DYNAMICAL MONGE-KANTOROVICH MASS TRANSPORTATION PROBLEM MODEL WITH WATER PERMEABILITY TERM

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DEDICATION

Dedicated to my beloved

family.

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ABSTRACT

This research focuses on the development of mass transportation problem model in urban planning. This study highlights the dynamical Monge-Kantorovich mass transportation problem model particularly on water permeability potential. It started from the problem that occurs when there is a decrease in the capability of water to infiltrate into the soil whenever there is an increase in population density in an area. This situation occurs when the development of the area is poorly planned such that the coverage of surface area disturbs the existing water infiltration process. Thus, this study aims to develop a model by considering the issues outlined above based on the dynamic Monge-Kantorovich mass transportation model, strengthened by theoretical analysis and support. Here, a new model is developed by extending the basic dynamical Monge-Kantorovich mass transportation model by incorporating a water permeability term. The resulting model shows that it satisfies a system of optimality and uniqueness conditions effectively. Also, the stability estimation of the new model are derived. In addition, the proximal splitting method specifically Douglas-Rachford method is implemented to solve the new model. The model is able to successfully identify areas suitable for development using its converged solution. As a conclusion, this investigation leads to an extension of the basic dynamical Monge-Kantorovich mass transportation model into a model with appended water permeability term, assessed and supported by theorems and propositions for validation. This model is useful for future research especially on the development of models in the field of water permeability.

ABSTRAK

Penyelidikan ini menumpu kepada pembangunan model bagi masalah pengangkutan jisim dalam perancangan bandar. Kajian ini mengetengahkan model masalah dinamik pengangkutan jisim Monge-Kantorovich khususnya pada potensi kebolehtelapan air. Ia bermula daripada masalah yang terjadi apabila keupayaan serapan air ke dalam tanah berkurangan disebabkan peningkatan kepadatan penduduk sesuatu kawasan. Keadaan ini terjadi apabila pembangunan sesuatu kawasan tidak dirancang dengan baik sehinggakan litupan permukaan kawasan tersebut mengganggu proses penyerapan air sedia ada. Maka, kajian ini bertujuan untuk membangunkan model dengan mengambil kira isu yang digariskan di atas berdasarkan model dinamik pengangkutan jisim Monge-Kantorovich, dikukuhkan oleh analisis teori dan sokongan. Di sini, model baharu telah dibangunkan dengan meluaskan model asas dinamik pengangkutan jisim Monge-Kantorovich dengan menggabungkan terma kebolehtelapan air. Model yang terhasil menunjukkan yang ia memenuhi sistem syarat optimum dan keunikan dengan berkesan. Di samping itu, anggaran kestabilan untuk model baharu telah diterbitkan. Sebagai tambahan, kaedah penguraian proksimal khususnya kaedah Douglas-Rachford telah digunakan untuk menyelesaikan model baharu ini. Model ini boleh mengenalpasti dengan jaya kesesuaian kawasan untuk pembangunan menggunakan penyelesaian bertumpu. Kesimpulannya, kajian ini menjurus kepada perluasan model asas pengangkutan jisim Monge-Kantorovich kepada model dengan penambahan terma kadar kebolehtelapan air, dinilai dan disokong oleh teorem dan usulan untuk pengesahan. Model ini berguna untuk penyelidikan masa hadapan terutamanya pada pembangunan model dalam bidang kebolehtelapan air.

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LIST OF SYMBOLS

$A(\rho, v)$	-	Total cost for transporting mass from with associated transport velocity.
$A(\rho, \nu, k)$	-	Total cost for moving the population density corresponding to velocity and water permeability rate.
B	-	Total benefit.
$c(x, y)$	-	Transportation cost.
C	-	Cost of the city.
C^1	-	Space of continuous function differentiable up to order 1.
$d(x, y)$	-	Kantorovich–Rubinstein cost function.
$d\pi(x, y)$	-	Measure the amount of mass transferred from location, *x* to location, *y*.
f	-	The infiltration capacity.
$f(\rho, k)$	-	Total water permeability rate based on the population density at area *x*.
F	-	The total volume infiltrated.
F_d	-	Kantorovich –Rubinstein optimal transportation cost.
h	-	Pressure head.
H	-	Total depth of ponded water above the surface.
i	-	Number of iterations.
$I[\pi]$	-	Total transportation cost associated to π.
$J(\rho, \nu)$	-	Transportation term of $A(\rho, \nu, k)$.
$J(\varphi, \psi)$	-	Dual formulation of transportation cost.
$k(t, x)$	-	Water permeability rate at location *x* time *t*.
K	-	Hydraulic conductivity (soil permeability).
L^n	-	Lebesgue spaces.
$m(t, x)$	-	Momentum at location *x* time *t*.
$n(r)$	-	Employment density at location *r*.
N	-	Spatial domain discretization.

