds. These capillaries then approach venules that are connected with minor veins. Minor veins are attached to major veins which are responsible to drain blood rich in carbon dioxide back to the heart from the organs and limbs that are being supplied by the major arteries. As depicted in Figure 1.2, the network of blood vessels in human body system which includes arteries, capillaries and veins can be clearly seen and differentiated from one another.

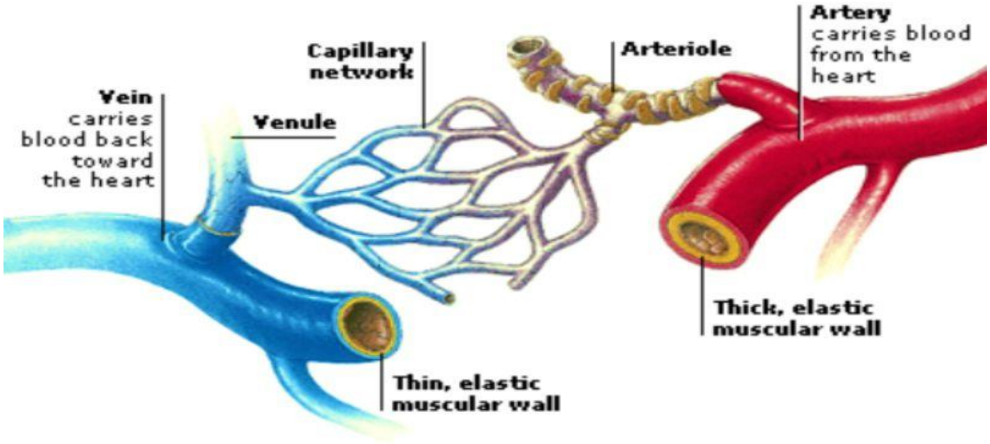

Figure 1.2 Human blood vessels (Fact Monster, 2017)

Atherosclerotic lesion is the arterial disease that is believed to be responsible for the cardiovascular system failure (Ikbal, Chakravarty, Wong, Mazumdar, and Mandal, 2009). It is formed due to the narrowing of the arterial lumen as a result of plaque deposition which consists of lipid, cholesterol and some other substances formed on the inner wall of the artery, which can be called as the endothelium (Rabby et al, 2014; Ikbal et al, 2009; Jahangiri, Saghafian and Sadeghi, 2015; Alimohamadi, Imani and Shojaeizadeh, 2014). This leads to the abnormal pattern of blood flow which severely reduced the flow of blood to the other organs and tissues (Stroud, Berger and Saloner, 2002). Plaque might behave differently and lead to different blood rheology. In some cases, plaque grows to a certain size and stops without eventually narrowing the lumen in which the arterial wall deformed their external diameter itself in order to provide sufficient space for the plaque (Ikbal et al, 2009). In such cases, this silent plaque can only be detected through intravascular ultrasound since it never caused any symptoms due to the plaque formation that does not even block blood flow. However, in most cases plaque acts differently where it grows and results in a significant blockage of blood circulation. As a consequence, patients of this case experience usual symptom like pain on exertion (in the chest or legs). Severe cases demonstrate that plaque deposition could suddenly rupture into emboli (particles) and lead to the formation of thrombus (blood clot) inside an artery (Ikbal et al, 2009; Stroud et al, 2002). Blockage of blood flow to the carotid artery and coronary artery particularly as a result of thrombus formation lead to neurological symptoms (stroke) and unstable angina or myocardial infarction (heart attack), respectively. According to Roh and Kim

(2003), atherosclerosis preferably develops at the abdominal aorta, the coronary arteries, the carotid arteries, and the peripheral arteries such as femoral and also iliac.

Health risks caused by the malfunction of cardiovascular system reveals that every human being are at risk of possibly being infected by these cardiovascular diseases. Once a mild stenosis has developed, the arterial lumen gets narrower and leads to the changes of regional rheological behaviour of blood which further influences these diseases to develop and become more severe (Rabby et al, 2014). Despite that, still lots of humans are unaware of this health problem. Dietary fat and blood cholesterol, high blood pressure and smoking are few risk factors that lead to the atherosclerosis infection, which in other words this disease development is much related to the human's way of living (Roh et al, 2003). Hence, there is a growing need to investigate the rheological behaviour of blood and identify the fluid dynamic properties of blood flow to capture the basic knowledge on diagnosis and treatment of cardiovascular and arterial diseases so that most of the human beings could be aware of the risk of these diseases.

