GRAPH POLYNOMIAL ASSOCIATED TO SOME GRAPHS OF CERTAIN FINITE NONABELIAN GROUPS

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DEDICATION

To my beloved husband, Alif Ehsan Hamzah,
and our children.
To my dearest parents, Najmuddin and Latifah.

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ABSTRACT

The study of associating the groups in group theory with the graphs in graph theory are widely done by many researchers. Since the algebraic properties of groups can be studied through the structures of graphs, then it is common to find certain graph invariants and graph properties. Nevertheless, the graph polynomials are also significant in the study of the graphs but have not yet determined for the graphs associated to groups. Graph polynomials, such as the independence polynomial, the clique polynomial, and the domination polynomial are used to store the combinatorial information of a graph. An independence polynomial of a graph is the polynomial in which its coefficients are the number of independent sets in the graph. A clique polynomial of a graph is the polynomial containing coefficients that represent the number of cliques in the graph. Meanwhile, a domination polynomial of a graph is the polynomial that contains coefficients representing the number of dominating sets in the graph. In the first part of this research, these three polynomials are determined for five types of graphs for three types of groups. The graphs considered are the conjugacy class graphs, the conjugate graphs, the commuting graphs, the noncommuting graphs, and the center graphs associated to the dihedral group, the generalized quarternion group, and the quasidihedral group. All these graphs are found and expressed in general in the form of the union and join of some complete graphs, complete bipartite graphs, and also complete multipartite graphs. Then, the graph polynomials associated to groups are obtained from these common types of graphs by using the properties of the graph polynomials. The results obtained are some polynomials of certain degrees. In the second part of this research, the roots of all the graph polynomials associated to the finite groups that have been computed are determined. The independence polynomial of the graphs associated to groups have real roots that are always negative. The clique polynomials have roots that are always real but may not be integers. Meanwhile, the domination polynomials always have a zero root and the other roots may be complex numbers. In the last part of this research, two types of new graph polynomials are defined and determined for the graphs mentioned earlier. The new graph polynomials are called the clique-independence polynomial and the clique-domination polynomial. The clique-independence polynomial of a graph is the polynomial containing coefficients that represent the number of clique-independent sets in the graph. The clique-domination polynomial of a graph is the polynomial in which its coefficients are the number of clique-dominating sets in the graph. The clique-independence polynomials are obtained for the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph, and the center graph associated to the dihedral group because these graphs contain clique-independent sets and are suitable to be expressed in the form of clique-independence polynomials. Meanwhile, the clique-domination polynomials are determined only for the noncommuting graph associated to the dihedral group since clique-dominating sets exist only for connected graphs.