P	-	Temporal domain discretization.
q	-	Average velocity.
R	-	Real number set.
$(r,0)$	-	Firm location.
(r,ϕ)	-	Location within the city.
s	-	Radius $s \in S$.
S	-	Radius.
(s,∅)	-	Worker location.
t	-	Time
T	-	Transport map from $U \to V$.
$\nu(t,x)$	-	Velocity at location *x*, time *t*.
u	-	Population density (mass).
U, V	-	Measurable set, as urban area.
w	-	Variable (ρ, m, k).
$\tilde{w}$	-	Corresponding variable $(\tilde{\rho}, \tilde{m}, \tilde{k})$.
$W_2(\rho_0, \rho_1)^2$	-	Wasserstein distance between ρ_0 and ρ_1,
x	-	Residential area.
$X(t,x)$	-	position of mass (population) at time *t*.
y	-	Production area.
z	-	Depth between soil and the wetting front.
$z(r)$	-	Production externality.
Z	-	Depth of wetting front.
μ	-	Probability measure defined as residents
υ	-	Probability measure defined as services.
ς	-	Weighting kernel.
Ω	-	Admissible class of mapping *T*.
$\rho(t,x)$	-	Population density (mass) at location *x*, time *t*.
ρ_0	-	Initial condition for population density.
ρ_1	-	Final condition for population density.
ϕ	-	Map.
λ	-	Lagrange multiplier.

ψ	-	The suction head of the wetting front.
θ	-	Water content.
$\vartheta(r)$	-	Fraction of land use for production at location *r*.
$\sigma(x)$	-	Number of workers per unit land at location, *x*.
π	-	Probability measure on the product space $U \times V$.
Π	-	Set of admissible transport plan.

CHAPTER 1

INTRODUCTION

1.1 Introduction

Monge-Kantorovich mass transportation problem has been in the time-light since the 1970s. The development of the mass transportation problem has become an important problem in various fields, such as in economy, fluid dynamics, transportations, statistical physics, shape optimization, automatic control and meteorology (Villani, 2003). There are three important papers that first described this problem. Mass transportation problem was originally developed by a great geometer, G. Monge in 1781 in his paper *Memoire sur la theorie des deblais et des remblais* (Villani, 2003). Kantorovich developed the second and third ones in his papers published in 1942 and 1948 (Gangbo and McCann, 1996).

From Villani (2003), Monge's research was concerned with the finding of an optimal way, to minimize the transportation in moving a pile of sand from some location, *x*, to location, *y*.

Basically, the idea came from the Figure 1.1,

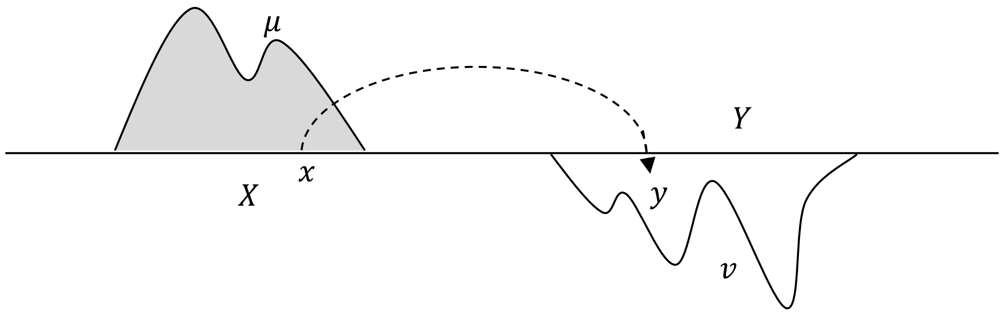

Figure 1.1 The mass transportation problem. (Villani, 2003)

Figure 1.1 shows the general idea of the problem. Moving the sand around from one location to another needs some effort and obviously, the pile and the void must be of the same volume. This situation can be modelled as the optimal transportation problem.