Most of the treatments of cardiovascular diseases are based on the application of an external magnetic field which is basically known as magnetic therapy. As discovered by Science Daily (2011), a lower magnetic field intensity could decrease the viscosity of blood by 20% to 30% which in turn reduce the possibilities of impaired vessels and risk of heart attack. As represented in Figure 1.3, the red blood cells aggregate and move in a streamline direction as the external magnetic field is employed that resulted into further reduction of blood viscosity. This allows the blood to move freely while at the same time lowers the friction against wall of the blood vessels (Science Daily, 2011). As magnetic field is applied on a moving blood, an electric and magnetic fields will be induced (Bhatnagar and Shrivastav, 2014). Since blood is composed of suspension of red blood cells that consist of haemoglobin and with the presence of ions in the plasma, blood can be classified as an electrically conducting fluid that exhibits the magnetohydrodynamics (MHD) principle (Bhatnagar et al, 2014; Bose and Banerjee, 2015).

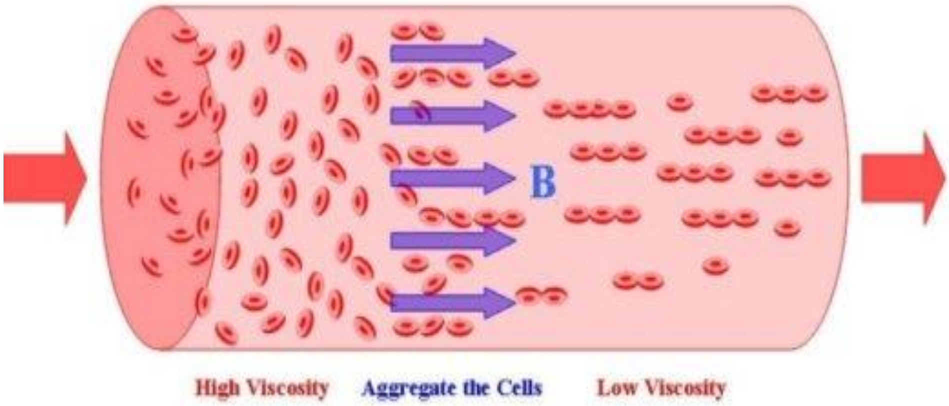

Figure 1.3 Aggregated red blood cells clump moving together in a streamline shaped (Science Daily, 2011)

When both fields are in contact with each other, a body force known as Lorentz force arises and slows down the fluid motion following the principle of MHD (Bhatnagar et al, 2014; Bose et al, 2015). This benefits especially for the pumping action of blood and magnetic resonance imaging (MRI). Another treatment of arterial occlusions that makes use of an external magnetic field is magnetic drug carriers (Bose et al, 2015). In this treatment, a magnetized drug will be injected to the blood flow while a magnetic field is placed at the target region that is facing an injury (Hasanzade, 2015). These drug particles move following the flow of blood until they reach the magnetic site and will be absorbed by the magnetic field (Hasanzade, 2015). Since this treatment does not affect any other body cells, hence it would be very efficient and safe to treat vascular diseases.

Apart from that, another haemodynamic variable to be considered for a better understanding on the rheological behaviour of blood is the arterial geometry. Arteries and arterioles that consist of curvatures, junctions and bifurcations were reported by Rabby et al (2014), Zarins, Giddens, Bharadvaj, Sottiurai, Mabon and Glagov (2015), Lou and Yang (1993), and Liu and Tang (2011) as the most favoured sites predisposed to atherosclerosis development. This suggests why the flow at certain region has high potential to be disturbed by flow disturbances like flow reversal and stagnation. Through an exposure of high or low shear stress at wall of these particular regions, the genesis as well as progression of the intimal thickenings could be predicted (Rabby et al, 2014; Zarins et al, 2015; Lou et al, 1993). Thus, an incident of thrombosis could be overcome.

