COUPLED FORMULATION OF NON-UNIFORM RATIONAL B-SPLINE AND RADIAL POINT INTERPOLATION METHOD FOR 2D PROBLEMS

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Civil Engineering)

School of Civil Engineering
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NOVEMBER 2020

DEDICATION

This thesis is dedicated to my mother, lovely wife and kids, and family.

ACKNOWLEDGEMENT

Praise to Allah, the Almighty who sparks my intuition to pursue my post graduate studies and provide me with invaluable guidance throughout my life.

I wish to express my deepest gratitude to my supervisor Dr. Ahmad Razin Zainal Abidin and my co-supervisor Associate Professor Dr. Airil Yasreen Mohd Yassin for their encouragement, guidance, critics and friendship. Without their continued support and interest throughout the duration of this study, this thesis would not have been the same as presented here.

Thanks to Universiti Selangor, for providing the financial support throughout the course of the research.

Special thanks to the closet person to me, Fadillah Hassan Berseri, whose love and support has always been a source of inspiration and strength to me. Also to my beloved children, Cinta and Suci, thanks for cheering up my life. To Emak, your second son loves you and thanks for teaching me about life. To my family, my deepest appreciation goes out to all of you.

My sincere appreciation also extends to all my colleagues and others who have provided assistance at various occasions. Their views and tips are useful indeed. Unfortunately, it is not possible to list all of them in this limited space. Finally, my gratefulness to all authors who wrote the books and papers that I had read.

ABSTRACT

Simulation-based design in engineering is becoming very important nowadays due to the advancement of computing technology. In this arena, computer-aided design (CAD) for modelling and computer-aided engineering (CAE) for analysis are the two major components. They evolve independently despite dealing with the same object-of-interest. The non-collaborative nature of CAD and CAE has resulted in more manpower and less computer time being used in the steps involved during data transfer for the modelling-analysis process, which can lead to many errors. Ideally, this process should be performed entirely by a computer without human intervention. In bridging the gap between the two, isogeometric analysis (IGA) was proposed to perform both modelling and analysis using the same basis functions, *i.e.*, non-uniform rational B-spline (NURBS). However, NURBS is formulated through the operation of tensor products, thus the refinement in the analysis process is found to be expensive due to excessive overhead of control points. This study presents the idea to develop more efficient methods by considering the NURBS only for modelling while the analysis is developed based on the Meshfree radial point interpolation method (RPIM). The main objective of this study is to construct and formulate a complete procedure for coupling NURBS and RPIM formulations, and written as N-RPIM. Computer code implementing N-RPIM is developed with MATLAB programming language. The N-RPIM is constructed based on Galerkin weak form formulation and possess the Kronecker delta property, hence enabling easy imposition of essential boundary conditions. Furthermore, parametric studies of two-dimensional planar analysis are conducted to determine the optimum range and value of parameters in ensuring the best performance of the N-RPIM method. The method is validated by employing heat transfer and plane stress problems, and is then extended to model a cellular beam with complex geometry due to the existence of web-holes along its span. Two types of performance are assessed; the convergence rate for displacements and stresses predictions. The presented result shows that, the N-RPIM works well and provides a favourable comparison against established numerical method, *i.e.*, finite element method (FEM). The converged solution is achieved faster and provides an exact solution of less than 90% of the number of nodes compared to FEM. The convergence of displacement is achieved when the total number of nodes reaches approximately 5,000 nodes with an error of 0.005%, while more than 20,000 nodes required for the FEM to converge. In the prediction of stresses throughout the beam, the N-RPIM stress functions are readily continuous over the domain, whereas some post-processing would be required in FEM for smoothing the stresses value over the domain. This shows the potential of N-RPIM as an alternative numerical method in bridging the substantial bottleneck between CAD and CAE. In addition, by taking advantage of the exact geometry presented by NURBS and, with the flexibility and adaptivity of RPIM to determine field variables, this new method promises highly effective solutions when dealing with irregular domain problems.

