LINGUISTIC TEMPORAL DISCRETE Z-NUMBERS AND ITS APPLICATION

MUJAHID ABDULLAHI

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy

Faculty of Science
Universiti Teknologi Malaysia

MAY 2020

DEDICATION

This thesis is dedicated to my beloved father Mal. Abdullahi Hudu and my mother Haj. Shaffa'u Shehu

ACKNOWLEDGEMENT

I owe my gratitude to ALLAH (SWT), the Most Gracious and the Most Merciful whose guidance and protection sustained me throughout my entire life and makes my educational career a reality with enormous success.

I would like to use this opportunity to express my sincere gratitude to my supervisors Professor Dr. Tahir Ahmad and Dr. Vinod Ramachandran for their fatherly advice, supports, guidance, encouragement and patience throughout the period of this research. Without their guidance, the research would have been impossible. Thank you so much to my beloved family particularly my father Mal. Abdullahi Hudu, my lovely mother Shaffa'u Shehu and my elder brother Dr. Aminu Abdullahi for their endless support. There are simply no words to describe my appreciation for all the help and guidance you have given me from the very beginning of my career until now. Without your guidance, I would not have been where I am today.

A special thank goes to my wife Khadijah Muhammad Abdullahi and my son Abdullahi Mujahid Abdullahi (Obaidullah) for their patience, tolerance, and sense of understanding.

Also I would like to thank the entire staff of Department of Mathematical Sciences, Faculty of Science, Universiti Teknologi Malaysia, for providing support and the necessary facilities. I am particularly grateful to the management of Sule Lamido University for nominating me to receive the TEDFUND intervention, which made the pursuit of this PhD degree possible.

Last but not least, I wished to thank all my siblings, relatives and friends for their understanding, continuous supports and motivations throughout this study.

ABSTRACT

The concepts of temporal intuitionistic fuzzy set and temporal fuzzy set were introduced by earlier researchers to model Spatio-temporal and dynamic motions of complex physical systems, respectively. However, the temporal intuitionistic fuzzy set has not been properly applied to solve systems with temporal information. Furthermore, both concepts do not address the problem of uncertainty with respect to the time of occurrence. In this thesis, the discrete and continuous Z-numbers are proven to be ordered by employing a linear ordering relation. This new concept of ordered discrete Z-number leads to the development of two families of temporal Z-number, namely, linguistic temporal discrete Z-number (LTDZ) and temporal discrete Z-number (TDZ). Some of the basic arithmetic operations for LTDZ are introduced and their properties are proven in this thesis. In relation to that, a method for measuring uncertainty for LTDZ with respect to its time of occurrence is proposed by modifying the method for measuring the uncertainty of discrete Z-number. The temporal discrete Z-number is developed for the purpose of analyzing the electroencephalographic (EEG) signal of an epileptic seizure. Numerical examples are included to show the feasibility of the proposed concepts.

ABSTRAK

Konsep set kabur intuitionistik temporal dan set kabur temporal diperkenalkan oleh penyelidik-penyelidik terdahulu untuk memodelkan Spatio-temporal dan gerakan dinamik bagi sistem fizikal yang kompleks. Namun begitu, intuitionistik temporal masih belum diaplikasikan dengan baik bagi menyelesaikan permasalahan sistem yang melibatkan maklumat temporal. Tambahan pula, kedua-dua konsep tersebut tidak menangani masalah ketakpastian berkenaan dengan masa kejadian. Dalam tesis ini, nombor-Z diskrit dan selanjar telah terbukti ketertibannya dengan menggunakan hubungan tertib linear. Konsep baharu ketertiban nombor-Z diskrit ini membawa kepada pengembangan dua kumpulan nombor-Z temporal, iaitu, Linguistic Temporal Discrete Z-number (LTDZ) dan Temporal Discrete Z-number (TDZ). Beberapa operasi asas aritmetik untuk LTDZ diperkenalkan dan sifat-sifatnya terbukti dalam tesis ini. Sehubungan dengan itu, kaedah untuk mengukur ketakpastian bagi LTDZ berkenaan dengan masa kejadiannya dicadangkan dengan mengubah kaedah untuk mengukur ketakpastian nombor-Z diskrit. Nombor-Z diskrit temporal dikembangkan untuk tujuan menganalisa isyarat Electroencephalography (EEG) dari serangan sawan. Contoh berangka disertakan untuk menunjukkan kebolehlaksanaan konsep-konsep yang dicadangkan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

BCC	-	Banker, Chames, Cooper
BPAs	-	Basic Probability Assignments
CCR	-	Charnes, Cooper, and Rhodes
CCR	-	Capacity Charging ratio
CPS	-	Centroid Point and Spread
CT	-	Continuous Time
CVaR	-	Conventional Value at Risk
CWW	-	Computing With Word
CZ	-	Continuous Z-number
DEA	-	Data Envelopment Analysis
DT	-	Discrete Time
DZ	-	Discrete Z-number
DMU	-	Decision Making Unit
EEG	-	Electroencephalography
FFLP	-	Fully Fuzzy Linear Programming
FPOs	-	Fuzzy Differential Equations
FMEA	-	Failure Model and Effects Analysis
FPO	-	Fuzzy Pareto Optimality
GA	-	Generic Algorithm
IFS	-	Intuitionistic Fuzzy Set
ILFS	-	Intuitionistic L-Fuzzy Set
INFS	-	Intuitionistic N-Fuzzy Set
LTDZ	-	Linguistic Temporal discrete Z-number
MCGDM	-	Multi criteria Group Decision Making
MCDM	-	Multi criteria Decision Making
NL	-	Natural Language
NLP	-	Natural Language Processing

ODEs	-	Ordinary Differential Equations
TDZ	-	Temporal discrete Z-number
TODIM	-	An acronym in Portuguese of interactive and MCDM
TIFS	-	Temporal Intuitionistic Fuzzy Set
UTM	-	Universiti Teknologi Malaysia
Z-DEA	-	Z-number Data Envelopment Analysis