In mathematical language, this problem is modelled by measurable cost function defined on $U \times V$. Informally, let $c(x, y)$ represents the cost (per unit mass) for transporting the mass from *x* to *y*. It is assumed that *c* is measurable and nonnegative as the masses are always with a value and can be counted. There is a probability measure π on the product space $U \times V$ with marginal μ and υ. The problem becomes

$$\text{minimize} \qquad I[\pi] = \int_{U \times V} c(x, y) d\pi(x, y) \quad \text{for} \quad \pi \in \Pi(\mu, \upsilon)\,. \tag{1.1}$$

where $d\pi(x, y)$ is to measure the amount of mass transferred from location *x* to location *y* and $I[\pi]$ is the total transportation cost associated to *π*. For a transferred plan $\pi \in P(U \times V)$ to be admissible, it is necessary that all the mass taken from *x* coincide with $d\mu(x)$.

The Equation (1.1) developed by Kantorovich is known as Kantorovich's optimal transportation problem. The model was developed in 1941 without the knowledge of the model by Monge developed in 1781. This model is known as the simplified version of the original mass transportation problem considered by Monge in the paper by Kantorovich in 1948. This model explains that the mass can be split to two different locations. In the other word, to each location, *x* is associated with non-unique destination *y*.

Monge basically proposed that the mass could not be split to two different locations *y*. Thus, based on Equation (1.1), model by Monge needs to have the transportation plan π in a special form,

$$d\pi(x,y) = d\pi_T(x,y) \equiv d\mu(x)\ \delta[y = T(x)],$$

where *T* is a measurable map from $U \to V$. The model by Monge known as Monge's optimal transportation problem is

$$\text{minimize} \qquad I[x] = \int_U c(x,T(x))d\mu(x)$$

over the set of all measurable maps *T* such that $T_{\#}\mu = \upsilon$. The name "Monge-Kantorovich mass transportation problem" is used for either Kantorovich's or Monge's minimization problem.

1.2 Research Background

Urban planning is a land usage planning in certain area in order to improve the structural, economic and social environments. Some of the models related to urban planning problem in various fields were highlighted in Table 1.1.

Table 1.1 Previous researches on urban planning model.

Models	**Main characteristics**	**Fields**
Opricovic and Tzeng (2002)	Model on global safety for area with potential hazard.	Fuzzy multi-criteria optimization method.
Awad and Aboul-Ela (2003)	Model on finding the cost of service connection from the street to the residential area.	Non-linear programming problem, Focus Search and Monte-Carlo Simulation.

Models	**Main characteristics**	**Fields**
Jung *et al.* (2007), Fujita *et al.* (1999))	Model on minimizing the cost of potential disaster.	Multi-criteria optimization model and ranking method.
Adhvaryu (2010)	Model on land use and transportation.	Simplified planning model.
Bigotte *et al.* (2010), Almandoz (2006)	Transportation network model based on historiography of the area.	Optimization model.
Camagni *et al.* (2013)	Model on maximizing the urban costs and benefits such as environmental quality and inter-urban network.	Optimization model.

Since there were numerous models involved in urban planning problem, this research focuses on Monge-Kantorovich mass transportation model, which could be interpreted as a movement of estimated population density from an already crowded city to newly undeveloped areas. Basically, the basic model for our research can be illustrated as follows,

$$A(\rho,\nu)=\int_0^1\left(\int_R \rho(t,x)\,|\,\nu(t,x)\,|^2\,dx\right)dt \qquad (1.2)$$

where R is real number, subject to the continuity equation

$$\frac{\partial\rho}{\partial t}+\nabla_x\cdot\rho\nu=0$$

The problem is bounded with initial and final conditions

$$\rho(0,.)=\rho_0 \text{ and } \rho(1,.)=\rho_1$$

where ρ is the estimated population density, and ν is the transport velocity that pushes the estimated density from initial location, ρ_0 to destination, ρ_1. Before proceeding to more specific scopes, discussion on the main reasons why this problem was chosen is in order.

There are various problems that could lead to discomforts to residents of a city. These are related to expenses and wellbeing of the residents, for example, denseness of the population, air and water pollution, poverty, as well as traffic congestion. As the world's population is constantly rising and more people are moving to cities, developers are trying to build new townships to solve overpopulated cities. Transportation and managements costs are among one of the major concerns. Hence, many researchers had developed ways to minimize this cost in order to reduce the spending of the masses. Table 1.2 highlighted previous researches related to mass transportation model in urban planning problem.