ABSTRAK

Kajian yang mengaitkan kumpulan dalam teori kumpulan dengan graf dalam teori graf telah dilakukan secara meluas oleh ramai penyelidik. Oleh kerana sifat aljabar bagi kumpulan boleh dikaji melalui struktur graf, maka ia adalah perkara biasa untuk mencari graf tak varian dan sifat graf yang tertentu. Walaupun begitu, polinomial graf juga adalah penting dalam kajian mengenai graf tetapi belum pernah ditentukan untuk graf yang berkaitan dengan kumpulan. Polinomial graf, seperti polinomial ketakbersandaran, polinomial klik, dan polinomial dominasi digunakan dalam penyimpanan maklumat kombinatorik bagi suatu graf. Polinomial ketakbersandaran bagi graf ialah polinomial yang pekalinya adalah bilangan set tak bersandar bagi graf. Polinomial klik bagi graf ialah polinomial yang mengandungi pekali yang mewakili bilangan klik dalam graf tersebut. Sementara itu, polinomial dominasi bagi graf ialah polinomial yang mempunyai pekali mewakili bilangan set berdominan bagi graf. Pada bahagian pertama penyelidikan ini, ketiga-tiga polinomial ini ditentukan untuk lima jenis graf bagi tiga buah kumpulan. Graf yang terlibat ialah graf kelas kekonjugatan, graf konjugat, graf berkalis tukar tertib, graf tak berkalis tukar tertib, dan graf pusat yang terkait dengan kumpulan dwihedron, kumpulan kuaternion teritlak, dan kumpulan kuasidwihedron. Kesemua graf ini telah dijumpai dan dinyatakan secara am dalam bentuk gabungan dan cantuman bagi beberapa graf lengkap, graf bipartit lengkap, dan juga graf multipartit lengkap. Kemudian, polinomial graf yang berkaitan dengan kumpulan diperoleh daripada graf-graf jenis biasa tersebut dengan menggunakan sifat-sifat bagi polinomial graf. Keputusan yang didapati adalah polinomial-polinomial yang mempunyai darjah tertentu. Pada bahagian kedua penyelidikan ini, punca-punca bagi kesemua polinomial graf yang berkaitan dengan kumpulan terhingga yang telah diperoleh awalnya ditentukan. Polinomial ketakbersandaran bagi graf yang berkaitan dengan kumpulan mempunyai punca nombor nyata yang sentiasa bernilai negatif. Polinomial klik mempunyai punca nyata tetapi tidak semestinya nombor bulat. Sementara itu, polinomial dominasi sentiasa mempunyai punca sifar dan punca yang selebihnya adalah nombor kompleks. Pada bahagian akhir penyelidikan ini, dua jenis polinomial graf baharu diperkenal dan ditentukan bagi graf-graf yang telah dinyatakan sebelumnya. Polinomial graf baharu tersebut dinamakan polinomial klik-ketakbersandaran dan polinomial klik-dominasi. Polinomial klik-ketakbersandaran bagi suatu graf ialah polinomial yang mempunyai pekali yang mewakili bilangan set klik-tak bersandar dalam graf. Polinomial klik-dominasi bagi suatu graf ialah polinomial yang pekalinya adalah bilangan set klik-berdominan dalam graf. Polinomial klik-ketakbersandaran diperoleh bagi graf kelas kekonjugatan, graf konjugat, graf berkalis tukar terib, graf tak berkalis tukar tertib, dan graf pusat yang berkaitan dengan kumpulan dwihedron kerana graf-graf ini mengandungi set klik-tak bersandar dan sesuai dinyatakan dalam bentuk polinomial klik-ketakbersandaran. Sementara itu, polinomial klik-dominasi ditentukan hanya bagi graf tak berkalis tukar tertib yang berkaitan dengan kumpulan dwihedron memandangkan set klik-berdominan hanya wujud untuk graf berkait.

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LIST OF SYMBOLS

$Z(G)$	-	Center of the group G
Γ_G^z	-	Center graph of G
$\mu(\Gamma)$	-	Clique-domination number of Γ
$C_D(\Gamma; x)$	-	Clique-domination polynomial of Γ
$\rho(\Gamma)$	-	Clique-independence number of Γ
$C_I(\Gamma; x)$	-	Clique-independence polynomial of Γ
$\omega(\Gamma)$	-	Clique number of Γ
$C(\Gamma; x)$	-	Clique polynomial of Γ
$N[v]$	-	Closed neighborhood of vertex v
Γ_G^{comm}	-	Commuting graph of G
$\overline{\Gamma}$	-	Complement of Γ
$K_{m,n}$	-	Complete bipartite graph with $m + n$ vertices
$K_{n_1,n_2,\ldots,n_t}$	-	Complete multipartite graph with $n_1 + n_2 + \ldots + n_t$ vertices
K_n	-	Complete graph with n vertices
Γ_G^{cl}	-	Conjugacy class graph of G
$cl(a)$	-	Conjugacy class of a
Γ_G^{conj}	-	Conjugate graph of G
D_{2n}	-	Dihedral group of order $2n$
$\delta(\Gamma)$	-	Domination number of Γ
$D(\Gamma; x)$	-	Domination polynomial of Γ
$E(\Gamma)$	-	Edge set of Γ
E_n	-	Empty graph with n vertices
Q_{4n}	-	Generalized quaternion group of order $4n$
Γ	-	Graph
G	-	Group
e	-	Identity of G