Computational Fluid Dynamics (CFD) uses numerical methods to solve the fundamental nonlinear differential equations that describe fluid flow (the Navier-Stokes and allied equations) for the predefined geometries and boundary conditions. It saves cost and turnaround time as well as acquires reliable result. In terms of the medical purposes, computational simulations help a lot in the prediction of surgeries outcome and in understanding the haemodynamic of blood flow (Sousa, Castro, Antonio and Chaves, 2011). Numerical modelling technique for flow characterization in the carotid artery has been carried out using CFD simulations which provide a source of references for further validation of other related studies (Ro and Ryou, 2010). Hence, simulations using CFD have significantly contribute in identifying sites which is prone to atherosclerotic lesions formation (Stroud et al, 2002).

1.2 Problem Statements of Research

Stenosis can cause the narrowing of blood vessels that may reduce the flow of blood supply to the organs and tissues. It disturbs the normal function of human circulatory system and is recognized as a leading cause to cardiovascular diseases such as heart attack and stroke. There are various geometrical shapes of stenosis to be proposed in blood flow studies which are either a single stenosis such as a bell shaped, mild, cosine or a multiple stenosis like overlapping and irregular stenosis. Multiple stenosis is usually found at the pulmonary and femoral arteries. Hence, by considering the presence of an overlapping stenosis in this study the stenosis representation would be more critical and almost realistic.

Geometry of an artery takes vital control on the behaviour of blood and with the presence of stenosis, the blood loading through the vessel is disturbed considerably. Bifurcated artery is claimed as the most favoured region to be exposed by atherosclerosis formation since it has a sudden change of area and curvature. It caused huge changes to the flow structure such as the formation of flow recirculation and stagnation. In fact, studies have shown that bifurcated geometry has the greatest influence compared to the fluid viscosity of blood into the flow field.

Another factor that should not be neglected in this haemodynamic analysis is the rheological behaviour of blood. For high shear rate region like a larger artery, representing the blood as a Newtonian fluid is acceptable. It is however not valid when it comes to region with low shear rate value especially downstream of stenosis and smaller arteries. Assuming the blood as a Newtonian fluid will lead to an inaccurate result which further causes incorrect risk estimation of haemodynamic diseases dependency. A few non-Newtonian models which can represent the blood are power-law, generalised power law, Casson, Herschel-Bulkley, and Carreau-Yasuda. The most general model for non-Newtonian blood viscosity over the other models is generalised power law since it includes the power law model at low shear rate, a Newtonian model at high shear rate as well as the Casson model for special cases.

Majority of the treatment for cardiovascular diseases utilised a static uniform magnetic field in magnetic therapy where the blood is considered as a magnetohydrodynamics (MHD) fluid. An exposure to the magnetic field could provide a relaxation to the blood vessel while also maintaining the flow of blood. Suitable application of magnetic field can also be practical for the conditions like muscles sprains, strains, joints pain and headaches. It is also useful for controlling the flow of blood during a surgery due to the production of Lorentz's force that retards the blood motion. However, an excessive exposure to a magnetic field can cause an irreversible alteration and can be risky to the heart. It might also lead to calcium dynamic changes. Therefore, further and careful studies on the influence of an external magnetic field should be employed to examine its effects towards the blood flow motion and progression of arterial diseases.

Finite element method (FEM) has been demonstrated as one of the most effective numerical methods for blood flow modelling that can be performed either in idealized or patient-specific models. The discretization of governing equations using Galerkin weighted residual (GWR) method that transforms the global coordinate to the local one is definitely essential for the solution of the problem within arbitrary geometry. This method also provides a faster rate of convergence for even large scale transient simulations. However, when the classical Galerkin finite element formulation is employed to solve highly viscous incompressible fluid problem, the numerical instabilities like locking and spurious oscillations emerged. Due to that, a stabilized finite element method is required to deal with such difficulties. A few stabilized finite element techniques that can be chosen to overcome those problems are Streamline upwind Petrov-Galerkin (SUPG) method, Galerkin least-squares (GLS), unusual stabilized finite element method (USFEM) and penalty finite element method.

1.3 Objectives of Research

The main aim of this study is to investigate the blood flow characteristics when flowing through a bifurcated artery that has a stenosis on the mother artery. Also, by considering the electrical conductivity property of blood driven by the MHD principles, changes and effects caused by this external forces are studied for further understanding. Hence, the proper governing equations with suitable boundary conditions are needed for that purpose. The resulting governing equations are then solved using a stabilized finite element method which is Galerkin least-squares formulation. To be more specific, the objectives are :

(a) To develop a mathematical model that describes the blood flow phenomenon in a bifurcated artery with the presence of stenosis at a mother artery characterised by generalised power law model with the influence of an external magnetic field.

