IMPACT OF FRACTIONAL DERIVATIVES ON UNSTEADY FLOWS OF BRINKMAN TYPE NANOFLUIDS, HYBRID NANOFLUIDS AND BLOOD

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DEDICATION

This thesis is wholeheartedly dedicated to my beloved Family and teachers for their love, endless support, and encouragement. Specially, to my mother, Bhabi, who has been a source of encouragement and inspiration to me throughout my life. She is the strongest woman I have ever known and the greatest influence in my life, who taught me to trust in Allah SWT and believe in hard work. Thank you very much for everything you have done from loving me unconditionally, raising me, and giving me the strength to reach the stars and chase my dreams.

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ABSTRACT

The boundary layer flows of nanofluids, hybrid nanofluids, and blood are usually studied in terms of classical partial differential equations instead of fractional partial differential equations to avoid complexities in the exact solutions. Fractional partial differential equations offer immense unconventional features to the research, making them potential mathematical tools for describing the complex behaviour of boundary layer flow. Therefore, the main objective of this thesis is to study unsteady convection flows of nanofluids, hybrid nanofluids, and magnetohydrodynamic blood based on the generalized fractional Brinkman type fluid model. The mixed convection boundary layer flows of water-based carbon nanotubes nanofluids and Ferro nanofluid with shape effect past a vertical plate are considered. The effects of thermal radiation, heat generation with ramped, and isothermal heating are studied. The natural convection boundary layer flows of water-based hybrid nanofluids in a channel and magnetohydrodynamic blood flow in a cylindrical tube are also examined. The dimensional boundary layer flow models are transformed into dimensional forms by using appropriate dimensionless variables. Then, the obtained dimensionless models are transformed into the fractional form by using Caputo and Caputo-Fabrizio fractional derivatives. The exact solutions are obtained by using the Laplace transform and joint Henkel and Laplace transform methods. The impacts of the fractional parameter, the volume concentration of nanoparticles, Brinkman type fluid parameter, magnetic parameter, thermal radiation, heat generation, and thermal Grashof number are studied graphically with physical interpretations. The empirical results reveal that for a shorter time, the temperature field, velocity field, blood velocity, and magnetic particles velocity are decreasing with increasing fractional parameters due to variations in temperature and velocity boundary layers. However, this trend revises for a longer time. Meanwhile, it is noticed that the temperature field is increasing with the increasing volume concentration of nanoparticles and hybrid nanoparticles due to the advanced thermal conductivity, but the velocity field behaves oppositely because of effective density. Besides this, the velocity field is decreasing with increasing Brinkman type fluid parameter due to resistive forces. Finally, in a limiting case, the general fractional solutions are reduced to the published classical solutions for the sake of correctness and validation.

ABSTRAK

Aliran lapisan sempadan bagi nanobendalir, nanobendalir hibrid, dan darah kebiasaannya dikaji dari segi persamaan terbitan separa klasik dan bukannya persamaan terbitan separa pecahan bagi mengelakkan kerumitan dalam penyelesaian tepat. Persamaan terbitan separa pecahan menawarkan ciri-ciri tidak konvensional yang besar terhadap penyelidikan, menjadikannya alat matematik yang berpotensi untuk menerangkan tingkah laku aliran lapisan sempadan yang kompleks. Oleh itu, objektif utama tesis ini adalah untuk mengkaji aliran perolakan tak mantap nanobendalir, nanobendalir hibrid, dan darah hidrodinamik magnet berdasarkan kepada model bendalir jenis Brinkman pecahan teritlak. Aliran lapisan sempadan perolakan campuran bagi nanobendalir karbon nanotiub dan nanobendalir Ferro berasaskan air dengan kesan bentuk mengalir melepasi plat menegak dipertimbangkan. Kesan sinaran terma, penjanaan haba dengan tanjakan, dan pemanasan isoterma dikaji. Aliran lapisan sempadan perolakan semula jadi nanobendalir hibrid berasaskan air dalam saluran dan aliran darah hidrodinamik magnet mengalir dalam tiub silinder juga diperiksa. Model aliran lapisan sempadan berdimensi diubah ke dalam bentuk tanpa dimensi dengan menggunakan pembolehubah tanpa dimensi yang bersesuaian. Seterusnya, model tanpa dimensi yang diperoleh telah diubah menjadi bentuk pecahan dengan menggunakan terbitan pecahan Caputo dan Caputo-Fabrizio. Penyelesaian tepat diperoleh dengan menggunakan kaedah transformasi Laplace dan gabungan Henkel dan transformasi Laplace. Kesan parameter pecahan, kepekatan isipadu nanopartikel, parameter bendalir jenis Brinkman, parameter magnetik, sinaran terma, penjanaan haba, dan nombor Grashof terma dikaji secara grafik berserta dengan interpretasi fizikal. Hasil empirik menunjukkan bahawa untuk masa yang lebih singkat, medan suhu, medan halaju, halaju darah, dan halaju partikel magnetik semakin menurun dengan peningkatan parameter pecahan disebabkan oleh perubahan dalam suhu dan halaju lapisan sempadan. Walau bagaimanapun, trend ini berubah untuk masa yang lebih lama. Sementara itu, diperhatikan bahawa medan suhu meningkat dengan peningkatan kepekatan isipadu nanopartikel dan nanopartikel hibrid disebabkan oleh kekonduksian terma termaju, tetapi medan halaju bertindak sebaliknya kerana ketumpatan yang berkesan. Selain itu, medan halaju menurun dengan peningkatan parameter bendalir jenis Brinkman disebabkan oleh daya tahan. Akhirnya, dalam kes terhad, penyelesaian pecahan itlak diturunkan kepada penyelesaian klasik yang telah diterbitkan bagi tujuan ketepatan dan pengesahan.

