HYBRID RUNGE-KUTTA METHOD FOR SOLVING LINEAR FUZZY DELAY DIFFERENTIAL EQUATIONS WITH UNKNOWN STATE-DELAYS

LIM RUI SIH

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy (Mathematics)

Faculty of Science
Universiti Teknologi Malaysia

JULY 2020

DEDICATION

Specially dedicated to

My beloved husband, Loh Kwok Lim and my mother, Koh Ah Moy, whose blessings, support, and patience;

My siblings, Jia Tong, Jia Hoe, and Lei Chin, whose care and encouragement;

My precious kids, Loh Jing Han, Loh Jing Xiang, and Loh Jing Cheng, whose endearing love and affections;

And

My supervisors, Associate Professor Dr. Yeak Su Hoe and Associate Professor Dr. Rohanin Ahmad, whose guidance;

Lead to the achievement of my doctoral degree.

ACKNOWLEDGEMENT

First of all, special thanks and deepest appreciation to my main supervisor, Dr. Yeak Su Hoe and co. supervisor, Assoc. Prof. Dr. Rohanin Ahmad for their endless support, guidance, practical advice, thoughtful suggestions and objective comments throughout the course of this study. Their willingness to allocate their precious time to advise me in completing this thesis has been extremely appreciated. This thesis would never have been completed without their knowledge and expertise.

My Ph.D. journey would not have been possible and remarkable without the assistance of so many people I met whose names may not all be mentioned, but their sharing and contributions towards my understanding and thoughts are fully acknowledged and sincerely appreciated.

Apart from this, I would like to acknowledge with great gratitude, the financial support from Ministry of Higher Education (MOHE) for offering me MyBrain15 (MyPhD) scholarship.

Last but not least, my heartiest love and gratitude goes to my family members: my husband, Loh Kwok Lim, my mother, Koh Ah Moy; my siblings, Jia Tong, Jia Hoe, and Lei Chin; my previous, Loh Jing Han, Loh Jing Xiang, and Loh Jing Cheng for their unlimited love and devotion, throughout my life. Their blessings and encouragement always enlighten me. Their continuous supports; mentally, financially or physically, has allowed me to complete this research.

ABSTRACT

In this research, a new method to solve the Fuzzy Delay Differential Equations (FDDEs) with unknown state-delays constrained optimization problem is introduced. This method is based on the coupling of second and third orders Runge-Kutta (RK) method called hybrid RK method. The main goal of this thesis is to identify the unknown state-delays using experimental data. RK methods are chosen because they are well-established and can be easily modified to overcome the discontinuities which occur in Delay Differential Equations (DDEs) especially outside uniform nodes with delay step-size. Numerical results of FDDEs from the hybrid RK methods are compared with exact solutions derived from stepwise approach using Maple software. The relative errors are calculated for the purpose of accuracy checking on these numerical schemes. In this study, a dynamic optimization problem in which the state-delays are decision variables is also imposed; with its formulated cost function. The gradient of the cost function is computed by solving auxiliary FDDEs. By exploiting the results, the state-delay identification problem can be solved efficiently and accurately using a gradient-based optimization method. In addition, a C program has been developed based on hybrid RK methods for solving these problems. Consequently, the results show that the new hybrid scheme is an efficient numerical technique in solving all the problems above with acceptable errors.

