CHARACTERIZATION OF THE SPHEROIDS FROM THE FIRST ORDER POLARZATION TENSOR

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DEDICATION

This thesis is dedicated to my parents.

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ABSTRACT

Polarization tensor is an essential concept used in science and engineering. It has appeared in many electric and electromagnetic applications such as electrical imaging in medical, material science and metal detection. It has also been adapted biologically to study electrosensing in fish. In these applications, the polarization tensors are used to describe conducting objects such as tumors in human bodies and metal components in landmines, presented in electric or electromagnetic fields. Here, polarization tensors can capture significant information about the objects such as their material, shape and orientation. Inspired by many interesting topics on polarization tensor, this research is conducted with the main purpose to investigate specifically the first order polarization tensor when the conducting object is a spheroid (prolate and oblate). In this study, a new formula of the first order polarization tensor for ellipsoid, consisting of depolarization factors is used. It is proven that the properties of depolarization factors are as follows. One, the value of each depolarization factor for ellipsoid is between 0 and 1. Two, the depolarization factors can be used to identify the semi axes of a spheroid and that the semi axes of the spheroid can also be used to describe the depolarization factors. Next, this study also describes the spheroid based on the first order polarization tensor by considering the properties of the depolarization factors above. Specifically, the spheroid can be classified as a prolate or oblate spheroid based on the first order polarization tensor. In addition, the semi axes of either prolate or oblate spheroid can be used to describe the related first order polarization tensor. Furthermore, the volume and the depolarization factors for the spheroid can be determined analytically based on this first order polarization tensor. The conductivity of the prolate or oblate spheroid also can be classified based on the components of such first order polarization tensor. This study also proposes a combined approach of an analytical and numerical to determine the semi axes of a spheroid based on a given first order polarization tensor at some specified conductivity. Numerical examples are provided to present the values of all semi axes of the spheroid. Finally, in order to verify the results, the values of semi axes obtained are used to compute the first order polarization tensor and then compared with the given first order polarization tensor. The comparison has shown that the computed first order polarization tensor is almost similar to the given first order polarization tensor.

ABSTRAK

Tensor pengutuban merupakan satu konsep penting yang digunakan dalam bidang sains dan kejuruteraan. Ia telah wujud dalam pelbagai aplikasi elektrik dan elektomagnetik seperti pengimejan elektrik dalam perubatan, sains bahan dan pengesanan logam. Ia juga telah diadaptasikan dalam biologi untuk mengkaji deria elektrik dalam ikan. Dalam aplikasi-aplikasi ini, tensor pengutuban telah digunakan untuk mencirikan objek yang mengkonduksikan elektrik seperti ketumbuhan dalam badan manusia dan komponen logam dalam periuk api, yang berada dalam kawasan yang mengandungi medan elektrik atau medan elektromagnetik. Di sini, tensor pengutuban boleh menjelaskan maklumat yang penting mengenai objek tersebut seperti bahan buatan, bentuk dan juga arah objek itu. Diinspirasikan oleh pelbagai topik yang menarik mengenai tensor pengutuban, kajian ini dijalankan dengan tujuan utama untuk mengkaji khususnya tensor pengutuban darjah satu apabila objek yang mengkonduksikan elektrik tersebut ialah satu sferoid (lonjong dan buntal). Dalam kajian ini, formula baru bagi tensor pengutuban darjah satu untuk elipsoid, yang terdiri daripada faktor penyahkutuban telah digunakan. Dapat dibuktikan bahawa ciri-ciri faktor penyahkutuban adalah seperti berikut. Pertama, nilai bagi setiap faktor penyahkutuban untuk elipsoid adalah antara 0 dan 1. Kedua, faktor penyahkutuban boleh digunakan untuk mengenalpasti semi paksi sesebuah sferoid dan seterusnya semi paksi bagi sferoid juga boleh digunakan untuk mencirikan faktor penyahkutuban. Seterusnya, kajian ini juga menjelaskan sesebuah spheroid berdasarkan tensor pengutuban darjah satu dengan mempertimbangkan ciri-ciri faktor penyahkutuban di atas. Khususnya, sesebuah sferoid boleh diklasifikasikan sebagai sferoid lonjong atau sferoid buntal berdasarkan tensor pengutuban darjah satu. Sebagai tambahan, semi paksi bagi sferoid lonjong atau sferoid buntal boleh digunakan untuk menggambarkan tensor pengutuban darjah satu berkenaan. Tambahan lagi, isipadu dan faktor penyahkutuban sesebuah sferoid boleh ditentukan secara analitikal berdasarkan tensor pengutuban darjah satu ini. Kekonduksian bagi sferoid lonjong atau sferoid buntal boleh diklasifikasikan juga berdasarkan komponen-komponen tensor pengutuban darjah satu ini. Kajian ini juga telah mencadangkan satu gabungan pendekatan analitik dan berangka untuk menentukan semi paksi sesebuah sferoid berdasarkan kepada satu tensor pengutuban darjah satu yang diberi pada beberapa kekonduksian tertentu. Beberapa contoh berangka telah disediakan untuk menunjukkan nilai-nilai semi paksi sferoid yang telah dikira. Akhirnya, untuk mengesahkan keputusan tersebut, nilai-nilai semi paksi yang diperoleh telah digunakan untuk mengira tensor pengutuban darjah satu dan kemudiannya, dibandingkan dengan tensor pengutuban darjah satu yang diberi. Perbandingan tersebut telah menunjukkan bahawa tensor pengutuban darjah satu yang diperoleh hampir serupa dengan tensor pengutuban darjah satu yang diberi.