ABSTRAK

Rekabentuk berasaskan simulasi dalam bidang kejuruteraan menjadi sangat penting pada masa kini disebabkan oleh kemajuan teknologi pengkomputeran. Dalam arena ini, reka bentuk terbantu-komputer (CAD) untuk kerja pemodelan dan kejuruteraan terbantu-komputer (CAE) untuk kerja analisis adalah dua komponen utamanya. Kedua-dua teknologi ini berkembang secara berasingan walaupun di dalam menyelesaikan permasalahan yang sama. Sifat bukan kolaboratif CAD dan CAE telah menyebabkan banyak tenaga manusia dan kurang masa pengkomputeran digunakan semasa pemindahan data untuk proses pemodelan-analisis, yang boleh menyebabkan banyak kesalahan berlaku. Secara ideal, proses ini harus dilakukan sepenuhnya oleh komputer tanpa campur tangan manusia. Bagi menghubungkan keduanya, kaedah analisis isogeometrik (IGA) telah dicadangkan iaitu kerja-kerja pemodelan dan analisis menggunakan fungsi asas yang sama, iaitu B-spline rasional tidak seragam (NURBS). Walau bagaimanapun, NURBS dirumuskan melalui pengoperasian produk tensor, oleh itu proses penambahbaikan ketika analisis menjadi sukar kerana lebihan titik kawalan. Kajian di dalam tesis ini mengemukakan idea untuk mengembangkan kaedah yang lebih cekap dengan mempertimbangkan NURBS hanya untuk pemodelan, sementara analisis dikembangkan berdasarkan kaedah tanpa jejaring interpolasi titik jejarian (RPIM). Objektif utama kajian ini adalah untuk membina dan merumuskan prosedur lengkap penggabungan formulasi NURBS dan RPIM, dan ditulis sebagai N-RPIM. Kod komputer yang menjayakan N-RPIM telah dibangunkan dengan menggunakan bahasa pengaturcaraan MATLAB. N-RPIM dibina berasaskan formulasi bentuk lemah Galerkin dan mempunyai ciri-ciri Kronecker delta, dengan itu membolehkan penyelesaian syarat sempadan dilakukan dengan mudah. Seterusnya, kajian parametrik untuk analisis dua dimensi dilakukan bagi menentukan nilai parameter yang paling optimum di dalam memastikan prestasi terbaik kaedah N-RPIM. Keberkesanan kaedah ini disahkan dengan menggunakan permasalahan pemindahan haba dan tekanan satah, seterusnya kajian diperluaskan dengan permodelan rasuk bersel dengan geometri kompleks kerana kewujudan lubang web di sepanjang rentangnya. Dua jenis prestasi telah dinilai; kadar penumpuan untuk anjakan dan ramalan tegasan. Keputusan menunjukkan bahawa N-RPIM memberikan perbandingan yang baik terhadap kaedah numerik yang biasa digunakan, iaitu kaedah unsur terhingga (FEM). Penumpuan dicapai lebih cepat dan memberikan penyelesaian tepat dengan bilangan titik kawalan 90% kurang berbanding FEM. Kadar penumpuan anjakan dicapai dengan jumlah titik kawalan mencapai kira-kira 5,000 dengan ralat 0.005%, sementara lebih daripada 20,000 titik kawalan diperlukan untuk FEM menumpu. Untuk ramalan tegasan, N-RPIM memberi kelebihan melalui fungsi bentuknya yang berterusan, tanpa memerlukan pasca-proses bagi tegasan tidak berterusan yang biasa dilakukan di FEM untuk menyempurnakan nilai tegasan di atas domain. Ini menunjukkan potensi yang baik untuk N-RPIM sebagai kaedah berangka alternatif dalam merapatkan perbezaan di antara CAD dan CAE. Di samping itu, dengan memanfaatkan geometri tepat yang ditunjukkan oleh NURBS dan, fleksibiliti dan adaptasi RPIM di dalam menentukan pemboleh ubah medan, kaedah baru ini juga menjanjikan penyelesaian yang sangat berkesan bagi menangani masalah domain yang tidak teratur.