LIST OF SYMBOLS

α	-	Alpha
β	-	Beta
$\in$	-	Element of
$=$	-	Equal to
$\equiv$	-	Equivalent
$\forall$	-	For All
$f : X \longrightarrow Y$	-	f is a mapping from X to Y
$>$	-	Greater than
$\geq$	-	Greater than or equal to
$\longrightarrow, \Longrightarrow$	-	Implies
$\Longleftrightarrow$	-	If and only if
$\cap$	-	Intersection
$\int$	-	Integral
∞	-	Infinity
Q^{-1}	-	Inverse of Q
$\vee$	-	Joint
$<$	-	Less than
$\leq$	-	Less than or equal to
$\wedge$	-	Meet
$\neq$	-	Not equal to
$\mathbb{N}$	-	Natural number
$\subset$	-	Proper subset
$R(X)$	-	Restriction on X
$\mathbb{R}$	-	Real number
$\subseteq$	-	Subset
$\vert, :$	-	Such that
$\sum$	-	Summation

$\exists$	-	There exist
$\prec'$	-	Temporal ordering

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

In 1965, Zadeh introduced the concept of fuzzy sets to model vague phenomena [1]. The notion of fuzzy sets provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects to the framework used in the case of ordinary sets. However, it is more general than the latter as it has wider scope of applicability, particularly in the fields of pattern classification and information processing. Also, fuzzy set theory provides a strict mathematical framework in which vague conceptual phenomena can be precisely and rigorously studied. Being a constitutive part of modern mathematics, fuzzy sets are not intended to replace traditional set theory but to provide a formal way of describing the real-world phenomena [2].

Goguen in [3] introduced the idea of L-fuzzy sets as a generalization of Zadeh's fuzzy sets. From another direction, Atanassov [4] introduced the concepts of intuitionistic fuzzy sets (IFS) and intuitionistic L-fuzzy sets (ILFS), as a generalization of fuzzy set and L-fuzzy set respectively.

Real-world information is flawed, and natural language is often used to represent this feature. On one hand, such information is often characterized by fuzziness, which implies that soft constraints are imposed on the values of variables of interest. Considering only fuzziness when dealing with real world, imperfect information is not enough. Partial reliability is another essential property of the information. Indeed, any estimation of values of interest, be it precise or soft, are subject to confidence in the sources of information. When one deals with knowledge, assumptions, intuition, envision, and experience, in general cannot completely cover the whole complexity of real-world phenomena. Thus, fuzziness from one side and partial reliability from the other side are strongly associated with each other [5].

In order to take into account this fact, Zadeh in [5] introduced the concept of Z-number as an adequate formal description of real-world information. Basically the concept of Z -number relates to the issue of reliability of information. The concept of Z-number is intended to provide a basis for computation with numbers which are not totally reliable [6].

1.2 Background and Motivation

Time is one of the significant features in some real world problems. It is a monotonic and a fundamental aspect for modeling dynamic information and systems, such as in the realms of decision making, traffic, weather, medicine, economics and signal analysis. Time has two forms of representation which are; time interval and time instance. Time interval represents the range between two points in time while time instance represents a point in time. Most of temporal models are based on either one of these forms [7]. It was mentioned in [7, 8] that temporal information is not crisp, but it's uncertain and vague. Therefore, both fuzzy and intuitionistic fuzzy sets have been used to represent temporal information.

Temporal intuitionistic fuzzy set (TIFS) was introduced in [9] by Atanassov to handle a spatio-temporal problem, where the degree of membership and non-membership of TIFS changes with respect to time moments. However, the study on TIFS is quite scares. Some works on theoretical concept of TIFS can be found in [7, 10, 11, 12, 13]. Chan and Tu in [14] suggested a time-validated intuitionistic fuzzy set in order to make an earlier decision based on the desired information level, which is based on TIFS.

In another direction, Kasanovic introduced the concept of temporal fuzzy set (TFS) to analyze dynamic motions of complex systems and their signals whenever there exists an observable structure or regularity in system behavior [2]. The concept of TFS, has been applied in the modeling of sleep onset dynamic from electroencephalography (EEG) signal. For instance, on the transition from awake to deep sleep that often occurs within the first hour following the 'light out' event. Moreover, it can also be applied to model problems in areas such as system identification, feature extraction and sensor fusion [15]. In short, temporal fuzzy sets are fuzzy sets constructed from universe whose elements are ordered in time [15]. However, the family of temporal fuzzy sets does not address the issue of uncertainty about a time of occurrence. It rather offers an answer to what happens and how much it happens at any time instant during the observation of a physical process.

By studying the two temporal sets through literature review, it was observed that the temporal intuitionistic fuzzy set is more suited to deal with linguistic temporal problems in some areas such as decision making, medical diagnosis, weather, traffic and others. However, the concept of temporal fuzzy set is constructed specifically for analyzation and modeling of complex physical systems, particularly EEG signals. Therefore, dynamic information or temporal information is often unreliable a

The concept of intuitionistic fuzzy, intuitionistic L-fuzzy and L-fuzzy sets are proven to be generalization of fuzzy sets. These concepts are all equivalent. However, the concept of intuitionistic N-fuzzy sets, proposed in [16] is not a generalization of IFS. In this study, the concepts of ILFS and intuitionistic N-fuzzy set (INFS) are proven to be equivalent. Moreover, two families of the temporal discrete Z-number are developed. The first one is linguistic temporal discrete Z-number, which is based on temporal intuitionistic fuzzy set. By utilizing the proposed concept, a model is developed to solve a linguistic temporal problem. The second family is the temporal discrete Z-number. It is constructed to analyze complex physical systems, particularly to serve as a tool for analyzing the EEG signal of an epileptic seizure. Both families have addressed the issue of uncertainty during the time of occurrence. In this study, the discrete Z-number is chosen over its counterpart continuous Z-number, because the natural language-based information has a discrete framework. Moreover, the discrete Z-number offers less computational difficulty.