ABSTRAK

Dalam kajian ini, kaedah baru untuk menyelesaikan Persamaan Terbitan Lengah Kabur (FDDE) dengan masalah pengoptimuman berkekangan lengah-keadaan tidak diketahui telah diperkenalkan. Kaedah ini berdasarkan gandingan kaedah Runge-Kutta (RK) peringkat kedua dan ketiga yang dipanggil RK terhibrid. Matlamat utama tesis ini adalah mengenal pasti lengah-keadaan tidak diketahui dengan menggunakan data ujikaji. Kaedah RK telah dipilih kerana ianya kaedah yang mapan dan boleh diubah suai dengan mudah untuk mengatasi ketidakselanjaran yang berlaku dalam Persamaan Terbitan Lengah (DDE) terutama di luar nod seragam dengan saiz-langkah lengah. Hasil berangka FDDE daripada kaedah RK terhibrid telah dibandingkan dengan penyelesaian tepat yang diperoleh daripada pendekatan berperingkat menggunakan perisian Maple. Ralat relatif telah dikira dengan tujuan memeriksa ketepatan skema berangka ini. Dalam kajian ini, masalah pengoptimuman dinamik di mana lengah-keadaan sebagai pembolehubah keputusan juga dikenakan; bersama fungsi kos yang dirumuskan. Kecerunan fungsi kos telah dikira dengan menyelesaikan FDDE bantuan. Dengan memanfaatkan hasil dapatan, masalah pengenalpastian lengah-keadaan dapat diselesaikan secara cekap dan tepat menggunakan kaedah pengoptimuman berdasarkan kecerunan. Di samping itu, program C telah dibangunkan berdasarkan kaedah RK terhibrid untuk menyelesaikan masalah ini. Justeru, hasil menunjukkan skema hibrid baharu ini adalah teknik berangka yang berkesan dalam menyelesaikan kesemua masalah di atas dengan ralat yang boleh diterima.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

FDDEs	-	Fuzzy Delay Differential Equations.
ODEs	-	Ordinary Differential Equations.
DDEs	-	Delay Differential Equations.
FDEs	-	Fuzzy Differential Equations.
RK	-	Runge-Kutta.
RK2	-	Second Order Runge-Kutta.
RK4	-	Fourth Order Runge-Kutta.
RK3	-	Third Order Runge-Kutta.
RK24	-	Second and Fourth Orders Runge-Kutta.
RK23	-	Second and Third Orders Runge-Kutta.
RK34	-	Third and Fourth Orders Runge-Kutta.
LMM	-	Linear Multistep Method.
NF	-	Newton Forward.
GCD	-	Greatest Common Divisor.
Im(z)	-	Imaginary part of z.
Re(z)	-	Real part of z.

LIST OF SYMBOLS

$\boldsymbol{J}\left(\boldsymbol{\tau}\right)$	-	Cost functions of time-delays, $\boldsymbol{\tau}$.
$\widehat{\boldsymbol{x}}$	-	System's output measurement.
t	-	Time.
τ_i	-	State-delays or time-delays.
α-cut	-	Alpha-cut.
$\underline{y}\left(t\right)$	-	Lower bound or minimum value for fuzzy number.
$\overline{y}\left(t\right)$	-	Upper bound or maximum value for fuzzy number.
ε	-	Stopping criterion.
h	-	Node-size.
N	-	Final iteration.
$\dfrac{\partial \boldsymbol{J}\left(\boldsymbol{\tau}\right)}{\partial(\tau_k)}$	-	Cost gradient of time-delays, $\boldsymbol{\tau}$.

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

In this chapter, the background of the study will be briefly introduced. Soon later, the statement and objectives of the problem will be clearly defined. In addition, the scope of the study will be discussed as well as the significance of the study. Last but not least, for clarity purpose, the layout of the thesis is briefly outlined.

1.2 Background of Study

Optimization and fuzzy set are the powerful tools in the analysis and modeling of uncertainty in physical systems and many areas of science. In this study, a class of optimal control problems where the state equation is the fuzzy delay differential equations (FDDEs) is considering. Ordinary differential equations (ODEs) and delay differential equations (DDEs) appear in many different contexts throughout the research in sciences and mathematics; while in ODEs the derivatives of unknown functions are dependent on the current value of the independent variables only, but in DDEs, the derivatives of the unknown functions are dependent on the values of the functions at current time and previous time (history). This signifies that the solution of DDEs requires the knowledge of the current state and also the certain past values/history. Moreover, the theory of fuzzy differential equations (FDEs) which is combination of ODEs and fuzzy number but without include the time-delay, has been rapidly growing and attracted widespread attention too. Coupling of DDEs and fuzzy number as called FDDEs, which play an important role in an increasing number of system models in engineering, biology, mechanics, economics and a wide range multitude of real-world phenomena, either they are linear or non-linear, the solutions to them must be list systematically out. These types of equations include a large number of dynamical systems. The exact solutions of DDEs and FDDEs are difficult to obtain and hence the numerical methods were proposed [1, 2]. Thus the numerical methods for solving DDEs and FDDEs are required, and work in this area is far less advanced. A variety of numerical methods have been developed for finding the solution of DDEs and FDDEs, that is, Adams-Bashforth method, linear programming method, predictor-corrector method, Taylor method and so on. However, there are many numerical methods have been studied and applied on it, the Runge-Kutta (RK) method is by far the most well-known and effective tool available to researchers with the increase in computing power, this has contributed to a growing of interest in numerical solution of DDEs and FDDEs. The RK method will be discussed and applied in this study.