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LIST OF ABBREVIATIONS

SABFD - Atangana Baleanu Fractional Derivative
Al_2O_3 - Alumina
Ag - Silver
BF - Biological Fluid
BFD - Bio-magnetic Fluid Dynamic
CNTs - Carbon Nanotubes
CFFD - Caputo Fabrizio Fractional Derivative
Cu - Copper
DRA - Duan-Rach Approach
Fe_3O_4 - Ferric Oxid
H_2O Water
MHD - Magneto-Hydro-Dynamic
MFD - Magneto-Fluid-Dynamic
MWCNTs Multi Wall Carbon Nanotubes
MoS_2 - Molybdenum Sulfide
OHAM - Optimal Homotopy Asymptotic Method
ODEs - Ordinary Differential Equations
PDEs - Partial Differential Equations
SWCNTs - Single Wall Carbon Nanotubes
TiO_2 - Titanium Oxide/ Titania

LIST OF SYMBOLS

$\underline{\mathbf{A}}_1$ - First Revilin-Ericksen tensor

$\mathbf{B}$ - Total magnetic field

$\mathbf{B}_0$ - Applied magnetic field

B_0 - Magnitude of applied magnetic field

$\mathbf{b}$ - Induced magnetic field

C_p - Specific heat at constant pressure /Heat capacitance

$\frac{D}{Dt}$ - Material time derivatives

$\mathcal{D}_t^{\alpha}(,.,)$ - Time fractional derivative of order α

$\mathbf{B}$ - Electric field

e - Internal energy per unit volume

$\mathbf{e_r}$ - Cylindrical unit vector in $r-$direction

$\mathbf{e_\theta}$ - Cylindrical unit vector in $\theta-$direction

$\mathbf{e}_z$ - Cylindrical unit vector in $z-$direction

$erfc$ - Complementary error function

$\mathbf{F}$ - Force

$\mathbf{g}$ - Gravitational acceleration

Gr - Thermal Grashof number

$H(t-1)$ - Heaviside unit step function

$\underline{\mathbf{I}}$ - Identity tensor

$\mathbf{I}_0$ - Interaction force of porous medium

$\mathbf{i}$ - Cartesian unit vector in $x-$direction

$\mathbf{J}$ - Current density

$\mathbf{j}$ - Cartesian unit vector in $y-$direction

$\mathbf{J}\times\mathbf{B}$ - Lorentz force

K - Stocks’ constant

k - Thermal conductivity

k_1 - Absorption coefficient

$\mathbf{k}$ - Cartesian unit vector in $z-$direction

p - Pressure

p_h - Hydrostatic pressure

p_d - Dynamic Pressure

P_c - Particle mass

P_c - Particles concentration

Pr - Prandtl number

q - The Laplace transform parameter

$\mathbf{q_r}$ - Radiative heat flux

q_r - Magnitude of radiative heat flux

$\mathbf{q}''$ - Heat conduction per unit area

q'' - Magnitude of heat conduction per unit area

$\mathbf{r}$ - Radial vector

r_0 - Radius

$\underline{\mathbf{T}}$ - Cauchy stress tensor

T - Temperature

t - Time

t_0 - Characteristic time

u - Velocity in $x-$direction

U_0 - Reference velocity

$\mathbf{V}$ - Velocity vector

V - Magnitude of velocity

$\mathcal{V}$ - Fixed control volume

Greek Letters

α - Fractional parameter

α_d - Positive coefficient of drag force

β_T - Volumetric coefficient of thermal expansion

β_b - Brinkman type fluid parameter

ρ - Density

ρ_d - Density of particles

μ - Dynamic viscosity

μ_m - Magnetic permeability

σ - Electric conductivity

σ_1 - Stefan-Boltzmann constant

ϕ - Volume concentration of nanoparticles

θ - Dimensionless temperature

λ_0, λ_1 - Amplitude of pressure gradient

τ - Convolution product variable

τ_v - Relaxation time for particle change

υ - Kinematic viscosity

ω - Frequency of oscillation

ωt - Phase angle

Subscripts

∞ - Ambient temperature condition

w - Wall temperature condition

nf - Nanofluid

hnf - Hybrid nanofluid

f - Base fluid

s - Solid nanoparticles

ch - Channel

cyl - Cylinder

Superscript

T - Transpose operation

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introductions

This chapter is intended to provide the research background of nanofluids, hybrid nanofluids, and blood based on fractional derivatives in Section 1.2. Problem statement and research objectives are specified in Sections 1.3 and 1.4 correspondingly. Scope of the study is given in Section 1.5 followed by significance of the study in Section 1.6. Finally, in Section 1.7, thesis organization is discussed.

1.2 Research Background