(b) To develop a source code based on the numerical discretization that has been performed by using Galerkin least-squares (GLS) method.

(c) To investigate the effects of shear-thinning, Newtonian and shear-thickening nature of blood passing through a stenotic vessel using generalised power law model.

(d) To examine the impacts of a uniform magnetic field into non-Newtonian characteristics of blood corresponding to its shear-thinning and shear-thickening nature using generalised power law model in the bifurcated artery with the presence of an overlapping stenosis.

1.4 Scope of Research

As stated in the objectives of research section, this study is concerned with the impact of stenosis and external magnetic field towards any potential haemodynamic disturbances and alterations it might cause either to the velocity, streamlines, wall shear stress or pressure. In this study, the fluid flow behaviour is modelled based on the Navier-Stokes equations with few assumptions imposed to the flow. These few assumptions are steady, laminar and two-dimensional considering a rigid wall with no-slip. Both Newtonian and non-Newtonian fluid models are employed to characterise the fluid viscosity of blood in this study. The existence of an overlapping stenosis at the mother artery and an externally applied uniform magnetic field are also demonstrated in the present investigation while a bifurcated artery is modelled as a bifurcated channel using Cartesian coordinates system. The blood flow will then be analysed using a stabilized finite element method (FEM) formulation where the Galerkin least-squares (GLS) method is utilised to discretize the governing equations and boundary conditions. To validate the developed MATLAB source code, a verification is performed on both previous studies and the result obtained from COMSOL Multiphysics software. Neither clinical nor experimental tests are involved in this research on the human or animal arteries.

1.5 Significances of Research

Outcome from this study on non-Newtonian fluid theory will help in a better understanding on the haemodynamic factors effecting towards the rheological behaviour of blood flow and also on the distraction that might be developed. Hence, study of blood flow through a stenosed bifurcated artery plays a vital role in understanding not only the basic idea but also on the diagnosis and treatment of the cardiovascular disease. A detailed evaluation on the haemodynamic alterations of blood flow is especially essential for the early detection of a highly stenosed artery and may have a very useful clinical values.

Computational fluid dynamics (CFD) simulations used in this study is definitely economically cheap for the physician to predict the outcome of alternative treatment plans for patients through a combination between physiologic and anatomic models. Thus, a construction of a computational model which reflects the true geometry of an artery with stenosis is predominantly needed as a comparison with the available vivo data in order to validate the resulting information on the occlusion caused by the plaque accumulation.

Since most of the medical diagnostic devices make use of a magnetic field in diagnosing cardiovascular diseases, this investigation would be very helpful for medical practitioners to analyse the impact of magnetic field in magnetic therapy towards patients. This study is also effective for the future designs of medical devices that utilise the magnetic field.

1.6 Outline of the Thesis

This thesis consists of six chapters namely introduction, literature review, problem formulation, finite element method implementation, results and discussion and ends with a conclusion. A brief description on the background of the research involving the composition of blood and its characteristics, blood vessels, genesis of atherosclerosis as well as the benefits of magnetic therapy are introduced in Chapter 1. This chapter then also presents the problem statements, objectives, scope and significances of research.

In Chapter 2, reviews on previous studies related to stenotic artery, rheological characteristics of blood, magnetic property of blood as well as the method of solution in solving the blood flow problem are demonstrated. From there, all the problems involved in this study are solved using one of the stabilization techniques of finite element method known as Galerkin-least squares method with the rheological characterisation of blood being represented by Newtonian and generalised power law models. In addition, the electrical conductivity of blood is also considered by including the MHD principles in this study.

Since the arterial geometry is portrayed as a bifurcated artery with the presence of an overlapping stenosis deposited on parent's arterial lumen, a detailed derivation on the arterial geometry involved is included in Chapter 3. Apart from that, the derivation on governing equations of this research for continuity and momentum equations driven by the physical principle of mass conservation and Newton's second law of motion, respectively are explained in the chapter. Based on the assumptions imposed, the mathematical model, boundary conditions and stress tensor are also prescribed.