The RK methods are attractive because they are much easier to start than other popular numerical methods and easily to modified for solving DDEs such as dde23 takes an approach which by extending the method of the Matlab ODEs solver ode23. Besides that, hybrid RK method is combining two different orders of RK methods to solving the time tracking problems in DDEs such as coupling the second with third-orders RK, second with fourth-orders RK, and third with fourth-orders RK. Thus, a new class of hybrid numerical methods for solving the DDEs and FDDEs will be discussed in Chapter 3.

For optimization section, time-delays estimations on DDEs and FDDEs will be studied in this study too. This is a control technique for the systems often depend heavily on accurate knowledge of the time-delays. In the problem of optimal control, generally the minimization of the functional cost is dealing. Here, the cost function required to minimize as follow [3]:

$$\boldsymbol{J}\left(\boldsymbol{\tau}\right) \triangleq \sum_{v=1}^{p} \left|\boldsymbol{x}\left(t_v \middle| \boldsymbol{\tau}\right) - \widehat{\boldsymbol{x}}^{v}\right|^2 , \tag{1.1}$$

where $\widehat{\boldsymbol{x}}^{v} \to \mathbb{R}^{q}$ denote the system's output measured at time $t = t_{v},\ v = 1,\ 2,\ \ldots,\ p$; a candidate state-delay is a dynamic optimization problem with two unique characteristics. At each time t, the non-linear delay differential system's instantaneous rate of change depends not only on its current state, but also on its state at times $t - \tau_{i},\ i = 1,\ \ldots,\ m,$ where each τ_{i} is a so-called state-delay. The study assumed the cost function (1.1) is instead depends on the state at multiple discrete time point. There are several steps involved: the class of admissible controls; the mathematical description (or model) of the system to be controlled; the specification of a performance criterion; then, the statement of physical constraints that should be satisfied [3]. Hence, this study will consider the above problem.

1.3 Statement of the Problem

RK methods have been numerous reported to solve various kind of problems with satisfactory results obtained. In dealing with time-delays, numerical errors may occur due to the abrupt history tracking and discontinuities. Hence, it is desired to obtain a good property of scheme to get more accurate solutions. Therefore, this research will concentrate on the use of the hybrid Runge-Kutta scheme for the problem of minimization of the fuzzy number index subject to a linear fuzzy delay differential system. This is the intention of this thesis and this research will clarify the following questions:

1.3.1 How to overcome the discontinuities in DDEs?

1.3.2 What is convergence of the RK method?

1.3.3 What is the hybrid RK scheme for DDEs and FDDEs?

1.3.4 Will the hybrid RK scheme be a more efficient tool compare with Maple in approximating the numerical solution of DDEs and FDDEs; and time-delays estimation?

Problem 1.3.1, 1.3.2, and 1.3.3 are covered in Chapter 3, whereas Chapter 4 provides the answer to Problem 1.3.4.

1.4 Objectives of the Study

Based on the research questions in Sub-Section 1.3, this research embarks on the following objectives:

1.4.1 To develop an algorithm for solving DDEs and FDDEs by using hybrid RK method and overcome the discontinuities problem.

1.4.2 To generalize the convergence properties of the developed algorithm.

1.4.3 To apply and implement the algorithm for optimal control problem with DDEs and FDDEs as constraints by using gradient based method.

