DISSIPATION EFFECTS WITH NATURAL CONVECTION FLOW OVER A NON-ISOTHERMAL VERTICAL CONE

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ABSTRACT

In this study, the effect of dissipation fully developed heat transfer by natural convection flow over a non-isothermal vertical cone is investigated. The governing equation consists of continuity, energy and momentum equations were derived. The purpose of the study is to investigate numerically the mathematical model describing the nature of natural convection flow over a vertical cone. The numerical result is obtained by using the Crank Nicolson method. This method is used due to the scheme is unconditionally stable and more accurate. The discretization equations were computed and numerical results were plotted using MATLAB software. The behaviour of velocity, temperature, skin friction and Nusselt number influenced by Prandtl numbers, viscous dissipation parameter as well as separation times of the flow were analyzed. Results showed that the behaviour of velocity and temperature profiles changed for every distinct values of Prandtl numbers, viscous dissipation parameter as well as separation time.

ABSTRAK

Dalam kajian ini, kesan lepasan yang dibangunkan sepenuhnya oleh aliran perolakan semulajadi melepasi kon menegak bukan bersifat isothermal disiasat. Persamaan menakluk terdiri daripada persamaan keselanjaran, tenaga dan momentum diterbitkan. Tujuan kajian ini adalah untuk mengkaji sefcara berangka model matematik yang menerangkan sifat aliran olakan semulajadi melepasi kon menegak. Keputusan berangka diperoleh dengan menggunakan kaedah Crank Nicolson. Kaedah ini digunakan kerana skema ini stabil tanpa syarat dan lebih tepat. Persamaan pendiskretan dihitung dan keputusan berangka diplot dengan menggunakan perisian MATLAB. Tingkah laku halaju, suhu, geseran kulit dan nombor Nusselt yang dipengaruhi oleh nombor Prandtl, parameter pelesapan likat serta masa pemisahan aliran dianalisis. Keputusan menunjukkan bahawa kelakuan profil halaju dan suhu berubah untuk setiap nilai nombor Prandtl, parameter pelesapan likat serta masa pemisahan yang berbeza.

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LIST OF ABBREVITIONS

PDE	-	Partial Differential Equation
LHS	-	Left Hand Side
RHS	-	Right Hand Side

LIST OF SYMBOLS

a,b,c,d	-	Constant
A,B	-	Tridiagonal Matrix
Cp	-	Specific heat at constant pressure
D	-	Mass diffusivity
Gr_L	-	Thermal Grashof number
g	-	Acceleration due to gravity
$\overline{h}$	-	Average heat transfer coefficient over surface of the cone
L	-	Reference length
n	-	Exponent in power law variation in surface temperature
Nu_x	-	Dimensional local Nusselt number (heat transfer rate)
$\overline{Nu_L}$	-	Average Nusselt number (heat transfer rate)
Nu_X	-	Non dimensional local Nusselt number (heat transfer rate)
Nu	-	Non-dimensional average Nusselt number (heat transfer rate)
Pr	-	Prandtl number
R	-	Dimensionless local radius
r	-	Local radius of the cone
T'	-	Temperature
T	-	Dimensionless temperature
t'	-	Time
t	-	Dimensionless time

U	-	Dimensionless velocity in X direction
u	-	Velocity component in x direction
V	-	Dimensionless velocity in Y direction
v	-	Velocity component in y direction
X	-	Dimensionless spatial co-ordinate along the cone generator
x	-	Spatial co-ordinate along the cone generator
Y	-	Dimensionless spatial co-ordinate along the normal to the cone generator
y	-	Spatial co-ordinate along the normal to the cone generator

Greek symbols

α	-	Thermal diffusivity
β	-	Volumetric coefficient of expansion with concentration
Δt	-	Dimensionless time step
ΔX	-	Dimensionless finite difference grid size in X direction
ΔY	-	Dimensionless finite difference grid size in Y direction
ε	-	Viscous dissipation parameter
μ	-	Dynamic viscosity
υ	-	Kinematic viscosity
θ	-	Cone apex half-angle
ρ	-	Density
τ_x	-	Dimensional shear stress

τ_X	-	Dimensionless shear stress
τ_L	-	Average shear stress
τ	-	Dimensionless average shear stress

Subscripts

i	-	Grid point along X direction
j	-	Grid point along Y direction
w	-	Condition on the wall
∞	-	Free stream condition
k	-	Time step level

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CHAPTER 1

INTRODUCTION

1.1 Background of the Research

Heat transfer occurs because of divergence in temperature. The thermodynamics first law illuminate the amount of preservation of energy and the second law illustrates on the direction of energy transfer. Separation flow is one of the important fluid behavior in thermodynamic problem to be explored in order to get its solution acceptably. The reason is that when flow separated, energy is lost. This problem is one of the problems that involved in viscous flow. The separation flow may occur in both situations such as external and internal flow. Flow separation was first introduced by Ludwig Prandtl in 1904. From the classical concept, flow separation occurred because of viscosity, where it can be indicated as boundary layer separation or boundary layer flow separation. In boundary layer theory, it stated that there is two condition for flow separation such as adverse pressure gradient and viscosity effect. This is the main factor where if one of these two factor missing, the flow will not separate. This statement hold for both incompressible and compressible fluids as well as laminar or turbulence viscosity effect. In mathematical view, the flow separation occur with three different situation of velocity which are when the velocity at the wall is zero or positive pressure gradient occurs in the direction of flow, and a negatively an inflection point exists in the velocity profile Brevdo and Merkine (1985).

Recently, the physical phenomena of flow separation around the vertical cone get much attention by researches to study both theoretically and numerically. From an investigation, it shows that the problem of natural convection flow over a cone placed vertically has been studied extensively in literature due to the cone shape's is suitable for laminar or turbulence viscosity effect. One of the reason is, this study can be a good model for problem of flow past blunt bodies such as cone.