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LIST OF ABBREVIATIONS

PT	-	Polarization tensor
WTMD	-	Walk Through Metal Detector
BEM	-	Boundary Element Method

LIST OF SYMBOLS

$>$	-	Greater than
$<$	-	Less than
$\geq$	-	Greater than or equal to
$\leq$	-	Less than or equal to
$=$	-	Equal
$\neq$	-	Not equal to
$\Leftrightarrow$	-	If and only if
$\in$	-	Element of
$\mathbb{R}$	-	Real number
∞	-	Infinity
ψ	-	Psi
φ	-	Varphi

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CHAPTER 1

INTRODUCTION

1.1 Introduction

Polarization tensor (PT) is a concept used in science and engineering, where, it has been applied in many areas especially in the application of electrics [1–3] and electromagnetics [4–8]. Previously, PT is studied to improve electrical imaging for examples in biomedical engineering [1]. On top of that, PT is also practiced by engineers for security scanning [9–12] and landmine clearance [13–15]. Recently, PT is explored further during the investigation of electrosensing fish in identifying objects [2, 3, 16–19].

Generally, PT is used to describe the perturbation in electric or electromagnetic fields due to the presence of conducting objects in for examples two or three dimensional space. According to Ammari and Kang [1], the perturbation caused by the conducting object can be represented by an asymptotic formulas, where, the dominant term of the formula can be expressed in terms of the PT called as the generalized polarization tensor (GPT). By using the explicit formula given in [1], the PT representing the perturbation can be determined if the geometry and the conductivity of the conducting objects are known. Hence, the PT is also referred as the PT for that object.

On the other hand, the depolarization factors are actually quite related to PT. Depolarization factors are commonly used by physicist while studying potential and magnetic problems. The study of depolarization factors for ellipsoid has been discussed in [20–22]. In this studies, some explicit formulas for the depolarization

factors for a few types of ellipsoids were presented. By using the general formula of the depolarization factors given by Stoner [21] and Milton [22], the formula of the first order PT was modified as given in [23].

1.2 Research Background

Investigating the properties of the PT is important in order to describe the object based on the PT itself. For this reason, there are many researches conducted to determine and study the properties of the PT such as in [1, 7, 24–29]. It is necessary to understand the PT before computing it so that it can be easily used to identify and characterize the object. However, there are some properties that have not been discovered yet.

This research focuses on investigating the simplest form of GPT called as the first order PT, when the electrical field is specifically perturbed by a conducting spheroid (see Figure 1.1 and Figure 1.2). The spheroids are highlighted in this research as previous study [25, 30] have shown that the first order PT for many objects can actually be related to the first order PT for spheroid. The formula of the first order PT where the depolarization factors are considered, is further investigated throughout this research. Some alternative ways in classifying the spheroid according to its first order PT are explained.

1.3 Problem Statement

According to Ammari and Kang [1], polarization tensor (PT) contains some significant information about material and geometry of an object. However, there are still lack of research on how to obtain the information as many previous research conducted focus on deriving the formula and finding the PT for a specified object.

