JORDAN-CHEVALLEY DECOMPOSITION OF RECORDED ELECTROENCEPHALOGRAPHY SIGNALS DURING EPILEPTIC SEIZURES

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ABSTRACT

Epilepsy is a disease that can be identified by its main features that are the recurrent and unpredictable electrical discharges of the cerebral cortex that trigger disturbances in brain functions. It can be diagnosed through a noninvasive tool called electroencephalography (EEG), which records the electrical signals emanated by the brain. The recorded signals provide important information about brain functions in terms of observed electrical potentials. This information is critical especially for diagnosing brain disorders, particularly epilepsy. In fact, EEG signals during epileptic seizures can form a semigroup of upper triangular matrices under matrix multiplication. This semigroup can be represented as the product of its elementary components through the Krohn-Rhodes decomposition technique. The representation of EEG signals in terms of the product of its elementary components is somehow connected to how any positive integer can be written as a product of prime numbers. In this study, the elementary EEG signals are shown to act as the building blocks of EEG signals, similar to the prime numbers being the building blocks of the positive integers. The primary goal of this research is to describe and view the elementary components of EEG signals during epileptic seizures as prime numbers. Firstly, the well-ordered property of prime numbers hints at a similar attribute to be found in the case of elementary EEG signals. Therefore, a new way of ordering matrices, namely the precede operator, is introduced to achieve this goal, and several theorems are developed in the process. Secondly, the elementary EEG signals are decomposed via the Jordan-Chevalley decomposition technique, which produces the summation of its simpler parts and reveals that it resonates with one of the main properties of prime numbers, particularly the Twin-Prime Conjecture. Finally, the decomposition method is implemented on the real EEG data of three patients. In this way, the theoretical framework of the EEG signals' building blocks is developed. In short, it is obtained that the ordered property of the elementary EEG signals and its representation through the Jordan-Chevalley decomposition exhibits similar properties with certain results in prime numbers as anticipated by the Krohn-Rhodes semigroup theory. The results hint that, due to their similar properties, the EEG signals could have a similar pattern to that of prime numbers.

ABSTRAK

Epilepsi ialah sejenis penyakit yang dapat dikenali melalui ciri utamanya iaitu pengaliran elektrik yang berulang dan tidak dapat diduga dari korteks serebrum yang mencetuskan gangguan fungsi otak. Ia boleh didiagnosis melalui suatu peranti tidak invasif yang dinamakan elektroensefalograf (EEG) yang merekodkan isyarat elektrik yang dipancarkan oleh otak. Isyarat yang direkodkan memberikan maklumat penting tentang fungsi otak dari segi potensi elektrik yang diperhatikan. Maklumat ini adalah kritikal terutamanya untuk mendiagnosis kecelaruan otak, khususnya epilepsi. Malah, isyarat EEG semasa sawan epilepsi boleh membentuk satu semikumpulan matriks segi tiga atas di bawah pendaraban matriks. Semikumpulan ini boleh diwakili sebagai hasil darab komponen asasnya melalui teknik penguraian Krohn-Rhodes. Perwakilan isyarat EEG dari segi hasil komponen asasnya mempunyai kaitan tertentu dengan bagaimana integer positif dapat ditulis sebagai hasil darab nombor perdana. Dalam kajian ini, isyarat asas EEG ditunjukkan berfungsi sebagai blok binaan isyarat EEG, sama seperti nombor perdana adalah blok binaan integer positif. Matlamat utama kajian ini ialah untuk menerangkan dan melihat komponen isyarat asas EEG semasa sawan epilepsi sebagai nombor perdana. Pertama, sifat nombor perdana yang tersusun membayangkan bahawa sifat yang serupa akan dijumpai dalam kes isyarat asas EEG. Oleh itu, satu kaedah baharu untuk mengatur matriks, dinamakan operator pendahulu, telah diperkenalkan untuk mencapai matlamat ini, dan beberapa teorem telah dikembangkan dalam proses ini. Kedua, isyarat asas EEG diurai melalui teknik penguraian Jordan-Chevalley, yang menghasilkan penjumlahan bahagian-bahagian yang lebih ringkas dan menunjukkan bahawa ia menyalun dengan salah satu daripada ciri utama nombor perdana, terutamanya Konjektur Prima Kembar. Akhir sekali, kaedah penguraian telah dijalankan berdasarkan data EEG sebenar tiga orang pesakit. Dengan cara ini, kerangka teori blok binaan isyarat EEG dikembangkan. Ringkasnya, didapati bahawa sifat isyarat asas EEG yang tersusun dan perwakilannya menerusi penguraian Jordan-Chevalley mempamerkan sifat-sifat yang serupa dengan hasil tertentu nombor perdana seperti yang dijangka oleh teori semikumpulan Krohn-Rodes. Hasil ini memberi petanda bahawa, disebabkan oleh sifat-sifat mereka yang serupa, isyarat EEG mungkin mempunyai corak yang serupa dengan corak nombor perdana.