In a previous studies, it has been detected that significant dissipation due to drippy fluid may occurs in natural convection flow in various equipment. In Sambath (2017), this is due to a high amount of deceleration with high speed of rotation. It can be identify the effects of viscous dissipation on gravitational fields for example, in high masses of gas in hemisphere, high velocity in bigger planets and industrial applications. The dissipation effects based on dissipative forces, for example the frictional force between two objects or the drag force in fluids flow, will rises the internal energy of the system and also increases the heat transfer to the surrounding. According to the second law of thermodynamic, this heat flow is said to be consistent which states that the entropy of the universe should get larger when those forces are present because of the irreversibility always accomplice with their actions. The effects of the dissipative forces are also included in the Newton's equations as impulses and pseudo-works.

1.2 Problem Statements

This project will focus into natural convection flow that past a cone placed vertically. The convection flow is studied with the effects of viscous dissipation. All problems will contribute on the development of the mathematical model as well as the approach applies to obtain the solutions. The mathematical model used in this project focuses on the flow over a vertical cone because this type of shape has been used extensively in order to study the behavior of laminar and turbulent flow over the surface. In the tremendous application in industry, heat transfer is one of the huge challenge that should been taken more seriously. Toward this issue, many researchers take an action to resolve it since heat transfer has important role when it comes to fluid flow. Temperature of fluid can influence the behavior of the fluid flow, especially when the temperature are not constant. Thus, in this research will considered non-isothermal vertical cone. There are three types of heat transfer occur in boundary layer region which are conduction, convection, and radiation. Each type has its own characteristic according to how the heat was transferred. In this case, convection has great possibilities or more frequent to occur in fluids. This problem is reproduce from Sambath (2017) with the same numerical approach.

The research conducted by using Crank Nicolson method and the results obtained will contribute on a better understanding on dissipation effect especially for separation time cases. In order to present the result, computational method is used to solve the model, hence this research contributes on the development of numerical programming which can solve the complex unsteady flows.

This project also will answer the following research question

a) How does the mathematical models describing the nature of free convection flow over a vertical cone?

b) How do the presence of Prandtl number, viscous dissipation parameter as well as separation times of the flow influence the numerical results of velocity, temperature, shear stress and heat transfer rate?

c) How to solve the dimensionless governing equations by using Crank Nicolson method.

1.3 Objectives of the Research

The study will explore on the effect of dissipation and heat transfer of unsteady natural convection flow over a cone placed vertically with constant heat flux as mentioned in problem statement. The objectives of this research are as follows:

1. To derive the mathematical models representing the problems.

2. To construct numerical schemes using Crank Nicolson method in order to obtain numerical solution.

3. To develop a computer programming in MATLAB environment to solve the problem.

4. To analyses the numerical results of velocity, temperature, shear stress and heat transfer rate influenced by Prandtl number, viscous dissipation parameter as well as separation times of the flow.

1.4 Scope of the Research

This study focused on a unsteady two dimensional incompressible natural convection flow over a non-isothermal vertical cone with the effects of dissipation. This problem is solved numerically via Crank Nicolson method. This method is the implicit finite difference scheme and is unconditionally stable. The numerical results are achieved via MATLAB algorithm and displayed graphically. The physical parameters taking into consideration are the Prandtl number Pr , viscous dissipation ε , and power law variation in surface temperature n .

1.5 Significance of the Research

In conducting this research, the results obtained will contribute on a better understanding on the dissipation effect with natural convection flow past a vertical cone. This research is considered of unsteady problem and will give focus on natural convection flow past a cone with non-isothermal surface temperature in the presence of viscous dissipation effect. The goal of the present work is to analyze the flow variables, coefficient of heat and mass transfer in the transient period for value of viscous dissipation parameter. The governing equations of the related problems in non-dimensional form are unsteady and non-linear. Therefore, no analytical method is available to solve for such problem. Hence, the numerical method particularly finite difference method paves the way to solve this problem. In present work, an implicit finite difference scheme which is Crank-Nilcoson is used to solve the problem and find the results. By using computational method to solve the model, hence this research contributes on the development of numerical programming. It also will give a clear picture in a form of graph presentation to other researchers when they are dealing with modelling, simulation and analysis.

1.6 Organization of the Research

This research is organized into five chapters. The first chapter provides the whole idea of this research. This chapter includes the introduction, the research background, problem statements, research objectives, research scope and research significance.

Chapter 2 consists of the literature review of this research. Some previous researches on heat transfer, thermodynamic's laws, convection flow and non-isothermal surfaces are included. Besides, some basic concepts of dissipation effects done by previous researchers are also stated.

The derivation of the equations is then introduced in Chapter 3. The derivation include Partial Differential Equation (PDE) of the motion of the boundary layer which due to conservation of momentum (Navier-Stokes equation), conservation of energy and mass diffusion equation. After the derivation is made, the equations is then presented in a form of non-dimensional equation. This chapter introduce some parameters and constants that related to the equations such as shear stress τ_x, heat transfer rate Nu_x and etc.

The solutions of dissipation effects with natural convection flow over a non-isothermal vertical cone by using MATLAB programming software are presented in Chapter 4. In order to apply numerical approach, the discretization is used for every equation by using Crank Nicolson method. The discussion and analyzation for the behavior of the numerical results of velocity, temperature, shear stress, heat transfer rate, influenced by Prandtl numbers, viscous dissipation parameter as well as separation times of the flow also included in Chapter 4.

Finally, the conclusion of this research is presented in the last chapter. Some suggestions for future research regarding the problem of natural convection over a non-isothermal vertical cone with some other effects are given in this chapter.

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