1.3 Problem Statement

Several approaches have been introduced to generalize fuzzy set, such as L-fuzzy set, IFS, and ILFS. However, the notion of intuitionistic N-fuzzy set was proposed in [16], such that *N* stands for negative. The only difference between IFS and INFS is that, in IFS the universe of discourse maps to the closed interval $[0, 1]$, while in INFS it maps to $[-1,0]$. Hence, the INFS is not a generalization of IFS as the authors suggested. In response to this issue, this study shows that IFS and INFS are equivalent.

On the other hand, the concepts of TIFS and TFS are introduced to model spatio-temporal problems and analyzation of complex physical systems respectively. However, TIFS has not been applied properly to solve problems which involve temporal information. Also, the concept of TFS lacks the capability to tackle the issue of uncertainty about the time of occurrence in a dynamic system. Therefore, based on this fact and the ability of discrete Z-number to handle fuzziness and uncertainty concurrently, the concepts of linguistic temporal discrete Z-number

1.4 Objectives of the Study

The objectives of this study are to

1. Prove that discrete and continuous Z-numbers can be ordered.
2. Introduce the concept of linguistic temporal discrete Z-number and to show that it can be used to solve temporal linguistic problems.
3. Introduce the concept of temporal discrete Z-number and to show that the proposed concept can serve as a tool for analyzing EEG signal of an epileptic seizure.

1.5 Scope of the Study

This study focuses on the construction of two families of temporal Z-number, the first is linguistic temporal discrete Z-number, which is based on the concept of temporal intuitionistic fuzzy set. It was introduced in [9] by Atanassov to handle spatiotemporal problems. The second family is the temporal discrete Z-number, which is based on the idea of temporal fuzzy set. It was introduced by Kosanovic in [2] to analyze dynamic motions of a complex system.

1.6 Significance of the Study

Ordered discrete or continuous Z-number is the most important concept needed to construct any form of temporal discrete Z-number. This research have proved that discrete and continuous Z-numbers can be ordered by creating a relation between set of discrete or continuous Z-number and any arbitrary ordered set in $\mathbb{R}$. Moreover, the concept of ordered discrete Z-number is utilized to construct two families of temporal discrete Z-number, namely the linguistic temporal discrete Z-number and the temporal discrete Z-number.

Some of the real-world problems in the areas of medicine, economics, traffic flows, decision making, and weather involve dealing with temporal information, where such information are uncertain and vague. The proposed linguistic temporal discrete Z-number could solve such problems efficiently. Another most significant advantage of this research is that the temporal discrete Z-number (TDZ) can be utilized as a tool for analyzation of the EEG signal, particularly the analysis of EEG signal of the epileptic seizure.

1.7 Research Methodology

Based on the research objectives and problem statement, the main aim of this research is to develop two families of temporal Z-number. The methodology employed for the purpose is outlined in this section. First of all, an ordered Z-number (discrete or continuous) is a prerequisite component for the construction of temporal Z-number. Therefore, the idea of an ordered fuzzy set is employed to construct an ordered discrete Z-number, whereby a linear ordering relation $\prec$ is used to establish the relation between a set of discrete or continuous Z-numbers with any arbitrary ordered set in R whereby the relation $\Longleftrightarrow$ is proven to be well defined between a set of discrete Z-numbers $(\bar{Z}_D, \prec)$ and arbitrarily ordered set $G \subset \mathbb{R}$. In short, the construction relies on a sequence of mapping and relations $\prec$: $(\bar{Z}_D \times G, \prec) \longrightarrow G \ni (Z_1, g_1) \prec (Z_2, g_2) \Longleftrightarrow g_1 \prec g_2$ which immediately implies $(\bar{Z}_D \times G, \prec)$ is totally ordered, hence, $(\bar{Z}_D, \prec)$ must be totally ordered too. Similar arguments are used for continuous Z-numbers.

Linguistic temporal discrete Z-number is developed based on the idea of temporal intuitionistic fuzzy set of Atanassov [4] and the proposed concept of ordered discrete Z-number above. Some of its basic arithmetic operations and properties are presented by converting LTDZ into temporal fuzzy set on the basis of fuzzy expectation. Some suitable examples are provided to show the feasibility of the proposed concept and model.

Similarly, temporal discrete Z-number is constructed based on the idea of temporal fuzzy set of Kosanovic [2]. To estimate the membership function of the components of temporal discrete Z-number, a relationship between type-2 temporal fuzzy set and temporal Z-number is established. Then a Z-number clustering algorithm is used to obtain the membership function of TDZ. For example, analyzing the EEG signal of an epileptic seizure is presented to show the implementation of the proposed concept.

1.8 Research Outline

This study consists of seven chapters as shown by Figure 1.1. The first chapter provides the general information about the study. Chapter 2 gives the literature review on fuzzy set, fuzzy numbers, L-fuzzy sets and dynamic systems, and comprehensive overview of Z-numbers. Chapter 3 presents the primary mathematical concepts that have been utilized in this study. Chapter 4 contains two theorems and a corollary that proved IFS, ILFS, INFS and L-fuzzy sets are equivalent. Moreover, it also discusses the idea of ordered discrete and continuous Z-numbers. Chapter 5 presents the concept of LTDZ and a model that is used to solve a linguistic temporal traffic flow problem. Chapter 6 introduces the notion of TDZ which can be served as a tool to analyze the EEG signal of an epileptic seizure. Finally, chapter 7 provides the conclusion and recommendation for future work.