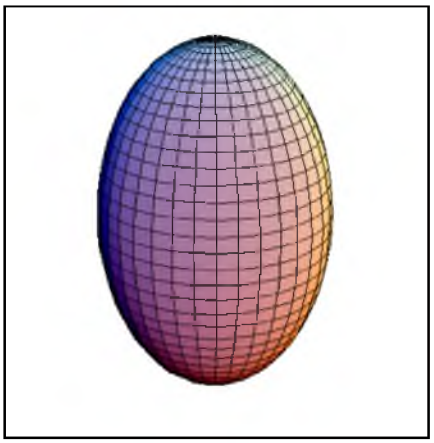

Figure 1.1 An example of a prolate spheroid

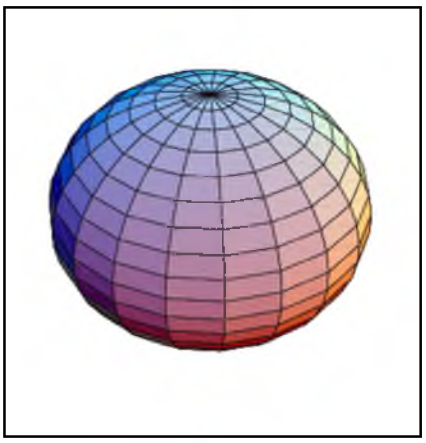

Figure 1.2 An example of an oblate spheroid

Hence, there are not much theories yet on describing the object based on its PT.

In order to describe the object, specifically a spheroid, the formula need to be understood first by investigating its properties. Since the formula of the first order PT consists of depolarization factors, it is also necessary to review and also explore the properties of depolarization factors as well. It is possible that some properties of the depolarization factors, related to this study, are not yet investigated.

1.4 Research Objectives

The objectives of this study are to:

i. determine some properties of the depolarization factors for spheroid.
ii. to classify whether a spheroid is a prolate or oblate spheroid based on the first order polarization tensor.
iii. determine the volume and depolarization factors of a spheroid based on the first order polarization tensor.

iv. characterize the conductivity of a prolate and oblate spheroid according to the first order polarization tensor.

v. determine the semi axes of a spheroid based on the first order polarization tensor.

1.5 Scope of the Research

This research focuses on the properties of the depolarization factors and the first order PT for spheroid. Spheroid is an ellipsoid that has two equal semi axes. In addition, there are two types of spheroid considered which are prolate spheroid (see Figure 1.1) and oblate spheroid (see Figure 1.2). Prolate spheroid is an ellipsoid with two equal semi axes and the two equal semi axes are shorter than the other semi axis. Meanwhile, an oblate spheroid is an ellipsoid with two equal semi axes where, the two equal semi axes are longer than the other semi axis.

This study focuses on spheroid that can be represented in the Cartesian coordinate system by $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1$, where the semi axes for prolate spheroid are either $a > b = c$, $b > a = c$ or $c > a = b$, while, for oblate spheroid, the semi axes are either $a < b = c$, $b < a = c$ or $c < a = b$.

1.6 Significance of the Research

This research is important as some useful properties of the depolarization factors and the first order PT for spheroid are revealed in order to use them for characterizing the spheroid. This might contribute to a new knowledge in the theoretical study on how to apply the first order PT. The study can also be the good references in the future for using the PT in any related problems or applications.

1.7 Research Methodology

In this research, some properties of the first order PT for spheroid are studied. In order to achieve that, the properties of the depolarization factors for ellipsoid are reviewed and further explored. The study on the properties of depolarization factors are important since the depolarization factors are implemented in the formula of the first order PT.

Hence, by using the properties of depolarization factors, the spheroid can be classified either as a prolate or an oblate spheroid according to the first order PT. Besides, it is shown that the depolarization factors and the volume of the spheroid can be determined based on the first order PT. Next, by considering the depolarization factors and the volume of the spheroid, the conductivity of each type of spheroids can be further described when the first order PT is given. Consequently, it is then possible to determine the semi axes of the spheroid from its first order PT.

The research framework of this study is illustrated in Figure 1.3.

1.8 Thesis Organization

This thesis is divided into eight chapters. In the first chapter, a brief introduction to the research is provided. The chapter consists of the research background, problem statement, research objectives, scope of the research, significance of the research and research methodology.