$\alpha(\Gamma)$	-	Independence number of Γ
$I(\Gamma; x)$	-	Independence polynomial of Γ
$\Gamma_1 \vee \Gamma_2$	-	Join of Γ_1 and Γ_2
Γ_G^{nc}	-	Noncommuting graph of G
$N(v)$	-	Open neighborhood of vertex v
QD_{2^n}	-	Quasidihedral group of order 2^n
$\Gamma_1 \cup \Gamma_2$	-	Union of Γ_1 and Γ_2
$V(\Gamma)$	-	Vertex set of Γ

CHAPTER 1

INTRODUCTION

1.1 Introduction

In algebra, the research on the graphs associated to groups have brought great interests to many researchers for the past few years. The algebraic properties of the groups from group theory can be represented by graph structures and usually correspond to certain types of graphs from graph theory, such as the complete graph, complete bipartite graph and multipartite graph. These graphs have certain properties that are used to class them. Graph properties that are being identified when investigating on certain graphs are the graph invariants which include the independence number, the clique number, the domination number and the chromatic number.

In group theory, various types of graphs associated to finite groups have been established. Some examples of those graphs are the conjugate graph, the commuting graph, the noncommuting graph, the orbit graph and the center graph. The names of these graphs are referring to certain properties of the group that are used in the definitions of the graphs. For example, the center graph of a group contains the elements of the group as its vertices and the edges are formed if and only if the product of any two vertices belong to the center of the group. Other than the graph invariants, other research topics that are also commonly discussed for the graphs associated to groups are the energy of graph, the topological index of graph and the spectrum of graph. Among all the studies related to the graphs of groups, none focus in finding the graph polynomials.

In general, graph polynomials from graph theory are established for analyzing various aspects of combinatorial graph invariants and characterizing the structure concerning graphs. They contain coefficients that represent certain informations and properties of the graphs. There are many types of graph polynomials that have been studied by other researchers such as the independence polynomial, the matching polynomial, the clique polynomial, the domination polynomial and the chromatic polynomial that are generalized for many common types of graphs from graph theory.

This research is concerning in obtaining the graph polynomials for some graphs associated to certain finite nonabelian groups. The graph polynomials included in this research are the independence polynomial, the clique polynomial and the domination polynomial that are determined for the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph and the center graph associated to three finite nonabelian groups, namely the dihedral group, the generalized quaternion group and the quasidihedral group. The roots of these graph polynomials are also found in this research.

The three groups that are considered are not isomorphic to each other since each of them has their own structures that differ between one another, such as the order of the elements in the group. However, the similarity between them is that they belong to the isomorphism classes of groups of order 2^n with nilpotency class $n-1$.

In addition to that, two new types of graph polynomials are also introduced in this research. The first one is the clique-independence polynomial of a graph and the second one is the clique-domination polynomial of a graph.

In this chapter, the introduction of the whole research is presented. The research background, problem statement, research objectives, scope of the study and significance of findings are stated.

1.2 Research Background

There are many types of graph polynomials that have been introduced and studied by many researchers since the past fifty years. Some examples of graph polynomials that are studied are the chromatic polynomial [1], the cycle polynomial [2], the dependence polynomial [3], the independence polynomial [4], the path polynomial [5], the domination polynomial [6] and the vertex polynomial [7]. In this research, three types of graph polynomials are of interest, namely the independence polynomial, the clique polynomial and the domination polynomial.

The three graph polynomials are brought into consideration as they are the most relevant types of polynomials for the five types of graphs of groups included in this research. Those graph polynomials have been expressed in general forms that are sufficient to be applied onto the graphs. Since this research is a pioneer in computing the graph polynomials associated to groups, then these three graph polynomials are the most suitable ones to be considered.

The concept of the independence polynomial of a graph is studied by Hoede and Li [4] in 1994, together with the concept of the clique polynomial of a graph. The independence polynomial of a graph is the polynomial in which the coefficient is the number of the independent sets of the graph. While the clique polynomial is defined as the polynomial in which the coefficient is the number of cliques of the graph. In 2005, Levit and Mandrescu [8] published a survey paper on the independence polynomial of graph that mentioned some ways to compute the independence polynomial, the unimodalitiy of the independence polynomials and other important results concerning the roots of the independence polynomials. Later, Ferrin [9] established the independence polynomials of some graphs including the complete graph, the complete bipartite graph, the cycle graph and the star graph.