On 30th September 1695, L'Hopital had written a letter to Leibniz. He asked about a particular notation that was used by Leibniz for n^{th} order derivative $\left(D^n / Dx^n f\left(x\right)\right)$ that what will be the answer if $n = 1/2?$. Leibniz responded that it is a paradox from which one day some useful consequences will be drawn (Leibniz, 1849). The communication between Leibniz and L'Hopital had given birth to fractional calculus. Fractional calculus is a branch of mathematics that generalizes the idea of conventional integral and derivatives of integer order to non-integer order integral and derivatives. For the last five decades, after the work of Caputo (Caputo, 1967), this field attained a great consideration of researchers due to its applications in real world problems by utilizing the usual initial condition in the application of Laplace transform in contrast to the unusual initial condition in case of Riemann–Liouville fractional derivative operator. Caputo was the first researcher, who understood the application of the Laplace transform to form a fractional derivative operator from the convolution product: the convolution of classical derivative of the function with power Law kernel which is given by

$$\mathcal{D}_t^{\alpha} f(y,t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{1}{t^{\alpha}} * \dfrac{\partial f(y,t)}{\partial t}; 0 < \alpha < 1 \\ \dfrac{df(y,t)}{dt};\ \alpha = 1, \end{cases} \tag{1.1}$$

or

$$\mathcal{D}_t^{\alpha} f(y,t) == \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \displaystyle\int_0^t \dfrac{1}{(t-\tau)^{\alpha}} \dfrac{\partial f(y,\tau)}{\partial \tau} d\tau; 0 < \alpha < 1 \\ \dfrac{\partial f(y,t)}{\partial t};\ \alpha = 1 \end{cases} \tag{1.2}$$

where $\Gamma(.)$ is the gamma function, * represents the convolution product, $\frac{1}{\Gamma(1-\alpha)} \frac{1}{t^{\alpha}}$ is the non-local singular power-law kernel, and $\partial f(y,t) / \partial t$ is the classical derivative of the function $f(y,t)$. The Laplace transform of Eq. (1.2) is given by

$$\mathcal{L}\left\{\mathcal{D}_t^{\alpha} f(y,t)\right\}(q) = q^{\alpha} \bar{f}(y,q) - f(y,0). \tag{1.3}$$

where $\mathcal{D}_t^{\alpha}(.)$ is the Caputo fractional derivative, α is the fractional order, q is the Laplace transform variable, $\bar{f}(y,q)$ is the Laplace transform of $f(y,t)$ and $f(y,0)$ is the usual initial condition. It can be clearly seen from Eq. (1.3) that the singularity in the power law kernel vanish in the application of Laplace transform. With this great first step, after the Riemann–Liouville fractional derivative given by

$$\mathcal{D}_t^{\alpha} f(y,t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dt} \displaystyle\int_0^t \dfrac{1}{(t-\tau)^{\alpha}} f(y,\tau) d\tau; 0 < \alpha < 1, \\ \dfrac{df(y,t)}{dt};\ \alpha = 1, \end{cases} \tag{1.4}$$

with the following Laplace transform

$$\mathcal{L}\left\{\mathcal{D}_t^{\alpha} f\left(y,t\right)\right\}\left(q\right)=q^{\alpha}\,\overline{f}\left(y,q\right)-\mathcal{D}_t^{\alpha-1} f\left(y,0\right), \tag{1.5}$$

where $\mathcal{D}_t^{\alpha-1} f\left(y,0\right)$ is the unusual initial condition. The problem of this unusual initial condition without physical meaning and tough to compute was fixed. In additions to this, the derivative of a constant is not zero in the case of Riemann–Liouville fractional derivative operator see for example

$$\mathcal{D}_t^{\alpha}\left(1\right)=\frac{t^{-\alpha}}{\Gamma\left(1-\alpha\right)};\alpha\geq 0,t\geq 0\,. \tag{1.6}$$

In the Caputo fractional derivative, the drawbacks of Riemann–Liouville fractional derivative were fixed and was successfully applied in many fields of science and technology which include biometric foods, extrusion of polymer fluid, colloidal solutions, cooling of metallic plates, exotic lubricants, glass fiber production, and glass blowing (Haque *et al.*, 2018).

