# DISCRETE HOMOTOPY ANALYSIS METHOD ON HEAT CONDUCTION WITH RADIATION PROBLEM VIA FREDHOLM INTEGRAL EQUATION

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# ABSTRACT

Science and engineering problems can be modelled in Fredholm integral equations (FIE) whether it is a linear or nonlinear problem. There is method called homotopy analysis method (HAM) can be used to solve even highly nonlinear equations ensuring convergence by introducing convergence control parameter, $\hbar$. HAM has been applied to solve linear and nonlinear Fredholm integral equations with high accuracy. There is also discretized version of HAM which evaluate definite integrals in FIE using numerical integration method which is called discrete Homotopy analysis method (DHAM). We applied DHAM on highly nonlinear Fredholm integral equation and compared the results with Nystrom method. It shows that convergence control parameter, $\hbar$ introduced in DHAM provides a convenient way of ensuring convergent series solutions. DHAM convergence is also faster than Nystrom method in solving nonlinear equations. We proved that accuracy of both DHAM and Nystrom method are dependent on numerical integration. Therefore, the DHAM is superior than Nystrom method in solving nonlinear Fredholm integral equations.

# ABSTRAK

Pelbagai masalah sains dan kejuruteraan boleh dimodelkan dalam bentuk persamaan kamiran Fredholm (FIE) samada masalah linear atau tidak linear. Kaedah analisis homotopy (HAM) boleh digunakan untuk menyelesaikan masalah tidak linear dengan memastikan penumpuan jawapan dengan parameter kawalan penumpuan, $\hbar$. HAM telah digunakan untuk menyelesaikan FIE linear dan tidak linear dan memperoleh jawapan yang tepat. Terdapat juga HAM versi diskrit yang menilai kamiran tentu menggunakan kaedah kamiran berangka yang dikenali kaedah analisis homotopy diskrit (DHAM). DHAM telah diaplikasikan pada FIE tidak linear tinggi and perbandingan jawapan telah dibuat dengan kaedah Nystrom. Parameter kawalan penumpuan, $\hbar$ telah terbukti merupakan cara mudah untuk memastikan penumpuan siri jawapan. Penumpuan jawapan DHAM juga lebih pantas berbanding kaedah Nystrom. Ketepatan jawapan bagi kedua-dua kaedah ini adalah terbukti bergantung pada kaedah kamiran berangka yang digunakan. Oleh itu, DHAM adalah kaedah yang lebih unggul berbanding kaedah Nystrom.

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# LIST OF ABBREVIATIONS

| | | |
|---|---|---|
| FIE | - | Fredholm integral equations |
| HAM | - | Homotopy analysis method |
| DHAM | - | Discrete homotopy analysis method |
| WRM | - | Weighted residual method |
| ODE | - | Ordinary differential equations |
| PDE | - | Partial differential equations |
| FEM | - | Finite element method |
| DADM | - | Discrete Adomian discomposition method |

## LIST OF SYMBOLS

| | | |
|---|---|---|
| $\hbar$ | - | Convergence control parameter |
| $p$ | - | Homotopy parameter |
| $L[\ ]$ | - | Linear operator |
| $N[\ ]$ | - | Nonlinear operator |
| $A_m$ | - | Adomian polynomial of order $m$ |

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# CHAPTER 1

# INTRODUCTION

## 1.1 Background of Study

Many physical phenomena in science and engineering can be modelled in terms of differential equations whether it is a linear or nonlinear. Among common problems that is of high interest are heat transfer problems which can be classified into conduction, convection and radiation. In recent years, there are researches that are trying to solve problems with at least two, if not all three of these heat transfer methods combined.

Solving linear problem with simple geometry, using classic analytical method is possible. However, in most cases, physical problem is of complex geometry to suit the need in functional design, attractive aesthetic, cost reduction and many more. For problems with complex geometry, analytical method is almost impossible to be implemented.

Finite element analysis (FEA) plays an important role in modern science and engineering especially in solving complex problem. It is a useful method to solve initial boundary value problem for differential equations by subdividing large model into smaller parts called finite element.

Instead of using the strong form of differential equation that governs a system, FEA normally uses the weak form of integral equation as solving the strong form of differential equation that satisfy its boundary condition at every point is very difficult. Among frequently used technique are weighted residual method (WRM) and variational method.

There are various methods to solve integral equations from the simple trapezoidal rule for linear to Adomian decomposition for nonlinear integral equation. There is a method called Homotopy analysis method (HAM) S.J Liao [1] introduced in his PhD theses. This method is a powerful method as it introduces a convergence parameter to assist convergence of a solution. There are cases where this method may not be able to be implemented as the integral parts associated to the equation is difficult to be solved if not impossible. Allahviranloo and Ghanbari [12] introduce discrete homotopy analysis method where he solves the integral part of the equation with an appropriate numerical method.

In this study, we will try to solve one dimensional steady state heat conduction with radiation equation which take the form of nonlinear one dimensional differential equation by converting it into Fredholm integral equation and solve the obtained Fredholm integral equation using discrete homotopy analysis technique. The result is then analysed to see the advantages of the method and to find out whether the benefits it brings are significant.

## 1.2 Problem Statement

Heat conduction with radiation problems can be modelled in the form of nonlinear differential equations with boundary conditions which is of a strong form. However, strong form numerical approach facing problem of less accuracy and stability problem. In addition, strong form requires the higher order of smoothness of solution. In the numerical solution of weak form, it also facing the problem of truncation error.

Strong form of a problem can also be transformed into a weak form of integral equation equivalent to the differential equation and its boundary conditions. There are analytical solutions for weak form of integral equation which will produce exact solution in series form which is homotopy analysis method (HAM). However, solving nonlinear integral equation using HAM is not always possible. Therefore, we should find solution using HAM with numerical approach which is called discretized homotopy analysis method (DHAM).

## 1.3 Objective of Study

The objectives of this study are as follows:

1. To apply discrete homotopy analysis method (DHAM) on one dimensional non-linear steady state heat conduction with radiation equation
2. To apply Nystrom method with Newton-Raphson iterations on one dimensional non-linear steady state heat conduction with radiation equation
3. To analyse and compare the numerical results between DHAM and Nystrom method.

## 1.4 Scope of Study

This study focuses on solving time independent one dimensional heat conduction with radiation which is a nonlinear ordinary differential equation. A nonlinear differential equation is chosen for its level of difficulty. Prior to solving the equation, it will be converted into Fredholm integral equation then discrete Homotopy analysis method is applied to the integral equation. Result obtained is then compared with Nystrom method in terms of their accuracy and whether it give a significant impact.

In this study, C++ programming will be used to present the numerical solutions of both methods. C++ is chosen for simplicity as both methods mentioned above do not require high level of programming language.

## 1.5 Significance of Study

The outcome of this study benefits both mathematicians and engineers in solving nonlinear equations. It will give new option to solve nonlinear equations with the idea of how significant the method's accuracy and stability and whether it is worth to be considered.

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