1.5 Scope of the Study

For this research, the basic of fuzzy concepts will be discussed, and also, understanding this numerical discretization extended of Runge-Kutta method in DDEs. Moreover, the focus is on the time-delays estimation of the FDDEs. The reason is, the numerical solutions of the FDDE are computationally demanding. The method that will be employed here is hybrid Runge-Kutta method. Microsoft Visual

C++ will be used for solving the fuzzy delay differential equations by using Runge-Kutta method.

1.6 Significance of the Study

The significances of this study include as follow:

1.6.1 New algorithm that will effectively to do their part in solving the mention problem that normally occurs in science and mathematics applications involving real world problems. It is therefore of utmost importance for algorithms to be reliable.

1.6.2 The algorithm is relied upon to help in decision-makings usually involving the signal processing problem and large sums of time.

1.6.3 The algorithm will produce the desired outcome with successfully.

Hence this study is specifically designed to handle such systems. The result will indicate more accurate and systematic solution.

1.7 Layout of the Thesis

This thesis consists of six chapters and it organized as follows:

Chapter 1 starts with the background, statement, and objectives of the study. Furthermore, the scope and significance of the study are also demonstrated.

Chapter 2 presents a detailed literature review of DDEs, FDDEs, convergence of RK method, and time-delays estimations.

Chapter 3, three new hybrids scheme of solving DDEs and FDDEs will be discussed. The convergence of the algorithm will be discussed.

Chapter 4, the hybrid scheme will be coupling with conjugate gradient method for time-delays estimations.

Chapter 5, the performance of hybrid scheme is validated with solving numerical example. The efficiency of the newly develop numerical schemes of hybrid RK method can be assured.

Finally, conclusions are drawn and recommendations for future research are illustrated in Chapter 6.

REFERENCES

1. Jayakumar, T., Muthukumar, T., and Kanagarajan, K. Numerical solution of fuzzy delay differential equations by Runge-Kutta Verner method, *Communications in Numerical Analysis*, 2015. 1:1-15.
2. Indrakumar, S. and Kanagarajan, K. Numerical solution of fuzzy delay differential equations under generalized differentiability by Euler's method, *Journal of Advances in Mathematics*, 2015. 10(7): 3674-3687.
3. Ryan Loxton, Kok Lay Teo, and Volker Rehbock. An optimization approach to state-delay identification, *IEEE Transactions on Automatic Control*, 2010. 55(9): 2113-2119.
4. Bellen, A. and Zennaro, M. *Numerical Methods for Delay Differential Equations.* Oxford: Oxford Science Publication. 2003.
5. Xu, R. and Chen, L. Persistence and stability for a two-species ratio-dependent predator-prey system with time delay in a two-patch environment. *Comput. Math. Applic.* 2000. 40(4-5):577-588.
6. Cao, Y. Y. and Frank, P. M. Analysis and synthesis of nonlinear time-delay systems via fuzzy control approach. *IEEE Trans. Fuzzy Sys*, 2000. 8(2):200-211.
7. Willé, D. R. and Baker, C. T. H. DELSOL – A numerical code for the solution of systems of delay-differential equations. *Appl. Numer. Math* , 1992. 9: 223-234.
8. Yaacob, Y., Yeak, S. H., Lim, R. S., and Soewono, E. A delay differential equation model for dengue transmission with regular visits to a mosquito breeding site. *AIP Publishing*, 2015. 1651:153-160.
9. Rosana M. Jafelice, Laecio C. Barros, and Rodney C. Bassanezi. A fuzzy delay differential equation model for HIV dynamics. IFSA Eusflat. 2009. 265-270.
10. Mokhtar A. Abdel Naby, Mohamed A. Ramadan, and Samir T. Mohamed. Spline Approximation for solving system of first order delay differential equations, Studis Univ. "Babes-Bolyai", *Mathematica*, June 2003. 48(2).
11. Fudziah Ismail, Raed Ali Al-Khasawneh, Aung San Lwin, and Mohamad Suleiman. Numerical treatment of delay differential equations by Runge-Kutta method using Hermite Interpolation, *Matematika*, 2002. 18(2): 79-90.
12. Tamas Kalmar-Nagy, Stability analysis of delay-differential equations by the method of steps and inverse Laplace transform Differential Equations and Dynamical System, Springer-Verlag, 2009. 17(1-2): 185-200.