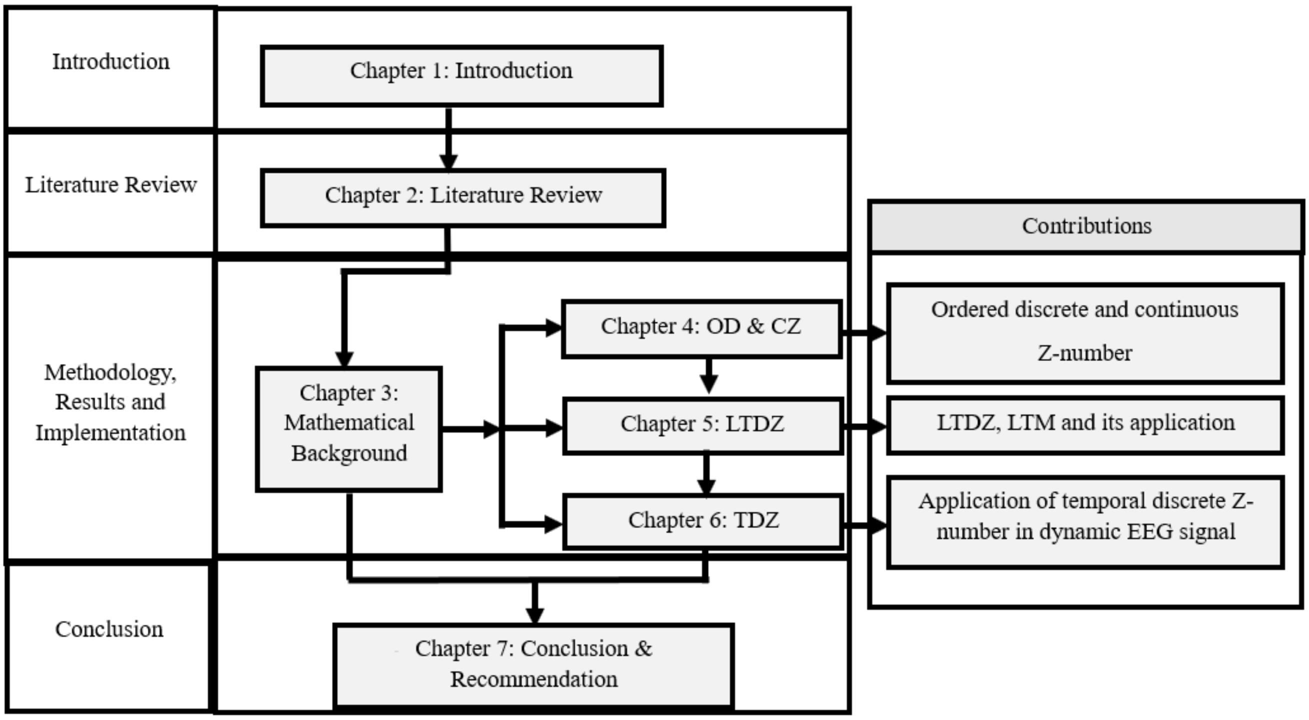

Figure 1.1: Research framework

REFERENCES

1. Zadeh, L. A. Fuzzy Sets. *Information and Control*, 1965. 8(3): 338–353.
2. Kosanovic, B. R., Chaparro, L. F. and Sclabassi, R. J. Signal Analysis in Fuzzy Information Space. *Fuzzy Sets and Systems*, 1996. 77(1): 49–62.
3. Goguen, J. A. L-fuzzy sets. *Journal of mathematical analysis and applications*, 1967. 18(1): 145–174.
4. Atanassov, K. T. Intuitionistic fuzzy sets. *Fuzzy sets and Systems*, 1986. 20(1): 87–96.
5. Zadeh, L. A. A note on Z-numbers. *Information Sciences*, 2011. 181(14): 2923–2932.
6. Zadeh, L. A. The concept of a Z-number-A new direction in uncertain computation. *Information Reuse and Integration (IRI), 2011 IEEE International Conference on*. IEEE. 2011. xxii–xxiii.
7. Parvathi, R. and Geetha, S. P. A note on properties of temporal intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets*, 2009. 15(1): 42–48.
8. Nagypal, G. and Motik, B. A fuzzy model for representing uncertain, subjective, and vague temporal knowledge in ontologies. *OTM Confederated International Conferences" On the Move to Meaningful Internet Systems"*. Springer. 2003. 906–923.
9. Atanassov., K. T. Temporal intuitionistic fuzzy sets. *Dokladi na Bolgarskata Akademiya na Naukite*, 1991. 44: 5–7.
10. Kutlu, F. and Bilgin, T. Temporal intuitionistic fuzzy topology in Sostak's sense. *Notes on Intuitionistic Fuzzy Sets*, 2015. 21(2): 63–70.
11. Kutlu, F., Atan, O. and Bilgin, T. Distance measure, similarity measure, entropy and inclusion measure for temporal intuitionistic fuzzy sets. *IFSCOM*, 2016: 130.
12. Kutlu, F., Ramadan, A. A. and Bilgin, T. On compactness in temporal intuitionistic fuzzy Sostak topology. *Notes on Intuitionistic Fuzzy Sets*, 2016. 22(5): 46–62.

13. Yilmaz, S. and Cuvalcioglu, G. On level operators for temporal intuitionistic fuzzy sets. *Notes on Intuitionistic Fuzzy Sets.*, 2014. 20(2): 6–15.

14. Chen, L.-H. and Tu, C.-C. Time-validating-based Atanassov's intuitionistic fuzzy decision making. *IEEE Transactions on Fuzzy Systems*, 2014. 23(4): 743–756.

15. Kosanovic., B. R. *Signal Analysis in Fuzzy Information Space*. Ph.D. Thesis. University of Pittsburgh. 1995.

16. Akram, M., Kavikumar, J. and Khamis, A. B. Intuitionistic N-fuzzy set and its application in biΓ-ternary semigroups. *Journal of Intelligent & Fuzzy Systems*, 2016. 30(2): 951–960.

17. Allenby, R. B. *Rings, fields and groups: An introduction to abstract algebra*. Hodder Arnold. 1983.

18. Russell, B. Vagueness. *Australasian J. Psychol.Philos*, 1923. 1: 84–92.

19. Zimmermann, H.-J. *Fuzzy Set Theory and Its Applications*. Springer Science & Business Media. 2001.

20. Rescher, N. *Many-Valued Logic*. McGraw Hill,New York. 1969.

21. Birkhoff, G. *Lattice theory*. vol. 25. American Mathematical Society New York. 1948.

22. Smuts, J. Holism and Evolution Macmillan. *Compass/Viking Press (1961) reprint*, 1926.

23. Black, M. Vagueness. An exercise in logical analysis. *Philosophy of science*, 1937. 4(4): 427–455.

24. Goguen. L-fuzzy sets. *Journal of mathematical analysis and applications*, 1967. 18(1): 145–174.

25. Goguen, J. A. The logic of inexact concepts. *Synthese*, 1969. 19(3): 325–373.

26. Chon, I. Fuzzy partial order relations and fuzzy lattices. *Korean J. Math*, 2009. 17(4): 361–374.

27. Kosanovic, B. R., Chaparro, L. F. and Scalabassi, R. J. Signal modeling with dynamic fuzzy sets. *IEEE International Conference on Acoustics, Speech, and Signal Processing Conference Proceedings*. 1996.