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LIST OF ABBREVIATIONS

$Aff_n^*(\mathbb{R})$	–	Affine group of EEG signals
APD	–	Average Potential Difference
BM	–	Base magnetic plane
BMI	–	Base magnetic image plane
EEG	–	Electroencephalogram
FFT	–	Fast Fourier Transform
FM	–	Fuzzy magnetic field
FMI	–	Fuzzy magnetic image field
FTTM	–	Fuzzy Topographic Toplogical Mapping
$ASMC_n^*(\mathbb{R})$	–	Group of affine scaling functions of EEG signals
$AMC_n''^*(\mathbb{R})$	–	Group of affine triangular functions of EEG signals
HKL	–	Hospital Kuala Lumpur
ILAE	–	International League Against Epilepsy
MC	–	Magnetic contour plane
MI	–	Magnetic image plane
PWE	–	Patients with epilepsy
$\beta_{ij}(z)_t$	–	Potential difference reading of EEG signals from a particular ij sensor at time t
$ASMC_n(\mathbb{R})$	–	Semigroup of affine scaling functions of EEG signals
$AMC_n''(\mathbb{R})$	–	Semigroup of affine triangular functions of EEG signals
$MC_n''(\mathbb{R})$	–	Semigroups of EEG signals
$MC_n(\mathbb{R})$	–	Set of square matrices of EEG signals
$Aff_n(\mathbb{R})$	–	The affine semigroup of EEG signals
TM	–	Topographic magnetic field
TMI	–	Topographic magnetic image field

LIST OF SYMBOLS

$\widetilde{X}$	–	Aperiodic monoid
$\tilde{X}$	–	Aperiodic semigroup
$\overline{A}$	–	Augmented monoid
$\times$	–	Cartesian product
$\circ$	–	Composition of function
$\bigoplus$	–	Direct sum
$\mid$	–	Divide
$\prec$	–	Divisibility between semigroups of EEG signals
$\in$	–	Element of
$=$	–	Equal to
$\forall$	–	For all
$>$	–	Greater than
$\geq$	–	Greater than or equal to
$\cong$	–	Homeomorphism
$\implies$	–	Implies that
$\cap$	–	Intersection between sets
$<$	–	Less than
$\rightarrow$	–	Map to
$\leq$	–	Less than or equal to
$\mathbb{R}^n$	–	n-dimensional Euclidean space
$\neq$	–	Not equal to
$\succ$	–	Precede partial order
Q	–	Orthogonal matrix
r	–	Radius of patient's head
$S_t : C_{EEG} \rightarrow MC$	–	S_t is a mapping from C_{EEG} to MC
$\propto$	–	Semidirect product
$\mathbb{N}$	–	The set of natural numbers

t	–	Time
$\mathbb{Z}^+$	–	The set of positive integers
$\mathbb{R}^+$	–	The set of positive real numbers
$\mathbb{R}$	–	The set of real numbers
$\exists$	–	There exists
$\wr$	–	Wreath product

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CHAPTER 1

INTRODUCTION

1.1 Background and Motivation

Epilepsy is a chronic neurological disease (previously defined as a disorder) that affects 60 million people worldwide [1]. Although several medical and surgical treatments have been developed, they cannot be employed to manage roughly one-quarter of all people with epilepsy (PWE) [2]. Epilepsy is caused by irregular, excessive, or sudden brain function disturbances that make patients lose control of their bodies, thus putting them at a high risk of suffering accidents and getting into possibly life-threatening scenarios [3]. Generally, PWE's brains function normally over 99.9% of the time. However, even though epileptic seizures make up less than 0.1% of a patient's lifespan, their palpable and unpredictable nature puts immense psychological strain on patients and jeopardizes their quality of life [2]. Accurate epilepsy diagnoses are paramount to avoiding all epileptic stages—such diagnoses would help protect PWE from injuries or death [4].