The clique polynomial is first introduced in 1990 by Fisher and Solow [3] but with a different name, namely the dependence polynomial which contains alternate sign coefficients. Later, the term clique polynomial appears in the research by Hoede and Li [4] with the condition that coefficients of the polynomial are always positive. The study of the clique polynomial is later extended by Hajiabolhassan and Mehrabadi in [10] that focused on the largest negative root of the clique polynomial of a graph and how it is related with its subgraph. Furthermore, the unique smallest root of the clique polynomial of graph is examined in a research by Goldwurm and Santini [11].

Another type of graph polynomial that is included in this research is the domination polynomial, introduced by Alikhani in [6]. The domination polynomial of a graph is the polynomial whose coefficient is the number of the dominating sets in the graph. In [12], Akbari et al. studied on the characterization of graphs by using the domination polynomials in which the graphs are characterized by how many roots their domination polynomials have. Few years later, in [13], Alikhani established the domination polynomials of some graph operations. The domination polynomial of the lexicographic product or the composition of specific graphs are then discovered by Alikhani and Jahari in [14].

Note that throughout this research, the graphs considered are all simple graphs, without loops or multiple edges, and from now on will be referred only as graphs. The graphs associated to groups that are included in this research are the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph and the center graph. These five types of graphs, when associated to the finite nonabelian groups considered in this research, can be expressed in specific general forms that are compatible with the graph polynomials selected.

The conjugacy class graph was first introduced by Bertram et al. in [15]. Later, Mahmoud et al. [16] obtained the general form of the conjugacy class graphs of some finite groups. In addition to that, Erfanian and Tolue [17] introduced the conjugate graph of finite groups and later, Erfanian et al. [18] extended the concept to generalized conjugate graph. Other than that, the commuting graph has been introduced by Segev [19]. Then, Raza and Faizi [20] discussed on certain properties of the commuting graphs of the dihedral groups. Furthermore, Neumann [21] has investigated the problem initiated by Paul Erdos that lead to the introduction of the noncommuting graph. Abdollahi et al. [22] and Talebi [23] studied on some properties of the noncommuting graph of certain types of groups. Meanwhile, Balakrishnan et al. [24] introduced the center graph of a group and later, this concept has been extended to n-th central graph of a group by Karimi et al. [25].

In association between the study of graphs with groups, the graph polynomials have not yet being considered to be determined for the graphs associated to groups. Therefore, in this research, the independence polynomial, the clique polynomial and the domination polynomial associated to the five types of graphs mentioned earlier that are associated to some finite groups are computed and to be expressed in general form. The finite groups include the dihedral group, the generalized quaternion group and the quasidihedral group. These three groups are not isomorphic to each other but certain types of graphs associated to them are isomorphic to each other. Thus, the graph polynomials together with their roots obtained in this research are analyzed to establish any connection among the groups.

Moreover, two new types of graph polynomials are defined and expressed in general forms for some graphs associated to the dihedral group. The clique-independence polynomial is established for the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph and the center graph of the dihedral group. Meanwhile, the clique-domination polynomial is determined only for the noncommuting graph of the dihedral group since the clique-dominating sets that are required to form the polynomial exist only in a connected graph. The new graph polynomials are determined only for the dihedral group because throughout this research, since the graph polynomials associated to groups with order 2^n are always equal, then it is deduced that the graph polynomials associated to the dihedral group is enough to represent the graph polynomials for the other two groups.

Many researchers state that it is quite obvious that if two graphs are isomorphic, then their graph polynomials will also be equal. Or, under certain conditions, if the graph polynomials of two types of graphs are equal, then usually the graphs are isomorphic. On the contrary, in this research, the graphs of two dihedral groups with different order that have different vertices and edges but they produce equal clique-independence polynomials.

1.3 Problem Statement

In previous researches, only several number of graphs associated to the dihedral group, the generalized quaternion group and the quasidihedral group have been expressed in general forms. Thus, for a few other graphs of groups that are not yet expressed generally, they are determined throughout this research. The general forms of those graphs are necessary in computing the graph polynomials in this research.