28. Torchinsky, A. *Real Variables*. Westview Press. 1994.

29. Davey, B. A. and Priestley, H. A. *Introduction to lattices and order*. Cambridge university press. 2002.

30. Hong, D. H. A note on operations on fuzzy numbers. *Fuzzy Sets and Systems*, 1997. 87(3): 383–384.

31. Casasnovas, J. and Riera, J. V. On the addition of discrete fuzzy numbers. *Proceedings of the 5th WSEAS international conference on Telecommunications and informatics*. World Scientific and Engineering Academy and Society (WSEAS). 2006. 432–437.

32. Zadeh, L. A. Calculus of Fuzzy Restrictions. *Fuzzy Sets, Fuzzy Logic, and Fuzzy Systems*, 1996. 6: 210.

33. Charles, M., Grinstead, J. and Snell, L. Introduction to probability. *American Mathematical Society*, 1997.

34. Zadeh, L. A. Probability measures of fuzzy events. *Journal of mathematical analysis and applications*, 1968. 23(2): 421–427.

35. Wang, G.-j. and He, Y.-Y. Intuitionistic fuzzy sets and L-fuzzy sets. *Fuzzy Sets and Systems*, 2000. 110(2): 271–274.

36. Pavelka, J. On fuzzy logic II. Enriched residuated lattices and semantics of propositional calculi. *Mathematical Logic Quarterly*, 1979. 25(7-12): 119–134.

37. Guo-Jun, W. Theory of topological molecular lattices. *Fuzzy sets and systems*, 1992. 47(3): 351–376.

38. Zadeh, L. A. Similarity relations and fuzzy orderings. *Information sciences*, 1971. 3(2): 177–200.

39. Dubois, D. J. *Fuzzy sets and systems: theory and applications*. vol. 144. Academic press. 1980.

40. Venugopalan, P. Fuzzy ordered sets. *Fuzzy sets and systems*, 1992. 46(2): 221–226.

41. Wang, G., Wu, C. and Zhao, C. Representation and Operations of Discrete Fuzzy Numbers. *Southeast Asian Bulletin of Mathematics*, 2005. 29(5).

42. Mendel, L. F., J.M. and Zhai, D. α-Plane Representation for Type-2 Fuzzy Sets: Theory and Applications. *IEEE Transactions on Fuzzy Systems,*, 2009. 5(17): pp.1189–1207.

43. Zadeh, L. A. The concept of a linguistic variable and its application to approximate reasoning. *Information sciences*, 1975. 8(3): 199–249.

44. Zadeh, L. A. Fuzzy sets as a basis for a theory of possibility. *Fuzzy sets and systems*, 1999. 100: 9–34.

45. Katok, A. and Hasselblatt, B. *Introduction to the modern theory of dynamical systems*. vol. 54. Cambridge university press. 1997.

46. Zakaria, F. *Dynamic Profiling of Electroencephalographic Data During Seizure Using Fuzzy Information Space.* Ph.D. Thesis. Universiti Teknologi Malaysia. 2008.

47. Mesarovic, M. D. and Takahara, Y. *General Systems Theory:Mathematical Foundations.* Academic Press,New York. 1975.

48. Bhatia, N. P. and Szegö, G. P. Dynamical Systems: Stability Theory and Applications. 1967.

49. Ogata, K. Modern Control Engineering. 2002.

50. Struble, R.-A. Nonlinear Differential Equations. 1962.

51. Pontryagin, L. *Ordinary Differential Equations*. Addison-Wesley,. 1962.

52. Aliev, R. A., Alizadeh, A. V. and Huseynov, O. H. The arithmetic of discrete Z-numbers. *Information Sciences*, 2015. 290: 134–155.

53. Aliev, R. A., Huseynov, O. H. and Zeinalova, L. M. The arithmetic of continuous Z-numbers. *Information Sciences*, 2016. 373: 441–460.

54. Castillo, O. *Type-2 fuzzy logic in intelligent control applications*. vol. 272. Springer. 2012.

55. Castillo, O. and Melin, P. Type-2 Fuzzy Logic: Theory and Applications. 2008.

56. Castillo, O., Melin, P. and Pedrycz, W. Design of interval type-2 fuzzy models through optimal granularity allocation. *Applied Soft Computing*, 2011. 11(8): 5590–5601.

57. Melin, P. and Castillo, O. A review on the applications of type-2 fuzzy logic in classification and pattern recognition. *Expert Systems with Applications*, 2013. 40(13): 5413–5423.

58. Aliev, R. A., Huseynov, O. H., Aliyev, R. R. and Alizadeh, A. A. *The arithmetic of Z-numbers: Theory and applications*. World Scientific. 2015.

59. Kang, B., Wei, D., Li, Y. and Deng, Y. A method of converting Z-number to classical fuzzy number. *Journal of information and computational science*, 2012. 9(3): 703–709.

60. Zadeh, L. A., Tadayon, S. and Tadayon, B. Methods and Systems for Applications for Z-numbers, 2014. US Patent App. 14/218,923.

61. Massanet, S., Riera, J. V. and Torrens, J. A New Vision of Zadeh's Z-numbers. *International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*. Springer. 2016. 581–592.

62. Massanet, S., Riera, J. V. and Torrens, J. On the Aggregation of Zadeh's Z-Numbers Based on Discrete Fuzzy Numbers. *International Summer School on Aggregation Operators*. Springer. 2017. 118–129.