Electroencephalography (EEG) is a well-established, inexpensive, and non-invasive clinical tool for diagnosing and managing PWE [5]. It played an important role in classifying and characterized seizure and seizure syndromes that support the clinical assessment of PWE [6]. The electrical activity of the brain can be measured via EEG recording, which gives crucial insight into the timing and location of the underlying activities [7]. Surgery is considered to be a more effective treatment for epilepsy than that of drugs [8]. Prior to the epilepsy surgery, the cortical and subcortical functions, such as the function of motor and language, can be monitored by stimulating the brain through the intracranial electrodes to provide the information necessary to assess the risk-benefit profile of the surgery [9]. The degree of reported seizure freedom post-surgery is indeed promising, but the fact that only a relatively small number of patients have surgically remediable epilepsy syndrome has impeded the option of surgical procedures as a large-scale treatment [10].

A practical way of anticipating seizure occurrence could significantly alleviate patients' physiological stress [11]. Seizure prediction offers an alternative way to treat epilepsy owing to its capability to generate a warning prior to the occurrence of a seizure. The PWE's quality of life could greatly improve with the aid of such a seizure prediction system [12]. However, this is a formidable task because the disabling aspect of epilepsy is that it strikes "like a bolt from the blue" and seemingly in random patterns [13]. Whether characteristic features can be derived from the recorded EEG signals that are predictive of an imminent seizure is a matter of particular interest. Clinicians and technologists can possibly rely on the dynamic changes extracted from the EEG of PWE to develop fully automated closed-loop seizure-prevention systems [14]. Therefore, doctors could prescribe an EEG-triggered on-demand treatment [15] or reset the brain dynamics to a state where the risk of a seizure is minimized through brain stimulation [16, 17].

Quantitative analysis and mathematical modeling of normal/abnormal brain function is an arduous challenge considering its state changes over time (dynamic). In fact, it is a common belief that the EEG signals of the brain, particularly during a seizure attack, are always considered to be frantic and disorderly [18]. Most commonly, statistical methods are employed to study this unpredictable nature of EEG signals (e.g., linear methods [19], non-linear methods [20, 21], wavelet transform [22, 23], and Independent Component Analysis (ICA) [24]). Besides that, nonlinear dynamics and chaos theory are other favored mathematical tools to study the complexity of the brain's function from the recorded EEG signals. Among them are the Largest Lyapunov Exponent (LLE) introduced by Iasemidis and Sckellares [25], wavelet-chaos method initiated by Adeli, Ghosh-Dastidar and Dadmehr [26], fractal dimension proposed by Easwaramoorthy and Uthayakumar [27], and dynamical similarity index established by Le Van Quyen et al. [28].

In 2008, Zakaria [29] developed a novel mathematical model, namely, Flat EEG, where EEG signals can be viewed on two-dimensional space allowing the EEG signals to be compressed and easier to be analyzed. Binjadhnan and Ahmad [30, 31] extended the model of Flat EEG where the EEG signals are written as upper triangular matrices and form a semigroup under matrix multiplication. This novel algebraic structure is referred to as a semigroup of EEG signals. Then, Binjadhnan [32] proved that the semigroup of EEG signals could be expressed in terms of a product of its elementary components through Krohn-Rhodes decomposition. The author suggested that the elementary components of EEG signals bear a resemblance to the prime numbers. In this research, the premise of viewing the elementary components of EEG signals as prime numbers is explored. The outcomes of this research are of great importance in the sense that they provide valuable information with regard to a better understanding of the dynamics of EEG signals during a seizure.