Chapter 4 describes the stabilization technique of finite element method implementation which involves spatial discretization, interpolation function, Gaussian quadrature to evaluate the numerical integrals and Newton-Raphson method to linearize the non-linear terms. Algorithms developed in solving the proposed problems using the Galerkin-least squares formulation are also explored and presented in this chapter. To validate the source code that has been developed, result on maximum velocity and its location is validated with previous work and simulation from COMSOL multiphysics that is found in a great agreement. Comparison of results obtained for pressure drop and skin friction at certain locations are also reported in a good agreement with the previous work by Xenos and Tzirtzilakis (2013).

In Chapter 5, focus has been made on the effects of different type of fluid characterisations (shear-thinning, Newtonian and shear-thickening) into the blood flow characteristics. The effects of Reynolds number, severity of stenosis as well as generalised power law index into the velocity profiles, pressure, wall shear stress and streamlines pattern are examined and discussed in the chapter. In addition, the second problem seek to examine the influences of external magnetic field into the blood flow behaviours where fluid is assumed to behave according to the nature of shear-thinning and shear-thickening only. Results are presented in a similar form as depicted in the previous problem with an additional effect of Hartmann number introduced for this second problem.

In the last chapter of this thesis, the whole outcome obtained from this study has been summarised as presented in Chapter 6. Furthermore, a few recommendations to improve this current research in the future are suggested.

REFERENCES

Achab, L., Mahfoud, M. and Benhadid, S. (2016) 'Numerical study of the non-Newtonian blood flow in a stenosed artery using two rheological models', *Thermal Science*, 20(2), 449-460.

Ahmed Bakheet Saeed (2017) *Generalized-Power Law Model of Magnetohydrodynamic Blood Flow in an Inclined Stenosed Artery with Body Acceleration*. PhD Thesis, Universiti Teknologi Malaysia, Skudai.

Alimohamadi, H., Imani, M. and Forouzandeh, B. (2015) 'Computational analysis of transient non-newtonian blood flow in magnetic targeting drug delivery in stenosed carotid bifurcation artery', *International Journal of Fluid Mechanics Research*, 42(2), 149-169.

Alimohamadi, H., Imani, M. and Shojaeizadeh, M. (2014) 'Non-Newtonian blood flow in a stenosed artery with porous walls in the present of magnetic field effect', *International Journal of Technology Enhancements and Emerging Engineering Research*, 2(8), 69-75.

Alshare, A., B. Tashtoush, B. and El-Khalil, H. H. (2013) 'Computational modeling of non-newtonian blood flow through stenosed arteries in the presence of magnetic field', *Journal of Biomechanical Engineering*, 135(11), 114503-114506.

Anderson, J. D. (1995) *Computational fluid dynamics : Basic with Applications*. Singapore: McGraw-Hill Book Co..

Bakheet, A., Alnussaiyri, E. A., Ismail, Z. and Amin, N. (2016) 'Blood flow through an inclined stenosed artery', *Applied Mathematical Sciences*, 10(5-8), 235-254.

Bali, R. and Awasthi, U. (2012) 'A Casson fluid model for multiple stenosed artery in the presence of magnetic field', *Applied Mathematics*, 3, 436-441.

Basicmedical Key. (2016, May 25). Composition of Blood. Retrieved from the Basic Medical Key website: http://basicmedicalkey.com/wp-content/uploads/2016/05/F500323f20-01-9780323096003.jpg

Behmardi, D. and Nayeri, E. D. (2008) 'Introduction of Frechet and Gateaux Derivative', *Applied Mathematical Sciences*, 2(20), 975-980.

Bhatnagar, A. and Shrivastav, R. K. (2014) ‘Analysis of MHD flow of blood through a multiple stenosed artery in the presence of slip velocity’, *International Journal of Innovative Research in Advanced Engineering (IJIRAE)*, 1(10), 250-257.

Bose, S. and Banerjee, M. (2015) ‘Magnetic particle capture for biomagnetic fluid flow in stenosed aortic bifurcation considering particle–fluid coupling’, *Journal of Magnetism and Magnetic Materials*, 385(2015), 32-46.

Brandon Pincombe (1999) *A Study of non-Newtonian Behaviour of Blood Flow through Stenosed Arteries*. PhD Thesis, University of Adelaide.

Chakravarty, S. and Mandal, P. K. (1994) ‘Mathematical modelling of blood flow through an overlapping arterial stenosis’, *Mathematical and Computer Modelling*, 19(1), 59-70.

Chakravarty, S. and Mandal, P. K. (1997) ‘An analysis of pulsatile flow in a model aortic bifurcation’, *International Journal of Engineering Science*, 35(4), 409-422.