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LIST OF ABBREVIATIONS

1D, 2D, 3D	-	1-dimensional, 2-dimensional, 3-dimensional
CAD	-	Computer-aided Design
CAE	-	Computer-aided Engineering
CPU	-	Central Processing Unit
DoF	-	Degree of Freedoms
EXP	-	Gaussian Shape Function
FEA	-	Finite Element Analysis
FEM	-	Finite Element Method
IGA	-	Isogeometric Analysis
MATLAB	-	Matrix Laboratory
MQ	-	Multi-quadrics Shape Function
NASTRAN	-	NASA Structural Analysis
NEFEM	-	NURBS-enhanced finite element method
N-RPIM	-	NURBS- Radial Point Interpolation Method
NURBS	-	Non-Uniform Rational B-Splines
PDE	-	Partial Differential Equations
PIM	-	Point Interpolation Method
RBF	-	Radial Basis Functions
RKPM	-	Reproducing Kernel Particle Method
RPIM	-	Radial Point Interpolation Method
TPS	-	Thin Plate Spline Shape Function
UB	-	Universal beam

LIST OF SYMBOLS

A	-	Area of Physical Space
b_f	-	Flange width
b_x, b_y	-	Specified known values
B_i	-	NURBS control points
$C(\xi)$	-	B-Spline curve function
$C^n(\xi)$	-	NURBS curve function
C_q	-	Specific heat
d	-	Dimension of the problem
d_i	-	Nodal displacement
d_c	-	Characteristic length in nodal spacing of support domain
D	-	Constitutive matrix
D_p	-	Beam depth
D_o	-	Hole diameter
e	-	Error
E	-	Young’s Modulus
F^e	-	Local force vector
F_x, F_y	-	Body forces
G	-	Moment matrix of RBFs
H	-	Height
I	-	Moment of Inertia
$J_{\tilde{\xi}}$	-	Jacobian of Lagrange shape function
$J_{\hat{\xi}}$	-	Jacobian of NURBS basis function
k_x, k_y	-	Thermal conductivity
K	-	Stiffness matrix
K^e	-	Local stiffness matrix
L	-	Length
m	-	Number of polynomial terms
n	-	Number of RBFs terms
n_x, n_y	-	Unit normal vector

$N_i(\xi), M_j(\eta)$	-	B-Splines basis function
p	-	Polynomial degree of B-spline
$P_i(x)$	-	Polynomial basis function
q	-	Heat generation
r	-	Radius
r_i	-	Distance between point of interest, x_Q and a node, x_i
R	-	Load vector
$R_i(x)$	-	Radial basis function
$R_{i,j}^{p,q}(\xi, \eta)$	-	NURBS basis function
S	-	Spacing between adjacent web-holes
S_e	-	Spacing between web-hole and end support
$S(\xi, \eta)$	-	B-Spline surface function
$S^n(\xi, \eta)$	-	NURBS surface function
t	-	Thickness of element
t_f	-	Thickness of flange
t_w	-	Thickness of web
T	-	Temperature
$u(x)$	-	Displacement function
$U, \widetilde{U}$	-	Displacements vector
w_i	-	Weighting of the control points
x_i	-	Location of node
x_Q	-	Point of interest (node)
∂	-	Differential operator
σ	-	Normal stresses
τ	-	Shear stresses
ε	-	Strain
Γ	-	Domain boundary
ρ	-	Density
ϕ	-	RPIM shape function
q_x, q_y	-	Heat flux
q_H	-	Internal heat generation
υ	-	Poisson's ratio

$\Xi, \mathcal{H}$	-	Knot vector
α_c, q, η	-	Dimensionless RBFs shape parameters
$\Omega(x, y)$	-	Physical space
$\tilde{\Omega}(\tilde{\xi}, \tilde{\eta})$	-	Parent space
$\hat{\Omega}(\hat{\xi}, \hat{\eta})$	-	Parametric space
$\{\ \}$	-	Vector
$[\ ]$	-	Matrix
$\|\ \|$	-	Determinant
$\|\ \ \|$	-	L^2-norm

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The development of digital computer and the growing number of challenges in engineering problems has spurred the growth of simulation-based design significantly, thereby changing the way engineers interact with engineering problems. Simulation-based design concept refers to the simulation of the entire cycle of the engineering solutions, from concept development to detail design through implementation and integration of computer technologies and associated software tools. The use of simulation-based design significantly shortens the cycle thus giving immediate results in solving engineering problems. It further provides the engineers with immediate feedback on design decision, in turn, promising more comprehensive exploration and improved final design.