13. Shampine, L. F. and Thompson, S. Numerical solution of delay differential equations, In: Delay Differential Equations. *Springer, Boston, MA*, 2009. 1-27.
14. Neves K. W., Automatic Integration of Functional Differential Equations: An Approach, *ACM Trans Math Softw*, 1975. 1:357-368.
15. Shampine, L. F., and Thompson, S. Solving delay differential equations in Matlab, *Applied Numerical Mathematics*, 2001. 37:441-458.
16. Paul, C. A. H. Developing a delay differential equation solver. *Numerical Analysis Report*, University of Manchester. 1991. 204: 1-12.
17. Baker, C. T. H., Paul, C. A. H., and Wille, D.R. Issues in the numerical solution of evolutionary delay differential equations. *Numerical Analysis Report*, Advances in Computational Mathematics, 1995. 3:171-196.
18. Burton, T. A. Stability theory for delay equations. *Funkcialaj Ekvacioj*, 1979. 22:67-76.
19. Eric A. Butcher, Haitao Ma, Ed Bueler, Victoria Averina, and Zsolt Szabo, Stability of linear time-periodic delay-differential equations via Chebyshev polynomials. *International Journal for numerical methods in engineering*, 2004. 59:895-922.
20. Xiangao Li, Shigui Ruan, and Junjie Wei. Stability and Bifurcation in delay-differential equations with two delays. *Journal of Mathematical Analysis and Applications*, 1999. 236:254-280.
21. Anatoli F. Ivanov and Musa A. Mammadov. Global stability, periodic solutions, and optimal control in a nonlinear differential delay model. *Electronic Journal of Differential Equations*, 2010. 19:177-188.
22. Paul, C. A. H. and Baker, C. T. H. Explicit Runge-Kutta methods for numerical solutions of singular delay differential equations. *Numerical Analysis Report*, University of Manchester. 1992. 212:1-39.
23. Barwell, V. K. On the asymptotic behaviour of the solution of a differential difference equations. *Utilitas Math*, 1974. 6:189-194.
24. Al-Mutib, A. N. Stability properties of numerical methods for solving delay differential equations, *J. Comput. Appl. Math*, 1984. 10:71-79.
25. Zennaro, M. P-stability properties of Runge-Kutta methods for delay differential equations, *Numer. Math*, 1986. 49:305-318.
26. Barwell, V. K. Special stability problems for functional differential equations. *BIT*, 1975. 15:130-135.

27. Torelli, L. Stability of numerical methods for delay differential equations, *J. Comput. Appl. Math*, 1989. 25:15-26.
28. Runge-Kutta methods. Available from: <https://en.wikipedia.org/wiki/Runge%E2%80%93Kutta_methods>. [3 June 2018].
29. Ralston, A. *A First Course in Numerical Analysis*, McGraw-Hill. 1962.
30. Lambert, J. D. *Computational Methods in Ordinary Differential Equations.* John Wiley and Sons, New York. 1973.
31. Christian, K. *Multiple Time Scale Dynamics*, Springer. 2015.
32. Kendall, A., Weimin, H., and David, E. S. *Numerical Solution of Ordinary Differential Equations.* John Wiley & Sons Inc. 2009.
33. Fadugba, S. E., and Nwozo, C. R. Crank Nicolson Finite Difference Method for the Valuation of Options. *The Pacific Journal of Science and Technology,* 2013. 14(2): 136-146.
34. Butcher, J. C. Coefficients for the study of Runge-Kutta integration processes. *J. Austral. Math. Soc.*, 1963. 3:185-201.
35. Butcher, J. C. Implicit Runge-Kutta processes. *Math. Comp.*, 1964. 18:50-64.
36. Jacobi method. Available from: <https://en.wikipedia.org/wiki/Jacobi_method>. [26 June 2018].
37. Gauss-Seidal method. Available from: < https://en.wikipedia.org/wiki/Gauss%E2%80%93Seidel_method>. [8 April 2018].
38. Zadeh, L. Fuzzy set. *Information and Control*, 1965. 8(3): 338-353.
39. Jayakumar, T., Maheskumar, D., and Kanagarajan, K. Numerical Solution of Fuzzy Differential Equations by Runge-Kutta Method of Order Five. *Applied Mathematical Sciences*, 2012. 6(60): 2989-3002.
40. Kanagarajan, K. and Suresh, R. Numerical solution of fuzzy differential equations under generalized differentiability by modified Euler method. *International Journal of Mathematical Engineering and Science*, 2014. 2(11): 2277-6982.
41. Rubanraj, S. and Rajkumar, P. Numerical solution of fuzzy differential equation by sixth order Runge-Kutta method. *International Journal Fuzzy Mathematics Archive*, 2015. 7(1): 35-42.