1.2 Statement of the Problem

The assertion proposed by Binjadhnan [32] that the elementary EEG signals during an epileptic seizure resonate with the prime number is of the main interest in this thesis. Therefore, certain properties of the distribution of prime numbers among positive integers, particularly the well-ordering principle of the positive integers, is explored so that similar property could be derived in the case of ordering square matrices. Furthermore, some important conjectures about prime numbers give the impression that the elementary EEG signals could be decomposed further into simpler parts; ergo, the befitting decomposition technique is determined. Contrariwise, certain properties of the EEG signals during an epileptic seizure can be found in prime numbers as well. This analogy is illustrated in Figure 1.1.

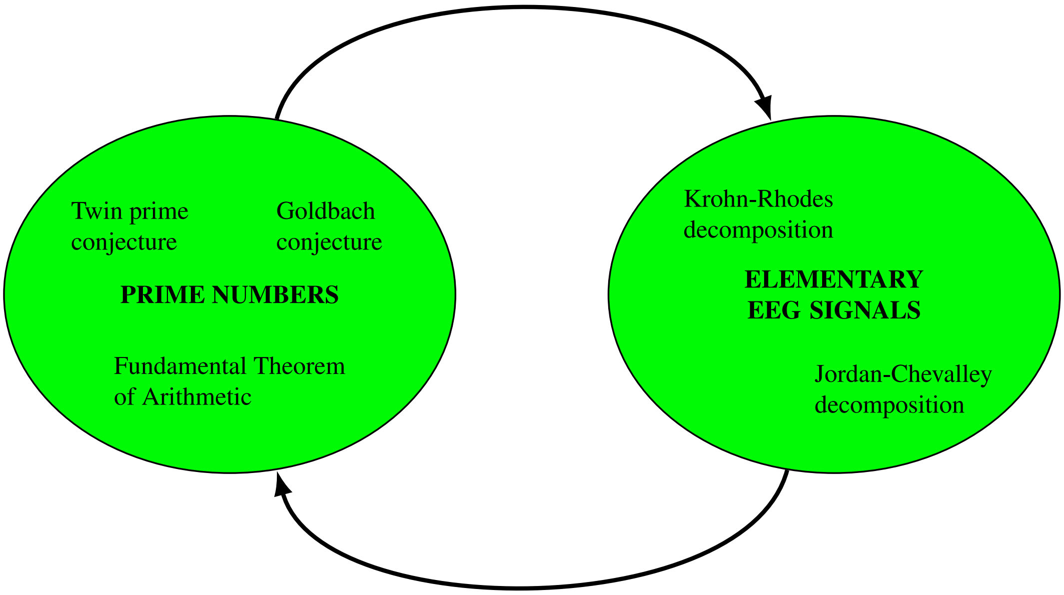

Figure 1.1 Analogy of elementary EEG signals as prime numbers

1.3 Methodology of Research

The premise of this research is viewing elementary EEG signals as prime numbers, as depicted in Figure 1.1. Therefore, a survey of the literature related to epilepsy, EEGs, and the distribution of prime numbers was conducted to gain deep insight into relevant theories and methods that have been used and developed. Another goal of the literature survey was to gain an understanding of the essential mathematical concepts and theories that have been used and to identify the appropriate technique for use in this research.

Next, the theoretical matrix partial order was developed to establish a further connection between EEG signals and prime numbers. This development proved several theorems and propositions for a certain type of square matrixces . Barja predicted that the Jordan–Chevalley decomposition on transformed EEG signals would resemble the Twin–Prime Conjecture of prime numbers. Hence, matrix decomposition was performed on the elementary EEG signals. Then, the overall technique and method were implemented on real data collected from three patients with epilepsy from the

Pediatric Institute of the Kuala Lumpur Hospital.

1.4 Objectives of the Research

The objectives of the research are:

1. To develop a new technique of ordering square matrices of EEG signals during a seizure.
2. To show that the elementary components of EEG signals can be written as a sum of semisimple and nilpotent parts via Jordan-Chevalley decomposition.
3. To build a theoretical foundation of the elementary components of the EEG signals, i.e., to find the relationship between the theory of prime numbers and elementary EEG signals.
4. To present three real-time examples of the Jordan-Chevalley decomposition of elementary EEG signals.