The simulation-based design consists of two (2) major components. One is the geometric model, which is formulated in the language of Computer-aided Design (CAD) software, and the other is the analysis, which is normally derived using Finite Element Analysis (FEA) and known as Computer-aided Engineering (CAE) software. The editorial of the third special issue of ‘Advances in Finite Element Method' reported that the global market growth for engineering software is worth about US$ 20 billion in 2014. Of that value, US$ 8-9 billion and US$ 4-5 billion are CAD and CAE software respectively, and others, *e.g.*, architecture, manufacturing, etc., valued at US$ 6-8 billion. The Compound Annual Growth Rate of the CAD software market is about 8-9%, while for CAE software is about 15% (Cen, *et al*., 2016).

CAD and CAE work independently despite dealing with the same object-of-interest. The non-collaborative nature of CAD and CAE has caused engineering designers and analysts tend to use a CAD software for design and CAE software for analysis separately (Roy, *et al*., 2008; Liu, *et al*., 2007). The construction of geometrical model is first represented using CAD software, the mesh is then generated from that CAD data, and lastly the analysis is performed using CAE software. The effort to convert CAD data to quality mesh is greater than that for analysis. It is estimated that about 80% of the entire time is devoted to mesh generation and analysis-suitable geometry, and 20% is devoted to analysis, in which the ratio of 80:20 for modelling-analysis seems to be a very common industrial experience (Cottrell, Hughes and Bazilevs, 2009). The concern is more manpower and less computer time in the steps involved due to the differences in the descriptions of the CAD and CAE system, which can lead to many errors. Therefore, ideally the process would be fully performed by computer without human intervention.

Many efforts have been made to bridge the gap between CAD and CAE. Several commercial CAD and CAE software have made significant improvements in the integration of both, but most of these software only allow cooperation within the authorized designated CAD or CAE systems and still did not fully integrate the two. Another attempt is the coupling CAD-CAE systems based on mathematical models. This method requires not only a wealth of knowledge, experience and effort, but also appropriate numerical techniques, and skills in scripting programming languages. Although the process is complex, the outcomes lead to obtain a global solution to the issue. As such, it has been a favourite topic for researchers lately, as well as the interest in this study.

1.1.1 Toward Optimization of CAD and CAE

Over the last few decades, FEA plays an important role in CAE software. FEA is a simulation concept developed from the theoretical basis established by the Finite Element Method (FEM). FEM history began in 1943 when Richard Courant introduced a method for solving certain boundary-value problems for the solution of PDE which is the basis of the FEM idea (Williamson, 1980). In 1968, Ergatoudis, Irons and Zienkiewicz introduced isoparametric representation in FEM. The main idea of isoparametric is the shape functions used for unknown values (displacements) are the same as the shape functions used for elemental geometric mapping. The first mathematical proofs on the properties of the FEM were published by Babuska and Aziz (1972).

FEM was created as numerical techniques for finding an approximate solution for Partial Differential Equations (PDE). In the formulation, a continuum with a complicated shape is divided into several elements. The individual elements are connected together by a topological map called a mesh. Due to its capability to simulate nonlinear behaviour, this numerical technique is well established and the most popular choice for analysis tools in the CAE system. However, the creation of a mesh and the non-smooth C^0-continuity across element boundaries in FEM has become a major drawback in FEM, leading to the developments of new numerical techniques, *e.g.*, the Meshfree methods.

The predominant technology used by CAD is the Non-Uniform Rational B-Splines (NURBS). The NURBS basis function is a mathematical model, which provided an efficient and numerically stable algorithm that can exactly represent all conic sections and allows very flexible modelling. NURBS properties are able to be refined through knot insertion, C^{p-1}-continuity for p^{th} order of NURBS, and is more robust regarding mesh distortion.

The above explanation clearly shows that the development and evolution of CAE analysis model is independent of the CAD model, and vice versa. Engineering designers and analysts tend to use CAD software for design and CAE software for analysis separately, thus analysis-suitable models are not automatically meshed from CAD geometry. Fundamental changes must be made to integrate both completely. Therefore, to optimize the current process, Hughes, *et al*. (2005) pioneered Isogeometric Analysis (IGA). IGA offers the possibility of coupling methods for modelling and analysis into a single process. The endeavour is to mutually utilize computational geometry technologies and computational engineering analysis technologies.