Corsini, A 1999, *Lecture Notes*, lecture notes distributed in A FE Method for the Computational Fluid Dynamics of Turbomachinery at Technical University of Budapest on Oktober 1999.

Deng, S 2010, *Lecture Notes*, lecture notes distributed in Math 5172 – Finite Element Method at UNC, Charlotte on 2010.

Dunavant, D.A. (1985) ‘High degree efficient symmetrical gaussian quadrature rules for the triangle’, *International Journal for Numerical Methods in Engineering*, 21, 1129-1148.

Fact Monster. (2017, Febuary 20). Types of Blood Vessel. Retrieved from Fact Monster website: http://i.infopls.com/images/ESCI353BLOVES003.gif

Franca, L. P. and Frey, S. L. (1992) ‘Stabilized finite element methods: II. The incompressible Navier-Stokes equations’, *Computer Methods in Applied Mechanics and* Engineering, 99(2), 209-233.

Franca, L. P., Hauke, G. and Masud, A. (2005) ‘Revisiting stabilized finite element methods for the advective–diffusive equation’, *Computer Methods in Applied Mechanics and Engineering*, 195(13), 1560-1572.

Ghaffari, S., Karimi, M. S. and Tonekaboni, S. A. M. (2015) ‘Investigation of magnetic field effect on pulsatile blood flow through the atherosclerotic right coronary artery', *Progress in Computational Fluid Dynamics, An International Journal (PCFD)*, 15(4), 236-246.

Gujral, K., Bhardwaj, R., Arora, J. K. and Singh, S. P. (2016) 'Experimental and analytical study for the radial variation of viscosity of non-Newtonian behavior of blood flow through overlapping stenosis : Computational modeling approach', *International journal of Advanced Scientific Technologies in Engineering and Management Sciences (IJASTEMS)*, 2(12), 96-103.

Harari, I. and Hughes, T. J. R. (1992) 'What are C and h? : Inequalities for the analysis and design of finite element methods', *Computer Methods in Applied Mechanics and Engineering*, 97(2), 157-192.

Hasanzade, A. (2015) 'Numerical study on the magnetic fluid delivering through a non-newtonian viscoplastic blood flow', *International Journal of Advanced Research in Science, Engineering and Technology*, 2(3), 550-556.

Hayat, T., Afsar, A., Khan, M. and Asghar, S. (2007) 'Peristaltic transport of a third order fluid under the effect of a magnetic field', *Computers & Mathematics with Applications*, 53(7), 1074-1087.

Heinrich, J. C. and Vionnet, C. A. (1995) 'The penalty method for the Navier-Stokes equations', *Archives of Computational Methods in Engineering*, 2(2), 51-65.

Huh, H. K., Ha, H. and Lee, S. J. (2015) 'Effect of non-Newtonian viscosity on the fluid-dynamic characteristics in stenotic vessels', *Experiments in Fluids*, 56(8), 1-12.

Ikbal, Md. A., Chakravarty, S., Wong, K. K. L., Mazumdar, J. and Mandal, P. K. (2009) 'Unsteady response of non-Newtonian Blood Flow through a Stenosed Artery in Magnetic Field', *Journal of Computational and Applied Mathematics*, 230(1), 243-259.

Jahangiri, M., Saghafian, M. and Sadeghi, M. R. (2015) 'Numerical study of turbulent pulsatile blood flow through stenosed artery using fluid-solid interaction', *Computational and Mathematical Methods in Medicine*, 2015(515613), 1-10.

Jakub Sistek (2004) *Stabilization of Finite Element Method for Solving Incompressible Viscous Flow*. Diploma Thesis, Czech Technical University in Prague.

Kefayati, S. and Poepping, T. L. (2010) 3-D Characterization and Shear Stress in a Stenosed Carotid Artery Bifurcation Model using Stereoscopic PIV Technique. *32nd Annual International Conference of the IEEE EMBS*. August 31-September 4, 2010. Buenos Aires, Argentina: IEEE, 3386-3389.

Kowalewski, T. A. 2005, *Lecture Notes 6*, lecture notes distributed in Blood Flow Modelling and Diagnostics at Institute of Technological Research, Poland on 20-23 June 2005.