The significance and efficiency of fractional derivatives to engineering applications are experimentally proven by many investigators. It was experimentally verified by Song and Jiang (1998) that the modified fractional Jeffery model is suitable to illustrate the behavior of Sesbania gel and xanthan gum. Jiang and Qi (2012) experimentally indicated that heat transfer in biological tissues can be consistently analyzed by using fractional wave models. Meral *et al.* (2010) experimentally confirmed that the fractional Voigt model is efficient in the simulation of the wave response of soft tissue such as phantoms. Chen *et al.* (2013) suggested that the data obtained from fractional derivative models with variable order showed a considerably improved contract with experimental data compare to the traditional model.

After the successful application of fractional derivatives in real-world problems, the mathematicians also tried to contribute further to this field. In 2015, Caputo and Fabrizio realized that the existing fractional operators have some limitations which lead to some misleading results particularly in the modeling of real-world problems. Some senior researchers claimed that the misleading results are due to the singularity in the power law kernel. This argument was very valid. so, Caputo-

Fabrizio fractional derivative (CFFD) operator was introduced which is based on the non-singular exponential kernel given by (Caputo and Fabrizio, 2015)

$$\mathcal{D}_t^{\alpha} f\left(y,t\right)=\frac{N\left(\alpha\right)}{1-\alpha}\exp\left(-\frac{\alpha t}{1-\alpha}\right)*\frac{\partial f\left(y,t\right)}{\partial t};0<\alpha<1, \tag{1.7}$$

or

$$\mathcal{D}_t^{\alpha} f\left(y,t\right)=\frac{N\left(\alpha\right)}{1-\alpha}\int_0^t\exp\left(-\frac{\alpha\left(t-\tau\right)}{1-\alpha}\right)\frac{\partial f\left(y,\tau\right)}{\partial \tau}d\tau;0<\alpha<1, \tag{1.8}$$

where $N\left(\alpha\right)$ is the normalization function and $N\left(\alpha\right)/1-\alpha\exp\left(-\alpha t/1-\alpha\right)$ is the non-singular exponential kernel. It is worth mentioning here that, integer order calculus has a precise physical explanation and is used in the description of numerous ideas of classical physics and applied mathematics. For example, the velocity of an object is the first derivative of functions, the second derivative of a function corresponds to the acceleration of the object, and so on. Fractional calculus is the generalization of classical calculus and is anticipated to have a wider meaning. Unfortunately, until now, there is no acceptable interpretation of fractional integrals and derivatives in the literature (Dalir and Bashour, 2010).

In the past few decades, fractional derivatives put forward immense novel features to the research community thereby extensively utilized in numerous applications of science and engineering. In recent decades, fractional derivatives became potential mathematical tools to describe real word problems in science and engineering. Fractional derivatives, particularly the models that involve fractional partial differential equations (PDEs), captured a rising concern in the various field which includes heat transfer, biology, physics, chemistry, quantum mechanics, diffusive transport, probability, electrical networks and electromagnetic theory, viscoelastic materials, rheology and fluid flow problems (Dalir and Bashour, 2010). In this thesis, the impact of fractional derivatives on unsteady fluid flows of Brinkman type nanofluids, hybrid nanofluids and blood have been analyzed. It is worth mentioning here that fluids are isotropic substances that deform continuously under the action of applied shear stress and is a substance that mainly included liquids and gases. Fluids can transfer matter, energy, and force from one place to another place, so, the study of fluid flow merge with other disciplines of science and technology. Fluid dynamics always attracted the interest of researchers right from its development in the sciences. From the past few centuries, fluid dynamics assisted in various industrial and technological applications as a vital tool. Many engineers, physicists, and mathematicians have been fascinated by the reputation and charm of fluid dynamics. Fluid dynamics has been blended from a highly developed research area to an exceedingly applicable field of broader scope.