A group that has many elements, when represented by a graph, will have a very large vertex set. This lead to difficulties in computing the graph invariants one by one in order to determine its graph polynomials. Instead of finding the properties of a graph associated to group direct from itself, the graph polynomials are practically useful in simplifying the process by determining the general forms of the graph polynomials using the general forms of graphs associated to the three finite nonabelian groups.

Even though it is common for other researchers when establishing a graph of group, the graph invariants such as the independence number, the clique number and the domination number are also determined. However, by finding the graph polynomials, more graph structures like the number of independent sets, the number of cliques and the number of dominating sets are also discovered.

Furthermore, other than the existing graph polynomials, there are two new types of graph polynomials that are introduced in this research. The clique-independence polynomial and the clique-domination polynomial are defined and determined for some graphs associated to the dihedral group. A few additional properties of those graphs are well-described through the new graph polynomials.

Moreover, numerous studies done by other researchers on the roots of graph polynomials are usually characterizing the roots as negative or positive, real or complex, and even or odd. Therefore, the roots for the graph polynomials obtained in this research are also found. The roots that have been determined represent important findings in graph theory in which certain conjectures from previous studies are proven to be true for some of the graphs polynomials associated to some finite groups.

1.4 Research Objectives

The objectives of this research are stated in the following:

1. To determine the general forms of the independence polynomial, the clique polynomial and the domination polynomial for the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph and the center graph associated to three types of finite nonabelian groups, namely the dihedral group, the generalized quaternion group and the quasidihedral group.
2. To construct two new types of graph polynomials associated to the dihedral group, namely the clique-independence polynomial for the five types of graphs as mentioned in Objective 1 and the clique-domination polynomial for the noncommuting graph.
3. To establish the roots for all the graph polynomials obtained in Objectives 1 and 2.

1.5 Scope of the Study

This research focuses on the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph and the center graph associated to three finite nonabelian groups, specifically the dihedral group of order $2n$ (denoted by D_{2n}), the generalized quaternion group of order $4n$ (denoted by Q_{4n}) and the quasidihedral group of order 2^n (denoted by QD_{2^n}). Three types of graph polynomials, namely the independence polynomial, the clique polynomial and the domination polynomial are determined for all those graphs mentioned. Additionally, two types of new graph polynomials are introduced, namely the clique-independence polynomial and the clique-domination polynomial that are established for the graphs associated to the dihedral group.

1.6 Significance of Study

Graph polynomial is an important concept in the study of graph theory as it has been applied in various fields of theoretical physics, theoretical chemistry, physical chemistry and computer science. For example, the concept of independence polynomial coincides with the concept of the lattice gas in physics as studied by Scott and Sokal in [26]. Banderier and Goldwurm [27] implemented the study of clique polynomial to distinguish some asymptotic properties on the number of prefixes in trace monoids. Furthermore, the domination polynomial has been studied by Dohmen and Tittmann [28] to measure new network reliability for some particular kind of service networks. The dominating concepts are studied by Eslahchi and Ansari in [29], to be applied in the assignment of structured domains in complex protein structures that are part of bio-informatics study.

Furthermore, the independence polynomial of a graph carry combinatorial informations related to the independent sets of the graph and also the independence number of the graph. By finding the graph polynomials for the graphs associated with group theory, its major contribution is in the group theory itself, in which the algebraic structures of groups that are described through the graph properties can be studied through the graph polynomials. Other than that, the graph polynomials may provide a link for group theory with other branches of sciences via graph theory. This research may also open opportunity to other group theorists and graph theorists to conduct further related research that may benefit other field of study.

Another major contribution in this research is in introducing two new types of graph polynomials which are called the clique-independence polynomial and the clique-domination polynomial, and to compute these new graph polynomials for some graphs associated to the dihedral group. The new polynomials are significant in describing several additional properties of the graphs, which give a supplementary knowledge regarding the information of the graphs associated to groups.

In addition, the roots found for the graph polynomials obtained in this research are substantial in the study of graph polynomials. Previous works on the roots of certain types of polynomials have appealing geometrical interpretation and have led to major progress in other study areas like numerical algebraic geometries [30], polynomial stability [31] and theoretical computer science [32]. Makowsky et al. [33] have deduced that the roots of graph polynomials received much concern and are significant when these polynomials model physical reality. Several preliminary works state that the location of roots of graph polynomials can give information about the structures or the families of the graphs such as in [34–36].