In Chapter 2, some basic knowledge related to this research are presented. The basic knowledge included in this chapter are the formula and some properties related to the depolarization factors. Besides, the formula of the first order PT and some of its properties as well as applications are reviewed.

Chapter 3 begins by exploring the properties of the depolarization factors for ellipsoid and spheroid. After that, the results obtained are used to classify the types of the spheroid based on the first order PT, which is discussed in Chapter 4. Moreover, Chapter 5 shows that the volume and depolarization factors can be determined according to the first order PT. Meanwhile, in Chapter 6, the conductivity of each type of spheroids is characterized based on the first order PT. In order to achieve that, the results from Chapter 3, Chapter 4 and Chapter 5 are applied.

On the other hand, Chapter 7 provides some numerical results for determining the size of the spheroid from a given first order PT. Considering the results from Chapter 3, Chapter 4, Chapter 5 and Chapter 6, it is possible to numerically calculate the semi axes of the spheroid and fit the first order PT by the spheroid.

In the last chapter, namely Chapter 8, the summary of the whole thesis is provided. Besides, some suggestions for further research in this topic are given. Figure 1.4 summarized the organization of this thesis.

1.9 Conclusion

In this chapter, the overview of the whole study is introduced. In addition, the problem statement, research objective and scope of the research have been stated. The significance of research and organization of thesis are also given in this chapter.