Li, J. and Huang, H. (2010) Effect of Magnetic Field on Blood Flow and Heat Transfer through a Stenosed Artery. *3rd International Conference on Biomedical Engineering and Informatics (BMEI 2010)*. 16-18 October. Academy of Frontier Science, China: IEEE, 2028-2032.

Liu, B. and Tang, D. (2011) 'Influence of non-Newtonian properties of blood on wall shear stress in human atherosclerotic right coronary arteries', *Mol Cell Biomech*, 8(1), 73-90.

Lou, Z. and Yang, W. –J. (1993) 'A computer simulation of the non-Newtonian blood flow at the aortic bifurcation', *J. Biomechanics*, 26(1), 37-49.

Machado, F., Zinani, F. and Frey, S. (2007) Galerkin Least-Squares Approximations for Herschel-Bulkley Fluid Flows through an Axisymmetric Abrupt Expansion. *19th International Congress of Mechanical Engineering*. 5-9 November. Brasilis; ABCM, 1-8.

Misra, J. C. and Shit, G. C. (2007) 'Effect of magnetic field on blood flow through an artery : A numerical model', *Tom*, 12(4), 3-16.

Mohammed Yahya (2010) *Three Dimensional Finite-Element Modeling of Blood Flow in Elastic Vessels : Effects of Arterial Geometry and Elasticity on Aneurysm Growth and Rupture.* MSc Thesis, Ryerson University.

Nadeem, S., Akbar, N. S., Hayat, T. and Hendi, A. A. (2012) 'Influence of heat and mass transfer on Newtonian biomagnetic fluid of blood flow through a tapered porous arteries with a stenosis', *Transport in Porous Media*, 91(1), 81-100.

Oliver Kayser-Herold (2005) *Least-squares Methods for the Solution of Fluid-structure Interaction Problems.* PhD Thesis, University Braunschweig.

Papafaklis, M. I. and Lampros, K. M. (2012) Intravascular Imaging and Haemodynamics: The Role of Shear Stress in Atherosclerosis and In-Stent Restenosis, in Intravascular Imaging. In: Tsakanikas, V. D., Michalis, L. K., Fotiadis, D. I., Naka, K. K. and Bourantas, C. V. (Ed.) *Current Applications and Research Developments.* (pp. 326-348). Hershey, PA, USA: IGI Global.

Rabby, M. G., Shupti, S. P. and Molla, M. M. (2014) 'Pulsatile non-Newtonian laminar blood flows through arterial double stenoses', *Journal of Fluids*, 2014(757902), 1-13.

Raptis, A., Xenos, M., Tzirtzilakis E. and Matsagkas, M. (2014) Finite element analysis of magnetohydrodynamic effects on blood flow in an aneurysmal geometry', *Physics of Fluids*, 26(10), 101901-101914.

Riahi, D. N., Roy, R. and Cavazos, S. (2011) 'On arterial blood flow in the presence of an overlapping stenosis', *Mathematical and Computer Modelling*, 54(11–12), 2999-3006.

Ro, K. C. and Ryou, H. S. (2010) 'Numerical study on turbulent blood flow in a stenosed artery bifurcation under periodic body acceleration using a modified k-ε model', *Korea-Australia Rheology Journal*, 22(2), 129-139.

Roh, H. -W. and Kim, J. -S. (2003) 'Effects of the velocity waveform of the physiological flow on the hemodynamics in the bifurcated tube', *KSME International Journal*, 17(2), 296-309.

Roy, R. and Riahi, D. N. (2013) 'On blood flow in an artery with an unsteady overlapping stenosis: mathematical and computer modeling', *International Journal of Fluid Mechanics Research*, 40(1), 9-26.

Sankar, A. R., Gunakala, S. R. and Comissiong, D.M.G. (2013) 'Two-layered blood flow through a composite stenosis in the presence of a magnetic field', *International Journal of Application or Innovation in Engineering and Management*, 2(12), 30-41.

Science Daily. (2011, June 8). Using magnets to help prevent heart attacks: Magnetic field can reduce blood viscosity, physicist discovers. Retrieved from ScienceDaily website: www.sciencedaily.com/releases/2011/06/110607121523.htm

Shaikh, A. A., Chandio, F. H. and Qureshi, S. (2015) 'Computation of Wall Shear Stresses across Various Stenosis Length in Common Carotid Artery', *Sindh University Research Journal (Science Series)*, 47(2), 367-370.