1.7 Research Methodology

This research begins with the study of some fundamental concepts in group theory and graph theory. The basic concepts of the graph polynomials from graph theory and the graphs associated with group theory are also studied. In this research, three types of graph polynomials are computed for five types of graphs associated to the dihedral group, the generalized quaternion group and the quasidihedral group.

The main results of this research are divided into four parts. To begin with, the first part is on the computation of the graph polynomials associated to the dihedral group, and is divided into two subparts. In the first subpart, the general form of the conjugate graph of the dihedral group is first constructed by using the definition of the conjugate graph, considering that the conjugacy class graph, the commuting graph, the noncommuting graph and the center graph of dihedral groups have been obtained in previous studies. In the second subpart, by using those general forms of the graphs associated to the dihedral group, the independence polynomials are computed for the five types of graphs. The computations are done by utilizing some existing properties such as the independence polynomials of some common types of graphs. The clique polynomials and the domination polynomials for the same five types of graphs are also computed, also by making use of the existing preliminaries from graph theory.

The second and third part of this research is on the computation of the graph polynomials associated to the generalized quaternion group and the quasidihedral group, respectively. The computations are using the same approaches as in the first part of this research which are through the use of existing properties of the independence polynomial, the clique polynomial and the domination polynomial.

In the fourth and last part of this research findings, based on the fundamental concepts in graph theory related to the clique-independent sets and the clique-dominating sets, two new types of graph polynomials are defined. The first new type of polynomial is the clique-independence polynomial and by the definition, some propositions concerning the clique-independence polynomials of some graphs from graph theory are determined. Then, by using those propositions, the clique-independence polynomials for some graphs associated to dihedral group are computed. The graphs involved are the same five types of graphs as in previous parts. The second new type of polynomial is the clique-domination polynomial that is defined and through the definition, the clique-domination polynomial of the noncommuting graph associated to dihedral group is obtained.

Simultaneously, for all the graph polynomials obtained, their roots are also computed and identified to be in the form of integers or non-integers. The computations are done by letting the graph polynomials to be equal to zero and the roots are determined through finding the solutions of the equations. In the case where the non-integer roots cannot be obtained in exact values, then the bounds of the roots are found using some fundamental concepts from Calculus, related to the Bolzano's Theorem and the first derivative test.

The research methodology of this thesis is illustrated in Figure 1.1.

1.8 Thesis Organization

There are seven chapters in this thesis and the thesis organization is illustrated in Figure 1.2.

Chapter 1 is the introduction chapter that explains the whole thesis. It contains the research background, problem statement, research objectives, scope of research, significance of research, research methodology and thesis organization.

Chapter 2 provides the literature review of this research and is divided into three sections. In the first section, some fundamental concepts in group theory and graph theory are presented. Some preliminaries on the graph polynomials that are used throughout this research are shown in the second section. The third section consists of some preliminaries on the graphs associated to the groups that are helpful in the computation of the graph polynomials associated to the groups.

In Chapter 3, by considering the conjugacy class graphs, the conjugate graphs, the commuting graphs, the noncommuting graphs and the center graphs associated to the dihedral group, this chapter is divided into three sections. Each section represents the independence polynomials, the clique polynomials and the domination polynomials of those five graphs, respectively. Those graph polynomials are computed and expressed in general forms and their real roots are determined.

Chapter 4 contains the same types of graphs and polynomials from previous chapter, but the results are computed for the generalized quaternion group. The roots of the graph polynomials obtained are also determined.

In Chapter 5, the three types of graph polynomials from the previous two chapters are computed for the graphs associated to the quasidihedral group, and their real roots are also established. The same five types of graphs from Chapter 3 and 4 are considered.

In Chapter 6, two new types of graph polynomials are defined and computed for the graphs associated to the dihedral group. The clique-independence polynomials are introduced for the conjugacy class graph, the conjugate graph, the commuting graph, the noncommuting graph and the center graph. Meanwhile, the clique-domination polynomial is introduced for the noncommuting graph of the dihedral group. The roots for these polynomials are also found.