42. Abbasbandy, Saeid. Numerical solutions of fuzzy differential equations by Taylor method. *Computational methods in Applied Mathematics*, 2002. 2(2):113-124.
43. Pederson, S. and Sambandham, M. Numerical solution of hybrid fuzzy systems. *Math Comput Model*, 2007. 45:1133-1144.
44. Pederson, S. and Sambandham, M. Numerical solution of hybrid fuzzy differential equation IVPs by Characterization theorem. *Inf Sci*, 2009. 179:319-328.
45. Hyunsoo, Kim and Rathinasamy Sakthivel. Numerical solution of hybrid fuzzy differential equations using Improved Predictor-Corrector method. *Commun Nonlinear Sci Numer Simulat*, 2012. 17:3788-3794.
46. Jayakumar, T. and Kanagarajan, K. Numerical solution of hybrid fuzzy differential equations by Adams fifth order Predictor-Corrector method. *International Journal of Mathematical Trends and Technology*, 2014. 10(10):2231-5373.
47. Farahi, M. H., and Barati, S. Fuzzy time-delay dynamical systems. *The Journal of Mathematics and Computer Science*, 2011. 2(1):44-53.
48. Khabat Barzinji, Normah Maan, and Noraini Aris. Linear fuzzy delay differential system: Analysis on Stability of Steady State. *Matematika*, 2014. 30(1a):1-7.
49. Fuzzy number. Available from: <https://en.wikipedia.org/wiki/Fuzzy_number>. 24 July 2019.
50. Shang Gao, Zaiyue Zhang, and Cungen Cao. Multiplication operation on fuzzy numbers. *Journal of Software*, 2009. 4(4): 331-338.
51. Nasseri, H. Fuzzy numbers: positive and nonnegative. *In International Mathematical Forum. Citeseer*, 2008. 3: 1777-1780.
52. Nagoor, G. A. A new operation on triangular fuzzy number for solving fuzzy linear programming problem. *Applied Mathematical Sciences*, 2012. 6(11): 525-532.
53. Yeh, C. 2019, *Chapter 5 Fuzzy Number*. [online] Slideplayer.com. Available at: http://tiny.cc/k4o9dz [Accessed 10 Oct. 2019].
54. Vasile Lupulescu, and Umber Abbas. Fuzzy delay differential equations. *Fuzzy Optim. Decis. Making*, 2012. 11: 99-111.