Mainly, fluids are categorized into two major classes; Newtonian and non-Newtonian fluids distinguished based on their viscosity trend. The class of fluids that follow Newton's Law of viscosity is referred to as Newtonian fluid. However, non-Newtonian fluids do not obey this Law. Among the class of Newtonian fluid, the one which is rarely investigated is the Brinkman type fluid. Brinkman type fluid is a viscous fluid introduced by Brinkman in his pioneer work while studying the fluid flow due to the exertion of viscous force, on dense swarm particles surface. Using Stokes' formula, it was noticed that the viscous force acting on dense swarm particles is significantly greater than that of on a remote particle (Brinkman, 1949a). Additionally, a spherical shaped solid particle was fixed inside the swarm particles and the impact of surrounded particles was considered as a porous medium. Based on this set up a formula for permeability as a function of porosity was derived. The mathematical formula was validated satisfactorily with experimental results. It was noticed that the formula was applicable only for a medium having porosity greater than 0.4. To fix this issue Brinkman (1949b), further polished his previous work and developed a formula with experimental validation for all range of values. Consequently, Brinkman established a governing equation for the flow of viscous fluid through a highly porous medium. The Brinkman equation is considered the extension of Darcy's equation which is typically referred to as Darcy's Law (Darcy, 1856). Rajagopal (2007), soothing the supposition made by (Brinkman, 1949a; Brinkman, 1949b), and developed a more general governing equation for the viscous fluid flowing through highly porous media. Fetecau *et al.* (2011) obtained the first exact solutions via Fourier sine transform for Stokes' incompressible Brinkman type fluid model. Ali *et al.* (2012), generated exact solutions for the velocity profile of Brinkman type fluid using the Laplace transform method.

Note that, traditional/regular fluids (pure fluids such as water, alcohol glycerine, polyester suspensions) have poor thermophysical properties and cannot transfer the required amount of heat in various engineering and industrial sectors. The heat transfer from one place to another place is referred to as heat convection which can be free, force, or mixed convection. In heat transport systems, convection heat transfer became a challenging task for engineers and industrialists. So, there is an urgent need to fix this problem. Heat transfer in convection flows of fluids can be enhanced by enhancing the thermal conductivity of the flowing fluids. In the past few decades, many procedures have been proposed by researchers to enhance the performance of convection heat transfer but failed due to scientific drawbacks. Investigators also tried to enhance the thermal conductivity of the fluid by dispersing micro-sized particles in the base fluid as the thermal conductivity of the solids is higher than liquids. Various theoretical and experimental studies were carried out on the suspension containing micro-sized solid particles. This research was initially conducted by Maxwell about 10 decades ago (Maxwell, 1881). However, this research was not commercialized because the suspension with large sized solid particles can cause; (i) increasing drop in pressure, (ii) erosion of pipelines, (iii) abrasion of surfaces, (iv) clogging micro-channels of devices, and (v) faster settling time due to higher density. Furthermore, the suspension causes supplementary resistance and possible erosion (Das *et al.*, 2007).

The new technique in nanotechnology affords a convenient process to form crystals from 1-100 nm on average. The suspensions of regular fluids and nano-meters size particles of metals, oxides, silica, carbide, and carbon nanotubes (CNT’s) are referred to as nanofluids. This term was firstly proposed by Choi and Eastman (Choi and Eastman, 1995). In nanofluids, various factors such as volume fractions, size, shape, clustering, PH value, and effect of particles materials of nanoparticles are responsible for improved thermal conductivity and viscosity. For a very little volume fraction of nanoparticles, a significant increase in the thermal conductivity occurs referred to as synergistic effect of nanoparticles. Nanofluids have remarkable thermophysical properties as compared to regular fluid and suspension containing micro-sized particles. The successful use of nanofluid supports the trend toward component minimization to implement the smaller and lighter design of the heat exchange system.

Nanofluids have been widely studied in the literature due to their industrial importance and applications included electronics, energy, heat exchanger, solar water heater, advancement in diesel engines, environment, biomedicine, and healthcare (Wang and Mujumdar, 2007). The contributions of nanofluids in science and technology exponentially increased and reached to next level by introducing hybrid nanofluids: the suspensions of two or more dissimilar nanoparticles in a single based fluid. The main motive of introducing hybrid nanofluids is to further improve the thermophysical properties of nanofluids. Hybrid nanofluids overcome the drawbacks of conventional nanofluids suspensions and connect the synergistic effect of nanoparticles. The newly branded hybrid nanofluids further advance the heat transfer capability of conventional/traditional fluids which leads to engineering and industrial applications with high performance and low cost (Bhattad *et al.*, 2018). Hybrid nanofluids are highly applicable in industrial and engineering processes with high efficiency. Hybrid nanofluids have wide range of applications in many fields which include transformer cooling, electric cooling, drug reduction, biomedical, generator cooling, refrigeration nuclear system cooling, and coolant in machining (Waini *et al.*, 2019c). It is important to highlight here that in many high voltage transformers, mineral oils were used as insulators and coolants. It was found that pure mineral oil cannot handle high voltage. The predicted 40-30-year life of transformers reduced to 18-17-years working life. Sumathi *et al.* (2019) experimentally proven that the working life of transfer was increased by 7.7 times than transformer oil by using oil base Al_2O_3-TiO_2-MoS_2 hybrid nanofluid and was increased by 3.7-time than magnetic nanofluid.