Table 1.2 Previous researches on mass transportation model in urban planning

Models	**Characteristics**	**Advantages**	**Limitation**
Melkote and Daskin (2001a)	Transportation network between residents and businesses.	The model minimized total cost for business location and network.	The model studies the transportation network.
Lucas and Rossi-Hansberg (2002)	Mass transportation cost consist of residence area and business.	The model reduces workers traveling cost to balance the external effect from placing businesses close from one another, contributed to dispersion of housing surrounding the city centre.	Residence and businesses compete for land at all locations, subject to no constraints other than ability to pay.

Models	Characteristics	Advantages	Limitation
Carlier and Ekeland (2004)	Mass transportation cost between residence and businesses.	The model maximizing total production and total consumption in a city.	The model aims to look for an equilibrium solution where there is no total performance that optimizes the utility criterion for the distribution of residences and businesses.
Rossi-Hansberg (2004).	Mass transportation model for business and housing.	The model studied on the city structure and the optimal size of urban area.	The model described monocentric city and the optimal land use has no mixed areas of business and housing. The model does not consider congestion of businesses.
Buttazzo and Santambrogio (2009)	Mass transportation cost for residents to move to business area, the unhappiness of the residents due to population density and the management cost for the businesses.	The model focuses on the equilibrium distribution of residential in the city with delocalization of businesses.	The model does not describe the shape of city where the optimal solution does not reflect what reality suggests (many disjoint and independent sub-cities with services only in the centre).

Monge-Kantorovich mass transportation model is a well-established mathematical model used by many researchers and can be widely applied in various field. In this study, this mass transportation model is use as a basis for our model. Similar to previous studies, the focus of this study is to minimize the total cost for the population by using Equation (1.2).

Table 1.2 highlighted the summary of several models done by previous researchers. Most these researchers focused on finding the best structure in the cities that would be beneficial to the economy. By using the basic model of mass transportation theory, different parameters were assigned for diverse objectives respective of their researches, which generally to find the optimal distribution of residences. However, this study does not contemplate most of their complex parameters since the population in this study is assume to be concentrated and will move from one location to a new undeveloped location.

Issues related to the environment such as rainwater runoffs, landslides and water supply security problems are neglected. Most of the characteristics used in previous models are not applicable in this research as they largely used economical parameters such as firm, market, worker, and mainly focused on management cost and production externalities. However, this study aims to focus on the application from the environmental aspect. Thus, these parameters are not in the scope of this study.

In order to develop new settlement, developers should not ignore the environmental issues. The problems resulting from these issues can cause discomfort to residents and high expenses to governments. The environmental problem focuses in this study is water problems. Water problems could be defined in many ways. In this research, the emphasis is on the infiltration rate of water in an area specifically in undeveloped new areas.

Infiltration of water into the soil is very important to maintain groundwater reservoir of the area. Maintaining natural water infiltration rate can avoid runoff and sink holes from happening. For example, no structure can be developed in an area with high infiltration rate of water (National Soil Centre and Agricultural Research Services, 1998). It is not recommended to develop any settlement in such area as it is not safe and high expenses are needed to make the area suitable for any development.

Figure 1.2 shows that stream discharge in urban area is very high whenever heavy rainfalls because the water infiltration rate of the area is low. This will cause surface ponding or runoffs in the area (Upper Parramatta River Catchment Trust, 2002).

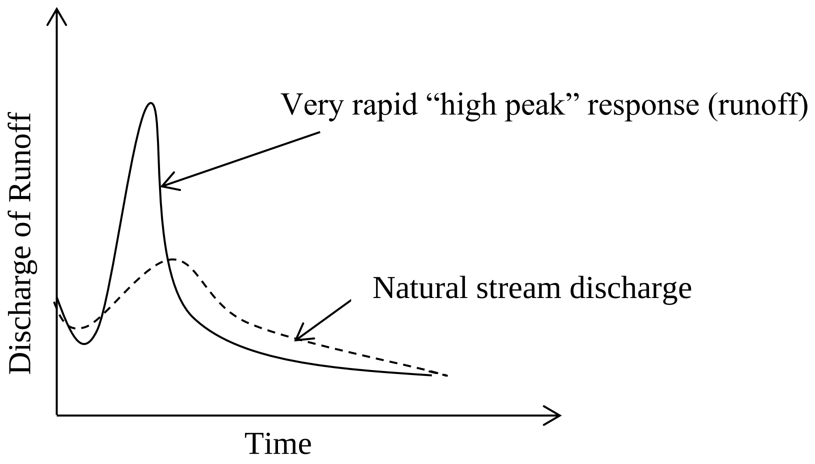

Figure 1.2 Hydrograph for stream discharge over time.