1.5 Scope of the Research

Two main areas—medical and mathematics—are covered in this study. Specifically, a brief discussion on the brain function during an epileptic seizure is presented on the medical side. Electroencephalogram used for the diagnosis of epilepsy is studied in detail. On the contrary, attention is devoted to the Jordan-Chevalley decomposition of elementary EEG signals on the mathematical side. Moreover, some properties of the prime number's distribution within the positive integers in connection with the elementary EEG signals are explored. The decomposition technique is implemented on the recorded EEG data of epileptic seizures obtained from three epileptic patients (Patient A, Patient B, and Patient C) aged between six months to nine years old from the Pediatric Institute, Hospital Kuala Lumpur (HKL) in 2004. The sample taken from the Patient A is the EEG data recorded at time $t = 1$ to $t = 15$, whereas the sample taken from Patient B and C, the data are recorded from $t = 1$ to $t = 15$. The results of this research are limited to the sample taken from these three patients.

1.6 Significance of the Research

This research adopts the technique of the Jordan-Chevalley decomposition in decomposing the elementary EEG signals during epileptic seizures. The decomposition technique is prompted by the prime numbers that can be written as a sum of two primes. The new approach provided a way to extract information from the magnetic contour plane. The technique justified Yun's [33] claim that: "Magnetic Contour (MC) is a plane containing information." The analogy of elementary EEG signals as prime numbers requires a method of ordering matrices, which is obtained in this thesis. Most importantly, this study presents a piece of supporting evidence that the elementary EEG signals behave like the prime numbers as the building blocks of the positive integers.

From a medical point of view, the results indicate that the EEG signals during an epileptic seizure contain patterns—which is similar to the pattern of prime numbers since they behaves like prime numbers—allowing the prediction of its occurrence. This discovery can serve as a stepping stone in developing a seizure-prediction method that will not only help the medical experts but, more importantly, also the PWE to lead a better life.

1.7 Outline of the Research

This thesis contains seven chapters. The introduction of this research is provided in Chapter 1, which includes the background and motivation, problem statement, objectives, scope, and significance of the research. This is followed by Chapter 2, in which the literature review of the research will be dealt with. It contains the discussion on epileptic seizures, Fuzzy Topographic Topological Mapping (FTTM), Krohn-Rhodes decomposition of EEG signals, the notions of ordering matrices, Jordan-Chevalley decomposition, and distribution of prime numbers.

Subsequently, a new method for ordering matrices is presented in Chapter 3. In Chapter 4, the elementary EEG signals during seizures are decomposed further via the Jordan-Chevalley decomposition. Furthermore, the relationship between the decomposition of elementary EEG signals during seizures with the prime numbers is established.

The entire procedure of decomposing elementary EEG signals during seizures is provided in Chapter 5. This chapter involves the decomposition of the main data of a patient suffering from epilepsy into its elementary components, which is done by Zakaria [29] and Binjadhnan [32], followed by the execution of Jordan-Chevalley on the data. Finally, the conclusion of the whole thesis along with some recommendations for further studies in this field of research are given in Chapter 6. The research outline is illustrated in Figure 1.2.