Besides this, the study of biological fluids (BF) under the influence of a magnetic field is referred to as bio-magnetic fluid dynamics (BFD). This area in fluid dynamics attracted numerous researcher (Carlton *et al.*, 2001; Haik *et al.*, 1999; Hussain *et al.*, 2018; Liu *et al.*, 2001; Voltairas *et al.*, 2002; Zeeshan *et al.*, 2018) due to the extensive applications proposed by medical science and bio-engineering which include the development of magnetic tracers, targeted transport of drugs using magnetic particles as drug carriers, development of magnetic devices for cell separation, provocation of occlusion of the feeding vessels of cancer tumors, cancer tumor treatment causing magnetic hyperthermia and reduction of bleeding during surgeries (Tzirtzilakis, 2005). The BF exists in all living organisms and their flow is significantly affected by the incidence of the magnetic field. Among various biological fluids, the one electrically conducting fluid is blood. The human blood is composed of plasma and red blood cells (RBC's) (Boyd, 1961). The RBCs consist of a high concentration of hemoglobin, the oxide of iron while the magnetic characteristics of blood are due to the state of oxygenation (Sharma *et al.*, 2015).

To study bio-magnetic fluids particularly blood, the mathematical model of BF developed by Haik *et al.* (1999) is frequently used in the literature. They suggested that Ferro-hydrodynamics (FHD) is like BFD in which the flow of electrically conducting fluid is influenced by the magnetization of the fluid in the presence of magnetic field with no induce current due to the small Reynold number. Hence, in the mathematical model of BF, the magnetization of the fluid is assumed in the formulation that corresponds to magnetohydrodynamic (MHD) or magneto-fluid-dynamics (MFD) (Haik *et al.*, 2002). The MHD or MFD focused on the interface among electrically conducting fluids and externally applied magnetic field. The fluid under the influence of the applied magnetic field experience electromagnetic so-called Lorentz forces which are like drag forces that change the flow behavior.

In various engineering and industrial devices such as accelerators, pumps, microfluidics, and mixer nuclear reactor, MHD is applied to control the boundary layer (Al-Habahbeh *et al.*, 2016). The MHD effect help in understanding and forecast and significant improvement in the fusion blanket system (Smolentsev *et al.*, 2010). In 1942, the idea of MHD was presented by Alfven for the first time and received Nobel Prize in 1972. MHD has plenty of applications in various field of science and technology such as electrochemical processes, chemical catalytic reactors, grain storage, beds of fossil fuels such as oil shale and coal, the extraction of geothermal energy, salt leaching in soils, solar power collectors, underground disposal of nuclear waste, the spreading of chemical pollutants in saturated soil, high-performance insulation for buildings, insulation of nuclear reactors, packed sphere beds, the migration of moisture in fibrous insulation and heat exchange between soil and atmosphere and sensible heat storage beds (van der Holst and Keppens, 2007).

1.3 Problem Statement

In the literature, the analytical studies based on the Brinkman type fluid for the boundary layer flows of nanofluids and hybrid nanofluids and blood are rarely reported due to the complexity in finding the exact solutions. These difficulties usually occur in the exact solutions treatment for the flow of nanofluids over an infinite plate with ramped heating, the flow of hybrid nanofluid in a channel, and MHD blood flow in a cylinder. Indeed, some studies are published in this direction but only restricted to integer-order derivatives. To fill this research gap, the following problems are addressed and solved for exact solutions using integral transforms such as the Laplace transform and the joint Laplace and Hankel transforms:

1. Convection MHD flow of water based CNT’S Brinkman type nanofluid with ramped heating.
2. Radiative convection MHD flow of nanofluid with heat generation and nanoparticles shape effect
3. Natural convection channel flow of hybrid nanofluid in the presence of heat generation.
4. Natural convection MHD channel flow of hybrid nanofluid in the presence of heat generation
5. MHD flow of fractional brinkman type blood with magnetic particles in a cylindrical tube

In this thesis, the Problems mentions in 1-5 are solved for exact solutions using integral transforms such as the Laplace transform and the joint Laplace and Hankel transforms. Additionally, this thesis is organized to answer the following questions:

1. How are the fractional Brinkman type fluid models formulated in different flow regimes such as flows past an infinite plate, flows in a channel, and blood flow in a cylinder?
2. How the fractional nanofluids and hybrid nanofluids behave in convection flow problems?
3. How do the fractional derivatives affect the velocity and temperature fields?
4. How do numerous effects such as MHD, thermal radiation, heat generation, ramped heating, isothermal heating, different shapes of nanoparticles, volume concentrations of nanoparticles and hybrid nanoparticles influence the velocity and temperature fields?
5. How do the exact solutions can be obtained from the fractional models of the flows of nanofluids past an infinite plate with ramped heating, the flows of hybrid nanofluids in a channel, and blood flow in a cylinder?