There are several reasons why the infiltration rate of water is of concern (Gregory *et al.*, 2006). Soil can be an excellent temporary storage of water. Thus, with proper management, the infiltration rate can be maximized and capture as much water as allowed. However, if water infiltrations are blocked, water does not enter the soil and caused surface ponding and runoffs as shown in Figure 1.2. This problem needs to be avoided in urban areas. Besides that, runoffs carry unnecessary particles that would end up in the place where they are not acceptable. The same thing would happen if the soil is too dry, as water can go through the soil rapidly. The soil will easily break; making it not suitable for any development without involving higher improvement cost.

Due to issues regarding movement of population to a new location and the cost needed to develop the new area, especially to relocate population with minimum cost, this research aims to enhance the existing Monge-Kantorovich mass transportation model particularly in the application to urban planning problem. It is important and interesting task to see the improvement made to the model after adding water permeability term into the model.

1.3 Statement of the Problem

From the above discussion, we identified one possible improvement to Monge-Kantorovich mass transportation model in urban planning problem is related to water issues.

Studies on urban planning problem had been done broadly. However, in regards to using Monge-Kantorovich mass transportation problem in urban planning, the studies were still lacking. Even though there were numerous types of urban planning models in various fields, this research focuses on Monge-Kantorovich mass transportation problem due to a wide application of the model.

It was observed that Monge-Kantorovich mass transportation in urban planning models focus on studies concerning the benefits for the residents, especially in minimizing the cost of spending for the masses such as transportation and managements costs. However, studies on cost of development for environmental aspect were still lacking. Particularly, there was no water permeability problem that was injected into the Monge-Kantorovich mass transportation model previously. Developing an urban area according to the environmental condition is vital as development itself could lead to harmful outcomes and loss to the residents in the future.

Also, Monge-Kantorovich mass transportation model without the water permeability term give a very wide application. The model applicable not only in environment application, but in economical application as in past researches depending on the objectives of the researches. Nevertheless, realistically, the model without the additional term is less representation of the real problem.

Therefore, this research is aimed to improve the existing Monge-Kantorovich mass transportation model in urban planning problem by adding the additional water permeability term. This research also aims to provide the relevant theoretical results to support the model. Several research questions need to be answered:

(1) How to formulate the water permeability term and hence incorporate term into existing Monge-Kantorovich model?
(2) What are the theoretical supports involved in validating the Monge-Kantorovich mass transportation model with water permeability term?
(3) How to solve Monge-Kantorovich mass transportation model with water permeability term?
(4) What are the effects of the water permeability rate to the new model?

1.4 Objectives of the Study

This research is focused in developing a new Monge-Kantorovich mass transportation with water permeability term as well as studying the effects of different values of the term to the model. Therefore, there will be several objectives for this research. The objectives are:

(1) To formulate the new Monge-Kantorovich mass transportation model with water permeability term.
(2) To establish the necessary theoretical prove and support for the new model.
(3) To solve the new Monge-Kantorovich mass transportation with water permeability term using proximal splitting method.
(4) To do sensitivity analysis on the effects of water permeability rate to the new model.

1.5 Scope of the Study

The mathematical modelling involved in this research is based on urban planning problem. Rather than considering the model in general cases, this research is focused only on water issues. Consequently, the water issues considered in this research is assumed to be the natural water absorption into soil of certain area; the destinations of the mass (estimated population) which is an undeveloped new area.

Besides that, development of a new area needs to involve a lot of other considerations. However, for the purpose of this research, the focus is only on water permeability rate. This water permeability rate will then be formed in mathematical term that can be used in the new model.

The mathematical model in this research are based on the Monge-Kantorovich mass transportation model which later on will be converted into partial differential form as proposed by Benamou and Brenier (2000). We consider the model by Benamou and Brenier (2000) as it is more applicable.

Currently there are a lot of first order optimization method, which can be considered in the research. However, for this research, only proximal splitting method is chosen for solving the Monge-Kantorovich mass transportation with water permeability term. Moreover, the proximal splitting method for this research to focus on is the Douglas-Rachford algorithm proposed by Lion and Mercier (1979).