1.1.2 IGA Merging Modelling and Analysis into One Model

IGA is being researched to fill up the existing gap between the worlds of CAD and CAE, with fully integrate the engineering design and analysis processes, and hence provide a better-approximated result. IGA's concept outlined by Hughes, *et al*. (2005) was to employ NURBS basis functions not only for exact geometry discretization but also directly adopt to approximate the unknown fields in numerical analysis. IGA has facilitated the design iteration and avoids geometry errors introduced by the FEM discretization at domain boundaries. IGA can be considered a successful merging of CAD and CAE. This has saved a lot of time taken by manpower for the entire process, thereby reducing more human errors.

However, NURBS basis functions are formulated through the operation of tensor products. This leads to an excessive overhead of control points and refinement which is found to be a global operation. Moreover, when the geometry is topologically complex, subdivision into multi-patch domains is needed, resulting in inconsistencies at patch boundaries. These deficiencies have a negative impact on numerical analysis.

1.1.3 Continuous Work of IGA

Many advanced techniques have been introduced to improve the issues in IGA. T-splines is the earliest method that has been developed to correct the deficiencies of NURBS basis functions by employing local refinement. However, the development of T-splines still relies on a structured grid in the parametric domains and this will still cause the *h*-refinement remains restricted.

Another attempt to improve the issues were proposed by Sevilla, *et al*. (2008, 2011). The method named NURBS-enhanced finite element method (NEFEM), considered the exact NURBS basis functions description only for the geometrical boundary of the domain while the solution of the numerical analysis was approximated with a standard piecewise polynomial interpolation. NEFEM preserved the computational efficiency of classical FEM analysis, thus allowed NURBS's exact geometry discretization. Nevertheless, creation of a mesh in FEM has always faced shortcomings including difficulties in adaptive analysis and low accuracy of stress. This situation did not seem to be a great advantage to the original issues in NURBS-based IGA.

On the other hand, integration of NURBS-based IGA with Meshfree method have been developed (Wang, *et al*., 2018; Zhang and Wang, 2017; Greco, *et al*., 2017; Chi and Lin, 2016; Valizadeh, *et al*., 2015; Wang and Zhang, 2014; Rosolen and Arroyo, 2013). The term 'Meshfree' was obtained from its ability to establish system algebraic equations for the whole problem domain without the use of a predefined mesh for the domain discretization, thus eliminating the FEM disadvantages as described in the previous paragraph. The formulation of Meshfree's shape function is based on nodes in support domains which is contrary to the FEM's procedure, and thus highly versatile and attractive approach for discretization. However, one of the major difficulties in the implementation is the non-interpolatory character of the approximation of the shape functions. The approximation does not pass through the nodal values in which interpolation of functions is not unity at nodes, thus does not possess Kronecker delta property. As a consequence, the imposition of essential boundary condition is less precise in comparison to a mesh based approximation.

1.1.4 Meshfree Method as a New Class of Numerical Methods

In recent years, a group of Meshfree methods have been developed and achieved remarkable progress. The inventions of Meshfree methods were motivated by the attempt to remove the need for predefined meshes which are required in FEM. A Meshfree method uses a pattern of nodes instead of mesh to discretize the analysis domain. Ease in programming due to no domain or surface meshing, make these methods very attractive.

Construction of shape function is one of the most important and fundamental tasks in developing a Meshfree method. In the early development of Meshfree methods, *i.e.*, Moving Least Squares or Reproducing Kernel, the shape functions are based on polynomial reproduction have been widely used. However, special treatments are needed for imposing the essential boundary conditions, because the approximation function does not possess Kronecker delta property.

A Meshfree point interpolation method (PIM) was proposed to address the issue (Liu and Gui, 2001a, 2001b; Wang, *et al.*, 2001). The PIM employed polynomials as its basis functions, in which the number of shape function is the same as the number of nodes. Hence, the PIM shape functions possess Kronecker delta property. However, PIM has weaknesses in which the moment matrix of the shape functions could be singular. Therefore special techniques are needed to overcome the issue. Wang and Liu (2002) proposed radial basis functions (RBF) to overcome the singularity issue and termed as Radial PIM (RPIM). It has been proven that the moment matrix of RBF interpolations is invertible for constructing shape functions in PIM. The RPIM has recently made remarkable progress in the Meshfree method of solutions. Its approximation function passes through each node point in the influence domain, thus makes the implementation of essential boundary conditions much easier and reducing complexity in numerical algorithms than other Meshfree methods.