Sharma, M. K., Nasha, V. and Sharma, P. R. (2016) 'A study for analyzing the effect of overlapping stenosis and dilatation on non-Newtonian blood flow in an inclined artery', *Journal of Biomedical Science and Engineering*, 9, 576-596.

Sherwood, J. M., Kaliviotis, E., Dusting, E. and Balabani, S. (2014) 'Hematocrit, viscosity and velocity distributions of aggregating and non-aggregating blood in a bifurcating microchannel', *Springer*, 13(2014), 259-273.

Siebert, M. W. and Fodor, P. S. (2009) Newtonian and non-Newtonian Blood Flow Over a Backward-Facing Step - A Case Study. *Proceedings of the COMSOL Conference 2009 Boston*. 30 March. Boston, 1-5.

Skála, J. and Bárta, M. (2012) ‘LSFEM Implementation of MHD Numerical Solver’, *Applied Mathematics*, 3(11A), 1842-1850.

Sousa, L., Castro, C., Antonio, C. and Chaves, R. (2011) ‘Computational techniques and validation of blood flow simulation’, *WSEAS TRANSACTIONS on BIOLOGY and BIOMEDICINE*, 8(4), 145-155.

Srinivasacharya, D. and Rao, G. M. (2017) ‘Micropolar fluid flow through a stenosed bifurcated artery’, *Nonlinear Analysis : Modelling and Control*, 22(2), 147-159.

Srivastava, V. P. and Rastogi, R. (2010) ‘Blood flow through a stenosed catheterized artery : Effects of hematocrit and stenosis shape’, *Computers & Mathematics with Applications*, 59(4), 1377-1385.

Stroud, J. S., Berger, S. A. and Saloner, D. (2002) ‘Numerical analysis of flow through a severely stenotic carotid artery bifurcation’, *Journal of Biomechanical Engineering*, 124(1), 9-20.

Tan Yan Bin (2017) The Effects of Gravitational Acceleration on Micropolar Fluid Model of Blood Flow in a Bifurcated Stenosed Artery. PhD Thesis, Universiti Teknologi Malaysia.

Tzirakis, K., Botti, L., Vavourakis, V. and Papaharilaou, Y. (2015) ‘Numerical modeling of non-Newtonian biomagnetic fluid flow’, *Computers & Fluids*, 126(2016), 170-180.

Tzirtzilakis, E. E. (2005) ‘A mathematical model for blood flow in magnetic field’, *Physics of Fluids*, 17(077103), 1-15.

Varshney, G., Katiyar, V. K. and Kumar, S. (2010) ‘Effect of magnetic field on the blood flow in artery having multiple stenosis : A numerical study’, *International Journal of Engineering, Science and Technology*, 2(2), 67-82.

Venkat Pradeep Vallala (2009) *Alternative Least-squares Finite Element Models of Navier-Stokes Equations for Power-law Fluids*. MSc Thesis, Texas A&M University.

Weddell, J. C., Kwack, J. H., Imoukhuede, P. I. and Masud, A. (2015) ‘Hemodynamic analysis in an idealized artery tree: differences in wall shear stress between Newtonian and non-Newtonian blood models’, *PLos ONE*, 10(4), 1-23.

World Health Organization 2017. (2016, September). Cardiovascular Diseases (CVDs). Retrieved from the World Health Organization website: http://www.who.int/mediacentre/factsheets/fs317/en/

Xenos, M. A. and Tzirtzilakis, .E. E. (2013) ‘MHD effects on blood flow in a stenosis’, *Advances in Dynamical Systems and Applications*, 8(2), 427-437.

Zaman, A., Ali, N., Sajid, M. and Hayat, T. (2015) ‘Effects of unsteadiness and non-Newtonian rheology on blood flow through a tapered time-variant stenotic artery’, *AIP Advances*, 5(3), 1-13.

Zarins, C. K., Giddens, D. P., Bharadvaj, B. K., Sottiurai, V. S., Mabon, R. F. and Glagov, S. (2015) ‘Carotid bifurcation atherosclerosis quantitative correlation of plaque localization with flow velocity profiles and wall shear stress’, *Journal of the American Heart Association*, 53(4), 502-514.

Zinani, F. and Frey, S. (2007) ‘Galerkin least-squares solutions for purely viscous flows of shear-thinning fluids and regularized yield stress fluids’, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 29(4), 432-443.