To achieve the goal of this study, primarily, a device with Mac Operating System (MacOS) High Sierra version 10.13.6 with central processing unit (CPU) of Intel Core i5 1.3 GHz is used. The device had a random-access memory (RAM) of 4 GB 1600 MHz with DDR3. This following device is used to produce the solution for the improved Monge-Kantorovich mass transportation model with water permeability term.

1.6 Significance of the Study

This research improves the established dynamical Monge-Kantorovich mass transportation model specifically for urban planning by including the water infiltration rate. This modification is very important as it might help in improving cost of development of an area specifically related with water permeability.

This research will certainly contribute to the body of knowledge, specifically in mass transportation theory concentrating on urban planning problem. This research will develop a robust model which is more representative of the real urban planning problem. Furthermore, it will supply a sound theoretical background on the said model, hence the validity of the work cannot be questioned.

The outcome of this research can definitely benefit the country from a few aspects; economic and environmental concerns. The result of this research can be used as an alternative method in solving urban planning problem in which the environmental issues of an area are not ignored. Besides that, the outcome of this research certainly can be used for further research in related areas.

1.7 Contributions of the Study

There are four main contribution presented in this research. The main contributions are mainly to the field of mass transportation problem. The contributions are noticeably in the development of the new model, supported by the theoretical analyses. With all the theoretical support, the dynamic Monge-Kantorovich problem model with water permeability term can be used as a basis for further analyses.

1.7.1 Contribution to Model Development of Monge-Kantorovich Mass Transportation Model in Urban Planning Problem

This research is an attempt to improve the dynamical Monge-Kantorovich mass transportation model so it will be more representative of the real problem.

1.7.2 Contribution to Theoretical Support for The Model

Theoretical analyses were added into the research. All the related theorem are proven in order to support the improved model.

1.7.3 Contribution to Numerical Solutions for Mass Transportation Model with Water permeability Term

This research introduced the use of proximal splitting method in solving the Monge-Kantorovich mass transportation model with water permeability term specifically using Douglas - Rachford method.

1.7.4 Contribution to the Field of Mass Transportation Theory

The subject matter of this research is Monge-Kantorovich mass transportation problem. Undeniable, all the development of model, even for the development of water permeability term gives progression to the field. The modification on the model with all supportive proving and implementation used in this research improved the application in this field. The problem in this thesis also adds diversity to the area of problems for the mass transportation problem.

1.8 Thesis Outline

The purpose of this research is to improve the Monge-Kantorovich mass transportation model with additional term, which is water permeability term. The improved model will be considered to see whether an area is suitable for future development based on water permeability of the soil in the area. Chapter 1 introduces the Monge-Kantorovich mass transportation theory. Chapter 1 also explained the research background, research problem and objectives. Next, the scope of the research and contributions are presented in the following section.

In Chapter 2, the related literature reviews of water permeability and urban planning models are discussed. Also, some issues on urban planning are reviewed, consequently, the importance of water permeability rate to urban planning are discussed. Finally, Chapter 2 reviews the literatures on Monge-Kantorovich mass transportation theory, Benamou and Brenier formulation of the theory. In the following section, the relationship between soil and water permeability rate and Douglas-Rachford method are reviewed.

In Chapter 3, the research methodology executed throughout the research is presented. Basically, the research involved from the formulation of water permeability term until the discussion on the numerical results. This chapter consists of two sections that are the operational and the theoretical frameworks.

In Chapter 4, the research starts by formulating the water permeability term and the relationship on water permeability term with the estimated population. After that, the formulation on Monge-Kantorovich mass transportation model with water permeability term is derived.

In Chapter 5, the theoretical support involved is analysed and proven. In this chapter the derivation for the theoretical proving is presented in detail.

Chapter 6 is where the new model is applied into the proximal splitting method. The discretization and the derivation of the dynamic Monge-Kantorovich problem model with water permeability term in proximal splitting scheme are presented. Then, the discretised model is applied into Douglas-Rachford method to produce solutions.

In Chapter 7, all the discussion on the numerical results is presented. The numerical results produced from different value of water permeability rate are illustrated and compared.

Lastly, Chapter 8 provides summary and conclusion of this research. In this chapter, some recommendations are given for further improvement of Monge-Kantorovich mass transportation problems.

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LIST OF PUBLICATIONS

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