63. Aliev, R. and Kreinovich, V. Z-Numbers and Type-2 Fuzzy Sets: A Representation Result. *Intelligent Automation & Soft Computing*, 2017: 1–5.

64. Aliev, R. Introduction to U-Number Calculus. *Intelligent Automation & Soft Computing*, 2017: 1–6.

65. Aliev, R. Approximate arithmetic operations of U-numbers. *Procedia Computer Science*, 2016. 102: 59–66.

66. Lala, Z. M. Identification of usual numbers. *Procedia Computer Science*, 2017. 120: 758–765.

67. Aliev, R., Alizadeh, A. and Huseynov, O. An introduction to the arithmetic of Z-numbers by using horizontal membership functions. *Procedia Computer Science*, 2017. 120: 349–356.

68. Piegat, A. and Landowski, M. Is the conventional interval-arithmetic correct? *Journal of Theoretical and Applied Computer Science*, 2012. 6(2): 27–44.

69. Aliev, R., Huseynov, O. and Aliyev, R. A sum of a large number of Z-numbers. *Procedia Computer Science*, 2017. 120: 16–22.

70. Aliev, R. A., Pedrycz, W. and Huseynov, O. H. Functions defined on a set of Z-numbers. *Information Sciences*, 2018. 423: 353–375.

71. Yager, R. R. On a view of Zadeh Z-numbers. *Advances in Computational Intelligence*, 2012: 90–101.

72. Yager, R. R. On Z-valuations using Zadeh's Z-numbers. *International Journal of Intelligent Systems*, 2012. 27(3): 259–278.

73. Aliev, R. Operations on Z-numbers with acceptable degree of specificity. *Procedia Computer Science*, 2017. 120: 9–15.

74. Valiev, A. A., Abdullayev, T. S., Alizadeh, A. V. and Adilova, N. E. Comparison of measures of specificity of Z-numbers. *Procedia Computer Science*, 2017. 120: 466–472.

75. Sotoudeh-Anvari, A., Najafi, E. and Sadi-Nezhad, S. A new data envelopment analysis in fully fuzzy environment on the base of the degree of certainty of information. *Journal of Intelligent & Fuzzy Systems*, 2016. 30(6): 3131–3142.

76. Sadi-Nezhad, S. and Sotoudeh-Anvari, A. A new Data Envelopment Analysis under uncertain environment with respect to fuzziness and an estimation of reliability. *Opsearch*, 2016. 53(1): 103–115.

77. Azadeh, A. and Kokabi, R. Z-number DEA: A new possibilistic DEA in the context of Z-numbers. *Advanced Engineering Informatics*, 2016. 30(3): 604–617.

78. Kang, B., Hu, Y., Deng, Y. and Zhou, D. A new methodology of multicriteria decision-making in supplier selection based on-numbers. *Mathematical Problems in Engineering*, 2016. 2016.

79. Agakishiyev, E. Supplier selection problem under Z-information. *Procedia Computer Science*, 2016. 102: 418–425.

80. Jabbarova, A. I. Application of Z-number concept to supplier selection problem. *Procedia Computer Science*, 2017. 120: 473–477.

81. Aliev, R. A., Huseynov, O. H. and Serdaroglu, R. Ranking of Z-numbers and its application in decision making. *International Journal of Information Technology & Decision Making*, 2016. 15(06): 1503–1519.

82. Babanli, M. and Huseynov, V. Z-number-based alloy selection problem. *Procedia Computer Science*, 2016. 102: 183–189.

83. Sharghi, P., Jabbarova, K. and Aliyeva, K. Decision making on an optimal port choice under z-information. *Procedia Computer Science*, 2016. 102: 378–384.

84. Mohamad, D. and Ibrahim, S. Decision Making Procedure Based on Jaccard Similarity Measure with Z-numbers. *Pertanika Journal of Science & Technology*, 2017. 25(2).

85. Peng, H.-g. and Wang, J.-q. Hesitant uncertain linguistic Z-numbers and their application in multi-criteria group decision-making problems. *International Journal of Fuzzy Systems*, 2017. 19(5): 1300–1316.

86. Salari, M., Bagherpour, M. and Wang, J. A novel earned value management model using Z-number. *International Journal of Applied Decision Sciences*, 2014. 7(1): 97–119.

87. Mohamad, D., Shaharani, S. A. and Kamis, N. H. A Z-number-based decision making procedure with ranking fuzzy numbers method. *AIP Conference Proceedings*. AIP. 2014, vol. 1635. 160–166.

88. Xiao, Z.-Q. Application of Z-numbers in Multi-criteria Decision Making. *Informative and Cybernetics for Computational Social Systems (ICCSS), 2014 International Conference on*. IEEE. 2014. 91–95.

89. Xiao, Z. A new approach to representing and defuzzifying a Z-number and Z-valuation. *Chinese Automation Congress (CAC), 2015*. IEEE. 2015. 797–801.

90. Alizadeh, A. V. and Serdaroglu, R. Application of Z-restriction-based Multi-criteria Choice to a Marketing Mix Problem. *Procedia Computer Science*, 2016. 102: 239–243.

91. Aliev, R. A. and Zeinalova, L. M. Decision making under Z-information. In: *Human-Centric Decision-Making Models for Social Sciences*. Springer. 233–252. 2014.

92. Marhamati, N., Khorasani, E. S. and Rahimi, S. Bayesian decision making using z-numbers. *Fuzzy Systems (FUZZ-IEEE), 2016 IEEE International Conference on*. IEEE. 2016. 2241–2248.

93. Zeinalova, L. M. and Mammadova, M. A. Decision making on oil extraction under z-information. *Procedia Computer Science*, 2016. 102: 168–175.

94. Soroudi, A. and Amraee, T. Decision making under uncertainty in energy systems: State of the art. *Renewable and Sustainable Energy Reviews*, 2013. 28: 376–384.

95. Kang, B., Wei, D., Li, Y. and Deng, Y. Decision making using Z-numbers under uncertain environment. *Journal of Computational Information Systems*, 2012. 8(7): 2807–2814.