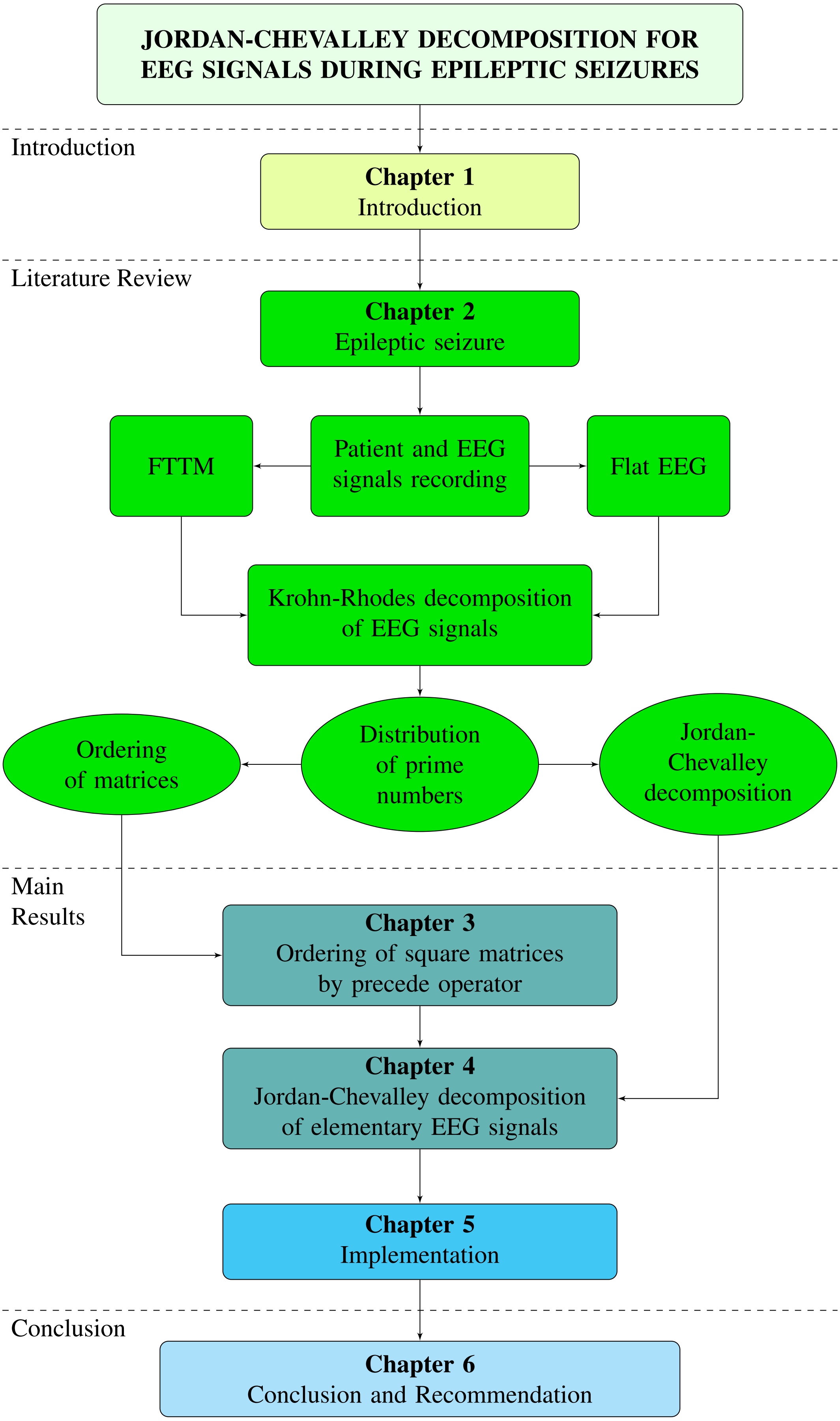

Figure 1.2 Research framework

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1. **Ahmad Fuad, A. A.**, and Ahmad, T. (2021). Ordering of transformed recorded electroencephalography (EEG) signals by a novel precede operator. *Journal of Mathematics*, 2021, 1–14. https://doi.org/10.1155/2021/6651445 (**Q2, IF: 0.971**).
2. **Ahmad Fuad, A. A.** and Ahmad, T. (2021). Decomposing the Krohn-Rhodes form of electroencephalography (EEG) signals using Jordan-Chevalley decomposition technique. *Axioms*, 10(1), 2021. 10(1). https://doi.org/10.3390/axioms10010010 **(Web of Science)**.

Indexed Conference Proceedings

1. **Ahmad Fuad, A. A.**, and Ahmad, T. (2019). The decomposition of electroencephalography signals during epileptic seizure. *AIP Conference Proceedings, 2184*. https://doi.org/10.1063/1.5136478 (**Scopus**).
2. **Ahmad Fuad, A. A.**, Ahmad, T., and Mohamad Nor, N. A. (2021). Elementary components of electroencephalography signals viewed as prime numbers. *Journal of Physics: Conference Series, 1988*(1), 12073. https://doi.org/10.1088/1742-6596/1988/1/012073 (**Scopus**).