1.2 Problem Statement

The key idea of IGA is to employ the same basis functions, *i.e.*, NURBS, used by CAD to model the geometry are also used to approximate the unknown fields in numerical analysis. Unfortunately, the basis function of NURBS is formulated through the operation of tensor products, thus it has certain weaknesses that make a significant impact on numerical analysis. Due to the tensor product nature, the control points are required to lie in a structured grid. That means, in the context of local mesh refinement, adding a new control point requires the addition of an entire row or column of control points to maintain the product structure of this tensor. This results in excessive overhead of control points which contain no significant geometric information. Therefore, a large percentage of nodes are only needed to satisfy topological constraints.

Other techniques have been developed to address the weaknesses of IGA, where the basis function of NURBS is only used to model the geometrical domains while the field approximations in the numerical analysis remain with standard FEM shape function. Nonetheless, due to the meshes concept and the piecewise continuous nature of the predefined mesh in FEM, this method does not seem to be of great advantage to the original issues in IGA. The non-smooth C^0-continuity basis functions at the interfaces of the boundaries in FEM elements affected the accuracy of stress analysis. The stress is a function of a gradient for the displacement, therefore C^0-continuity across the boundaries results in stress values being undefined at these boundaries, thus required special techniques in the post-processing stage in order to achieve better accuracy.

In an effort to improve the weaknesses of the IGA and the coupled of NURBS-FEM, another attempt has been made to couple NURBS with Meshfree approximation. It follows a similar approach to the one employed in a coupling NURBS and FEM, but a Meshfree method is used in the field approximations. A Meshfree method is a group of numerical methods that sharing the same techniques, *i.e.*, do not require predefined meshes information to construct the approximation function. The shape functions can achieve essentially arbitrary order of continuity and completeness. It breaks the mesh-based view in IGA and FEM analysis further introduces the idea of getting rid of meshes concept in coupling CAD and CAE, therefore highly versatile for discretization and becomes a more promising technique.

Even though the benefits of Meshfree concept in domain discretization are visible, current studies on coupling NURBS and Meshfree have shown difficulties in imposing essential boundary conditions because of the boundary description of domain discretized by Meshfree particles is less precise. This is due to the character of the selected shape functions are not unity at nodes, thus does not possess the Kronecker delta property, in turn, the analysis needs special schemes to ensure these desired features. Therefore, more studies, in particular to improve the Meshfree shape functions in the coupled CAD and CAE should be enhanced.

1.3 Research Goal

The Meshfree RPIM has been proven to possess the Kronecker delta property. Its approximation function passes through each node point in the influence domain, thus makes the implementation of essential boundary conditions can be imposed directly. The ability of the RPIM to fit the domain nodes exactly will be able to improve the efficiency of the coupled NURBS and Meshfree, and in turn provide better analysis. Therefore, the main goal of this study is to integrate between CAD and CAE through coupling the formulations of NURBS basis functions and the Meshfree RPIM approximation. To reinforce the idea of this newly developed method, intensive studies will be conducted for two-dimensional (2D) planar analysis of heat transfer and plane stress problems. For engineering applications, this new method will be extended to model a cellular beam with complex geometry due to the existence of web-holes along its span.

1.4 Research Objectives

The objectives of this research are summarized as follows:

1) To construct and formulate a coupled formulation of NURBS and Meshfree RPIM for 2D elastic solid problems, and to establish the corresponding source codes.

2) To determine the optimum range and value of parameters in ensuring the best performance of the developed formulation by conducting parametric studies and validate the results against benchmark problems.

3) To evaluate the performance of the developed formulation in engineering applications by performing analysis on cellular beams with various boundary conditions and validate the result against established numerical method, *i.e.*, FEM.

1.5 Research Scope and Limitation

The formulation is applied for 2D planar analysis of heat transfer and plane stress equations. Heat transfer is a scalar field problem, therefore, it will be used to facilitate the discussion on the development of the formulation. The effectiveness of the developed formulation is then tested by analysing the plane stress equations. All assumptions in heat transfer and plane stress equations holds. The material of the element assumed linearly elastic. The deflection of the element is considered small and the plane sections remained plane after deformation.