96. Aliev, R. R., Mraiziq, D. A. T. and Huseynov, O. H. Expected utility based decision making under Z-information and its application. *Computational intelligence and neuroscience*, 2015. 2015: 2.

97. Zeinalova, L. M. Expected utility based decision making under Z-information. *Intelligent Automation & Soft Computing*, 2014. 20(3): 419–431.

98. Sadikoglu, G. Modeling of Consumer Buying Behaviour Using Z-Number Concept. *Intelligent Automation & Soft Computing*, 2017: 1–5.

99. Aliyev, R. R. Multi-attribute decision making based on z-valuation. *Procedia Computer Science*, 2016. 102: 218–222.

100. Wang, J.-q., Cao, Y.-x. and Zhang, H.-y. Multi-criteria decision-making method based on distance measure and choquet integral for linguistic Z-numbers. *Cognitive Computation*, 2017. 9(6): 827–842.

101. Azadeh, A., Saberi, M., Atashbar, N. Z., Chang, E. and Pazhoheshfar, P. Z-AHP: A Z-number extension of fuzzy analytical hierarchy process. *Digital Ecosystems and Technologies (DEST), 2013 7th IEEE International Conference on*. IEEE. 2013. 141–147.

102. Aliev, R. A., Pedrycz, W., Kreinovich, V. and Huseynov, O. H. The general theory of decisions. *Information Sciences*, 2016. 327: 125–148.

103. Eyupoglu, S., Jabbarova, K. and Aliyeva, K. The Identification of Job Satisfaction under Z-Information. *Intelligent Automation & Soft Computing*, 2017: 1–5.

104. Aliev, R. A., Huseynov, O. H. and Zulfugarova, R. Z-Distance Based IF-THEN Rules. *The Scientific World Journal*, 2016.

105. Kang, B., Deng, Y. and Sadiq, R. Total utility of Z-number. *Applied Intelligence*, 2018. 48(3): 703–729.

106. Lorkowski, J., Kreinovich, V. and Aliev, R. Towards decision making under interval, set-valued, fuzzy, and Z-number uncertainty: a fair price approach. *Fuzzy Systems (FUZZ-IEEE), 2014 IEEE International Conference on*. IEEE. 2014. 2244–2253.

107. Aliev, R., Huseynov, O. and Aliyeva, K. Z-valued T-norm and T-conorm Operators-based Aggregation of Partially Reliable Information. *Procedia Computer Science*, 2016. 102: 12–17.

108. Bakar, A. S. A. and Gegov, A. Multi-layer decision methodology for ranking z-numbers. *International Journal of Computational Intelligence Systems*, 2015. 8(2): 395–406.

109. Bakar, A. S. A. and Gegov, A. Ranking of fuzzy numbers based on centroid point and spread. *Journal of Intelligent & Fuzzy Systems*, 2014. 27(3): 1179–1186.

110. Mohamad, D., Shaharani, S. A. and Kamis, N. H. Ordering of Z-numbers. *AIP Conference Proceedings*. AIP Publishing. 2017, vol. 1870. 040049.

111. Ezadi, S. and Allahviranloo, T. New Multi-layer Method for Z-number Ranking using Hyperbolic Tangent Function and Convex Combination. *Intelligent Automation & Soft Computing*, 2017: 1–7.

112. Abbasbandy, T., Saand Hajjari. An Improvement in Centroid Point Method for Ranking of Fuzzy Numbers. *Journal of Sciences*, 2011.

113. Qiu, D., Xing, Y. and Dong, R. On Ranking of Continuous Z-Numbers with Generalized Centroids and Optimization Problems Based on Z-Numbers. *International Journal of Intelligent Systems*, 2018. 33(1): 3–14.

114. Farina, M. and Amato, P. A fuzzy definition of" optimality" for many-criteria optimization problems. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 2004. 34(3): 315–326.

115. Aliev, R. A., Pedrycz, W., Alizadeh, A. V. and Huseynov, O. H. Fuzzy optimality based decision making under imperfect information without utility. *Fuzzy Optimization and Decision Making*, 2013. 12(4): 357–372.

116. Jiang, W., Xie, C., Luo, Y. and Tang, Y. Ranking Z-numbers with an improved ranking method for generalized fuzzy numbers. *Journal of Intelligent & Fuzzy Systems*, 2017. 32(3): 1931–1943.

117. Wang, Y.-M., Yang, J.-B., Xu, D.-L. and Chin, K.-S. On the centroids of fuzzy numbers. *Fuzzy sets and systems*, 2006. 157(7): 919–926.

118. Guo-chun, T. The Area Degree of Fuzziness in Fuzzy Scheduling. *Journal of Shanghai second polytechnic university*, 1999. 2.

119. Razvarz, S. and Tahmasbi, M. Fuzzy equations and Z-numbers for nonlinear systems control. *Procedia Computer Science*, 2017. 120: 923–930.

120. Jafari, R., Yu, W. and Li, X. Numerical solution of fuzzy equations with Z-numbers using neural networks. *Intelligent Automation & Soft Computing*, 2017: 1–7.

121. Jafari, R. and Yu, W. Uncertain nonlinear system control with fuzzy differential equations and Z-numbers. *Industrial Technology (ICIT), 2017 IEEE International Conference on*. IEEE. 2017. 890–895.

122. McKay Curtis, S. and Ghosh, S. K. A variable selection approach to monotonic regression with Bernstein polynomials. *Journal of Applied Statistics*, 2011. 38(5): 961–976.

123. Davis, P. J. *Interpolation and approximation*. Courier Corporation. 1975.

124. Abiyev, R. H. Number Based Fuzzy Inference System for Dynamic Plant Control. *Advances in Fuzzy Systems*, 2016. 2016.

125. Aliev, R. and Memmedova, K. Application of Z-number based modeling in psychological research. *Computational intelligence and neuroscience*, 2015. 2015: 11.

126. Koczy, L. T. and Hirota, K. Approximate reasoning by linear rule interpolation and general approximation. *International Journal of Approximate Reasoning*, 1993. 9(3): 197–225.