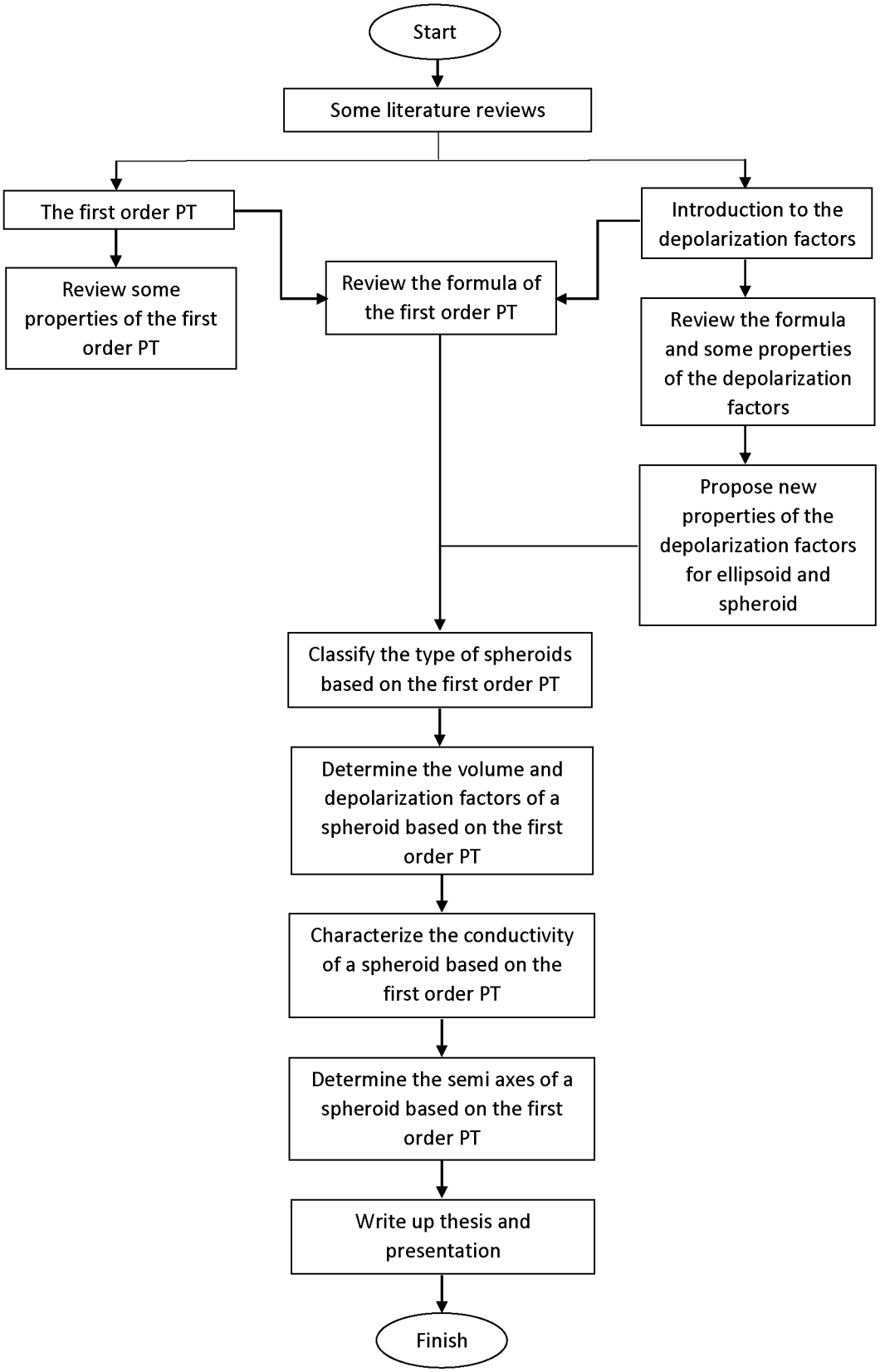

Figure 1.3 Flow chart for conducting the research

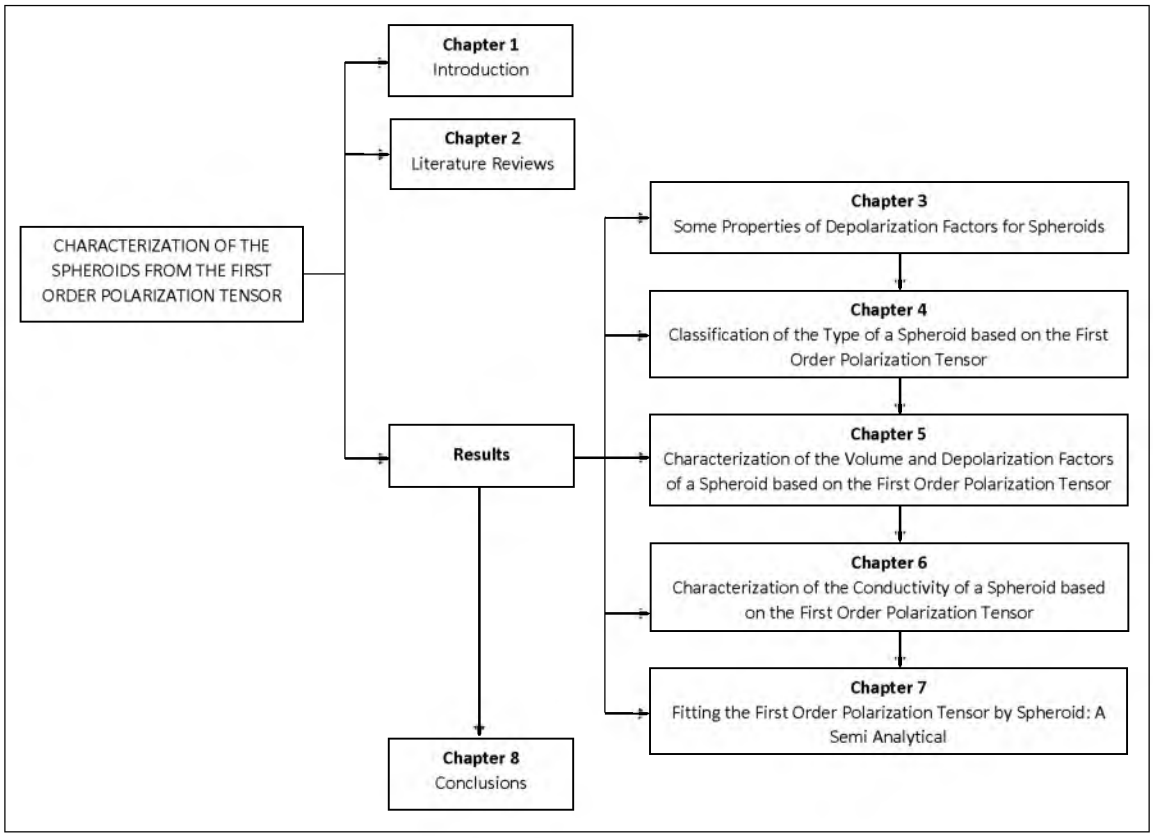

Figure 1.4 Thesis Organization

Figure 1.4 Thesis Outline.

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