1.4 Research Objectives

This study examines the mixed convection flows of nanofluids past an infinite plate with ramped heating, natural convection flows of hybrid nanofluids in a channel and MHD blood flow in a cylinder. The exact solutions are obtained via the Laplace transform and joint Hankel and Laplace transform methods. The obtained solutions are computed and plotted. In a limiting sense, the obtained solutions are reduced to classical form and validated with published work. The following are the main objectives of this research.

1. To formulate the problems of convection flows of nanofluids, hybrid nanofluids, and blood in three different geometries such as the flows past an infinite plate, flows in a channel, and blood flow in a cylinder.
2. To generalize the classical model using Caputo-Fabrizio, Caputo fractional derivatives.
3. To obtain and validate in limiting cases the exact solutions for the proposed problems.
4. To investigate the influence of various pertinent flow parameters on velocity, temperature, and magnetic blood particles velocity fields physically.

1.5 Scope of the Study

This research focused on the unsteady, incompressible boundary layer flows of Brinkman type nanofluids, hybrid nanofluids, and blood. This research is based on the following assumption and limitations.

1. Unidirectional and one-dimensional boundary layer flows are considered in Cartesian and cylindrical coordinate systems.
2. The incompressible and laminar flows are considered.
3. In the case of MHD flows, the electrically conducting fluids are considered and the induced magnetic field is neglected.
4. The Brinkman type fluid model together is fractionalized by using the Caputo-Fabrizio fractional derivative in the first four problems while the last problem is fractionalized via the Caputo fractional derivative.
5. This thesis investigates nanofluids and hybrid nanofluids theoretically and mathematically. In this thesis, nanofluids or hybrid nanofluids are not physically prepared/characterized.

6. In the first four problems, the energy equation is partially coupled with the momentum equation by using the Oberbeck-Boussinesq approximation.
7. The exact solutions are computed and plotted by using the MATHCAD software.

1.6 Significance of the Study

The significance of the study is listed as follows.

1. The results obtained from this research will help to enhance the knowledge of boundary layer flows in nanofluids, hybrid nanofluids, and blood in different flow regimes.
2. The results obtained from this study will help in understanding trend and features of various boundary layer flow of nanofluids, hybrid nanofluids, and blood.
3. The results obtained from this study can be used as a base for complex non-linear flow problems recurrently taking place in engineering and applied sciences.
4. The concept of generalization/fractionalization of classical models with an integer order can be further advanced for highly non-linear non-Newtonian nanofluids and hybrid nanofluids to achieve additionally accurate and realistic results.

1.7 Thesis Organization

This thesis comprises eight Chapters. Chapter 1 discussed a detailed research background go after problem statement, research objective, scope of the study, and significance of the study. Chapter 2 provides a detailed literature survey relevant to the problems established in research objectives and highlighted in the problem statement. Chapter 3 addressed the problem of convection heat transfer of MHD flow of CNTs nanofluids past an oscillating infinite vertical plate. In this Chapter, the detailed derivation of the continuity equation, momentum equation, and energy equation are provided. The constitutive equation of Brinkman type fluid in conjunction with certain appropriate assumptions, the governing equations of the problem are developed in terms of partial differential equations (PDEs). The velocity field is subjected to oscillating boundary conditions whereas, the ramped heating conditions are considered for the temperature field. To eliminate units and reduce the number of variables, some suitable non-dimensional and non-similarity variables are introduced into the governing equations and initial and boundary conditions to make the system dimensionless. The dimensionless model is then transformed into time-fractional form by using the time-fractional Caputo-Fabrizio fractional derivative. The time-fractional model is solved for exact solutions by using the Laplace transform method. The obtained solutions for velocity and temperature fields are computed and presented in numerous graphs to study the effect of various pertinent flow parameters. Besides this, the obtained solutions are reduced to classical form and validated with published work.

Chapter 4 examined the influence of thermal radiation, heat generation, and nanoparticles shape effect in MHD flow of water-based Ferro nanofluid (Fe_3O_4-H_2O) past an oscillating infinite vertical plate. In a similar procedure as in Chapter 3, the exact solutions are developed for the Caputo-Fabrizio time-fractional Fe_3O_4-H_2O nanofluids by using the Laplace transform method and plotted in various graphs with a physical explanation. The idea of nanofluids is further advanced in Chapter 5 and Chapter 6. More exactly, Chapter 5 discussed the flow of Cu-Al_2O_3-H_2O hybrid nanofluid in and channel subject to homogenous boundary conditions and isothermal heating. This idea is further extended in Chapter 6 by considering the MHD effect in the flow of Ag-TiO_2-H_2O hybrid nanofluid. Both the problems presented in Chapter 5 and Chapter 6 are solved for exact solutions by following the same procedure as in Chapter 3 and Chapter 4.