127. Aliev, B. F. and Gardashova, L. A. Selection of an optimal treatment method for acute pulpitis disease. *Procedia Computer Science*, 2017. 120: 539–546.

128. Wu, D., Liu, X., Xue, F., Zheng, H., Shou, Y. and Jiang, W. A new medical diagnosis method based on Z-numbers. *Applied Intelligence*, 2018. 48(4): 854–867.

129. Jiang, W., Zhuang, M., Xie, C. and Wu, J. Sensing attribute weights: A novel basic belief assignment method. *Sensors*, 2017. 17(4): 721.

130. Dempster, A. P. Upper and lower probabilities induced by a multivalued mapping. In: *Classic works of the Dempster-Shafer theory of belief functions*. Springer. 57–72. 2008.

131. Shafer, G. *A mathematical theory of evidence*. vol. 42. Princeton university press. 1976.

132. Sahrom, N. A. and Dom, R. M. A Z-number extension of the hybrid Analytic Hierarchy Process-Fuzzy Data Envelopment Analysis for risk assessment. *Research and Education in Mathematics (ICREM7), 2015 International Conference on*. IEEE. 2015. 19–24.

133. Jiang, W., Xie, C., Wei, B. and Tang, Y. Failure Mode and Effects Analysis based on Z-numbers. *Intelligent Automation & Soft Computing*, 2017: 1–8.

134. Zou, B., Zhou, Y., Hu, J., Wen, F., Dong, Z.-Y., Zheng, Y. and Zhang, R. Z-number-based negotiation model for determining two-part transmission tariffs of cross-regional transmission projects. *IET Generation, Transmission & Distribution*, 2017. 11(7): 1829–1838.

135. Patel, P., Rahimi, S. and Khorasani, E. Applied Z-numbers. *Fuzzy Information Processing Society (NAFIPS) held jointly with 2015 5th World Conference on*

Soft Computing (WConSC), 2015 Annual Conference of the North American. IEEE. 2015. 1–6.

136. Patel, P., Khorasani, E. S. and Rahimi, S. Modeling and implementation of Z-number. *Soft Computing*, 2016. 20(4): 1341–1364.

137. Pal, S. K., Banerjee, R., Dutta, S. and Sarma, S. S. An insight into the Z-number approach to CWW. *Fundamenta Informaticae*, 2013. 124(1-2): 197–229.

138. Banerjee, R. and Pal, S. K. The Z-number enigma: a study through an experiment. In: *Soft computing: state of the art theory and novel applications*. Springer. 71–88. 2013.

139. Conde-Clemente, P., Alonso, J. M., Nunes, É. O., Sanchez, A. and Trivino, G. New types of computational perceptions: Linguistic descriptions in deforestation analysis. *Expert Systems With Applications*, 2017. 85: 46–60.

140. Farina, M. and Amato, P. A fuzzy definition of" optimality" for many-criteria optimization problems. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 2004. 34(3): 315–326.

141. Aliev, R. A., Pedrycz, W., Alizadeh, A. V. and Huseynov, O. H. Fuzzy optimality based decision making under imperfect information without utility. *Fuzzy Optimization and Decision Making*, 2013. 12(4): 357–372.

142. Kang, B., Deng, Y., Hewage, K. and Sadiq, R. A method of measuring uncertainty for Z-number. *IEEE Transactions on Fuzzy Systems*, 2018. 27(4): 731–738.

143. Kosko, B. Fuzziness vs. probability. *International Journal of General System*, 1990. 17(2-3): 211–240.

144. Aliev, R. and Guirimov, B. Z-Number Clustering Based on General Type-2 Fuzzy Sets. *International Conference on Theory and Applications of Fuzzy Systems and Soft Computing*. Springer. 2018. 270–278.

145. Alizadeh, A. V. and Huseynov, O. Minimum and maximum of discrete Z-numbers. *11th Int. Conf. on Appl. of Fuzzy Syst. and Soft Comp.* 2014.

146. Krohling, R. A., Pacheco, A. G. C. and dos Santos, G. A. TODIM and TOPSIS with Z-numbers. *Frontiers of Information Technology & Electronic Engineering*, 2019. 20(2): 283–291.

147. Herrera, F. and Herrera-Viedma, E. Linguistic decision analysis: steps for solving decision problems under linguistic information. *Fuzzy Sets and systems*, 2000. 115(1): 67–82.

148. Boyd, S. and Vandenberghe., L. Convex Optimization. Cambridge University Press, 2004.

149. Sharmila, A. and Geethanjali, P. A review on the pattern detection methods for epilepsy seizure detection from EEG signals. *Biomedical Engineering/Biomedizinische Technik*, 2019.

LIST OF PUBLICATIONS

Indexed Journal

1. Abdullahi M, Ahmad T. and Ramachandran V. 2018. Intuitionistic L-fuzzy sets and Intuitionistic N-fuzzy sets Malaysian Journal of Fundamental and Applied Sciences Vol. 14, No. 1 (2018) 125-126
2. Abdullahi M, Ahmad T. and Ramachandran V. 2020. Ordered discrete and continuous Z-numbers. Malaysia Journal of Fundamental and Applied Sciences (in press).
3. Abdullahi M, Ahmad T. and Ramachandran V. Temporal Discrete Z-number and Its Application in Assessing EEG Signal Data of Epileptic Seizure. Sains Malaysiana ICUW 2019 Special Issue (in press)
4. Abdullahi M, Ahmad T. and Ramachandran V. A Review On Some Arithmetic Concepts Of Z-Number And Its Application To Real World Problems. International Journal of Information Technology & Decision Making. (in press)

Non-Indexed conference proceedings

1. Abdullahi M, Ahmad T. and Ramachandran V. 2019. A Radical Approach For Assessing EEG Signal of Epileptic Seizure Using Temporal Discrete Z-number. International conference on Universal Wellbeing, 4-6 December. Kuala Lumpur.

Copyright

1. A coding program for subtraction of two discrete fuzzy numbers (Ref: IP/CR/00459 filed on 30 April 2020)