55. Nor Atirah Izzah Zulkefli, Normah Maan. The existence and uniqueness theorems of fuzzy delay differential equations. *Malaysian Journal of Fundamental and Applied Sciences*, 2014. 10: 139-143.
56. Narayanamoorthy, S., and Yookesh, T. L. Approximate method for solving the linear fuzzy delay differential equations. *Hindawi Publishing Corporation Discrete Dyamics in Nature and Society*, 2015. 1-9.
57. Khabat Barzinji. Numerical solution of fuzzy delay predator-prey system. *International Journal of Mathematical Analysis*, 2017. 11(12): 595-603.
58. Tuch, J., Feuer, A., and Palmor, Z. J. Time delay estimation in continuous linear time-livariant systems, *IEEE Transactions on Automatic Control*, 1994. 39(4): 823-827.
59. Fei Gao, Yibo Qi, Qiang Yin, and Jiaqing Xiao. An artificial bee colony algorithm for unknown parameters and time-delays identification of Chaotic systems. *Proceedings of the 5th International Conference on Computer Sciences and Convergence Information Technology*, 2010. 659-664.
60. Qinqin Chai, Ryan Loxton, Kok Lay Teo, and Chunhua Yang. Time-delay estimation for nonlinear systems with piecewise-constant input. *Applied Mathematics and Computations*, 2013. 219(7): 1-27.
61. Ling Yun Wang, Wei Hua Gui, Kok Lay Teo, Ryan Loxton, and Chun Hua Yang. Time delayed optimal control problems with multiple characteristic time points: computation and industrial applications. *Journal of Industrial and Management Optimization,* 2009. 5(4): 705-718.
62. Zhang, X. X., Xu, J., and Huang, Y. Experiment on parameter identification of a time delayed vibration absorber. *International Federation of Automatic Control,* 2015. 48(12): 057-062.
63. Li Juan Li, Ting Ting Dong, Shu Zhang, Xiao Xiao Zhang, and Shi Ping Yang. Time-delay identification in dynamic processes with disturbance via correlation analysis. *Journal of Control Engineering Practice*, 2017. 62: 92-101.
64. Qinqin Chai, Ryan Loxton, Kok Lay Teo, and Chunhua Yang. A unified parameter identification method for nonlinear time-delay systems. *Journal of Industrial and Management Optimization*, 2013. 9(2): 471-486.
65. Lin Qun, Ryan Loxton, Xu Chao, and Kok Lay Teo. State-delay estimation for nonlinear systems using inexact output data. *Proceeding of the 33rd Chinese Control Conference*, 2014. 6549-6554.

66. Shaalini, J. V., Kanaga Pushpam, A. E. Analysis of composite Runge-Kutta methods and new one-step technique for stiff delay differential equations. *International Journal of Applied Mathematics*, 2019. 49(3):359-368.
67. Friedman, Ma. M, and Kandel, M. A. 1999. Numerical solutions of fuzzy differential equations, *Fuzzy Sets and System*, 105: 133-138.
68. Abbasbandy, S., and Allah, V. T. Numerical solution of fuzzy differential equation by Runge-Kutta method. *J. Sci. Teacher Training University*, 1(3), 2002.
69. Smoothness. Available from: < https://en.wikipedia.org/wiki/Smoothness>. 29 September 2019.
70. Ahmed, N. U. Dynamic system and control with application. Singapore: World Scientific, 2006.
71. Straeter, T. A. On the extension of the Davidon_Broyden Class of rank one, Quasi-Newton minimization methods to an infinite dimensional Hilbert Spare with applications to optimal control problems. *NASA Technical Reports Server*, 2011.

Appendix D

LIST OF PUBLICATIONS

1. R. S. Lim, Rohanin A, and S. H. Yeak. Implicit Second and Third Orders Runge-Kutta for Handling Discontinuities in Delay Differential Equations. *Proceeding of 2nd International Science Postgraduate Conference 2014 (ISPC 2014)*, 2014. 1-13.

2. Y. Yaacob, S. H. Yeak, R. S. Lim, and E. Soewono. 2015. A Delay Differential Equation Model for Dengue Transmission with Regular Visits to a Mosquito Breeding Site. *Symposium on Biomathematics. AIP Conference Proceedings*. 1651(1), 153-160. DOI: 10.1063/1.4914447.

3. R. S. Lim, Rohanin A, and S. H. Yeak. 2017. History Tracking Ability of Hybrid Second and Fourth Orders Runge-Kutta in Solving Delay Differential Equations. *Jurnal Teknologi*. 79(6), 29-35. DOI: https://doi.org/10.11113/jt.v79.9899. (Indexed by SCOPUS)

4. R. S. Lim, S. H. Yeak, and Rohanin A. 2018. Fuzzy Delay Differential Equations with Hybrid Second and Third Orders Runge-Kutta Method. *Journal of Engineering Science and Technology*, 13(11), 3795-3807. (Indexed by SCOPUS)