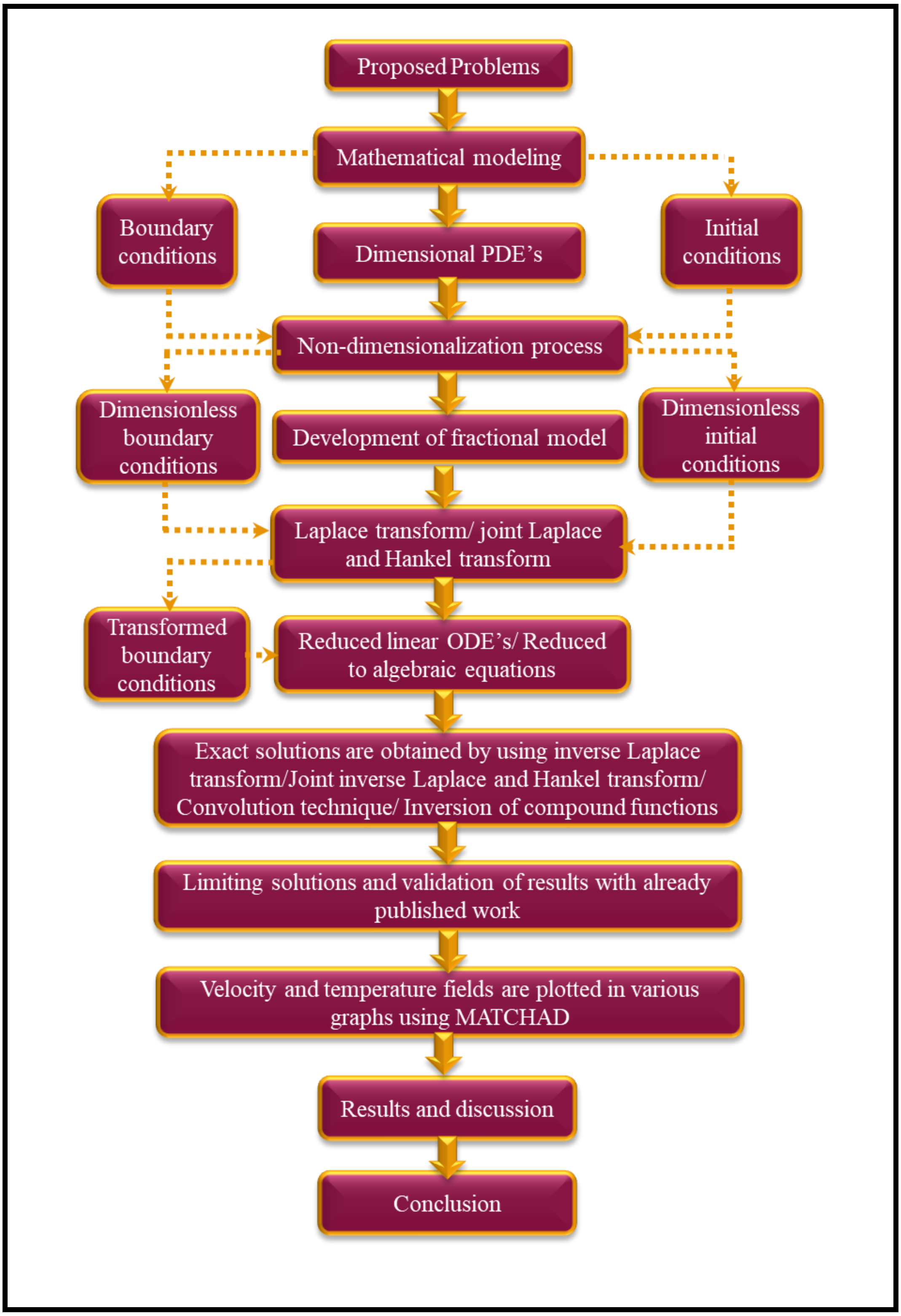

Figure 1.1 Operational framework

Chapter 7 deals with the two-phase MHD blood flow based on the Caputo time fractional Brinkman type fluid model in a horizontal cylinder. In this Chapter, the detailed derivation of the continuity equation, momentum equation, and equation of motion of the magnetic particle in the blood is presented in the cylindrical coordinates system. The joint Hankel and Laplace transform method is employed to find the exact solutions for blood and magnetic particle velocities. Classical solutions for blood and magnetic particle velocities are recovered. For the sake of correctness, the obtained solutions are validated with already published work. Finally, in Chapter 8 the whole thesis is summarized, and an adequate conclusion is drawn. Future recommendations and suggestions are also included in this chapter. The operational framework and solutions methodology is given in Figure 1.1.

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