For engineering applications, the formulation is used to analyse cellular beam problems with multiple web openings. Since the study focusses on the accuracy of irregular domain analysis, the geometric modelling of the cellular beam only consider the web region with openings without the flanges.

Although one of the main advantages of the Meshfree method is the ease of treating the irregular arrangement of nodes in the refinement process, due to the structured nature of NURBS mapping, only uniform distribution of nodes is considered. Three types of Radial Basis Functions (RBFs) are considered in the construction of the Meshfree RPIM shape functions *i.e.* Multi-quadrics (RBF-MQ), RBF-Gaussian (RBF-EXP), and RBF-Thin Plate Spline (RBF-TPS).

The study strictly involves mathematical derivations and computer programming, therefore the validation and verification of the works are carried out against standard benchmarking numerical method *i.e.* test patch and cantilever beam subjected to distributed vertical load. For cellular beam analysis, as no closed-form solution is available, the exact solution is estimated from established commercial FEM software *i.e.* COMSOL, set up with very fine mesh. The source codes of the developed formulations are written in MATLAB (Matrix Laboratory) programming language developed by MathWorks.

1.6 Outline of Thesis

This present chapter gives a brief introduction to the issues of coupling methods for analysis and modelling into a single process. The concept of IGA and the role of NURBS basis functions have been briefly explained. An overview of the Meshfree methods and the shortcomings issues have also been discussed in detail within the body of the text. The proposed idea of coupling RPIM and NURBS formulation is highlighted and objectives of the study are presented at the end of the chapter.

In Chapter 2, an extensive review of the development of CAD and FEM in the simulation-based design and the contribution in the construction industry is reviewed. The review starts with early development of the technology. Till then, the study area has been extended to the recent development of IGA and Meshfree methods. The second part of the review touches on the development of RPIM in various attempts in solving structural mechanics problems and a review on the development of cellular beams with multiple web openings.

The shape function of numerical solution is the main parameters that give numerical methods their own characteristic. Therefore, in Chapter 3, the construction of RBFs and NURBS basis function are derived in a step-by-step manner. The discussion of the RBFs is based on three (3) typical RBFs, that is, Multi-quadrics (MQ), the Gaussian (EXP), and the Thin Plate Spline (TPS). The Kronecker delta property and the satisfaction of the partition unity are also demonstrated. The discussion on NURBS formulation is based on the B-spline basis functions. Some common objects in engineering are also used to demonstrate the advantages of NURBS basis functions.

Chapter 4 presents the development of a novel formulation of Meshfree method based on RPIM with the integration of NURBS basis functions, namely N-RPIM. To facilitate the complexity of the discussion, the steady heat transfer differential equation will be used as a mathematical model. Finally, through a numerical example, *i.e.*, quarter ring element, the concepts of the proposed method are reinforced. The code is developed using MATLAB programming language and the results were verified against the established numerical methods, FEM.

In Chapter 5, the formulation of N-RPIM is extended to plane stress problems. The chapter begins with the discretization of the plane stress partial differential equation (PDE) in to algebraic equations by Galerkin Weighted-residual Method. Numerical testing of various numerical parameters have been conducted and validated against benchmark problems *i.e*. cantilever beam subjected to distributed vertical load published by Timoshenko and Goodier (1951). Then, using the optimum values of the numerical parameters, the convergence rate of the N-RPIM methods is assessed.

This is followed by Chapter 6, to use the methods developed in real engineering application. The method was extended to model cellular beams problems with multiple web openings. The beam is produced from an original I-section of $UB1016 \times 305 \times 222$. Then, it was designed using *Cellbeam* software by *Westok Ltd*. The beams analysis is performed by dividing the domain into patches known as unit cells. Therefore, the geometrical modelling of the cellular beams is based on unit cells, and connectivity array is used for assembling of the stiffness matrices and forces vectors. Results are compared with FEM for validation purposes.

In Chapter 7, the capabilities of developed formulation and numerical modelling are studied and discussed. Summarises the development of the formulation and its capability in terms of applications. Effects of numerical parameters on the formulation accuracy are concluded and several recommendations for future works are suggested.

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