Finally, the summary of the whole thesis is provided in Chapter 7. Some suggestions for future research are also proposed.

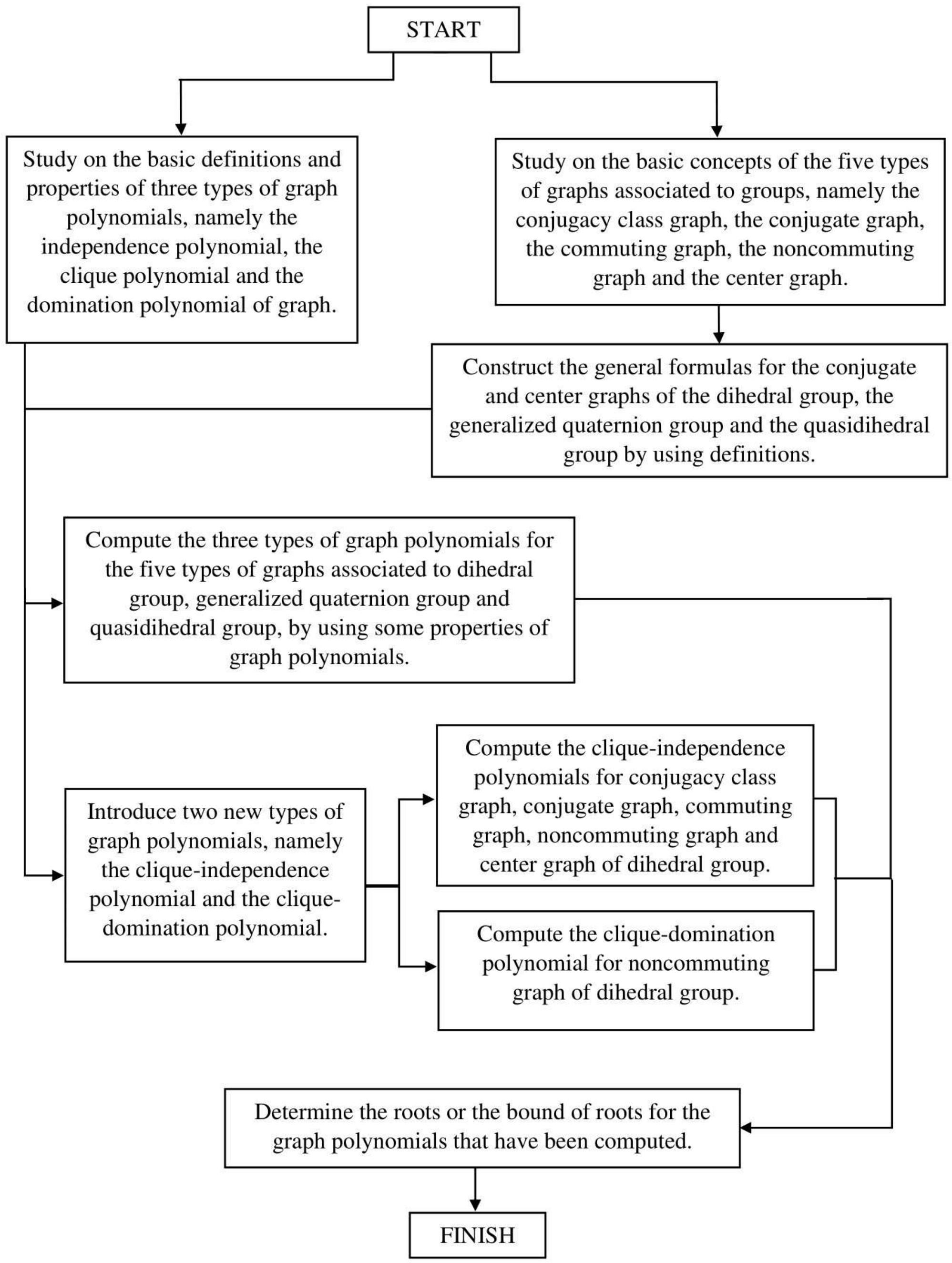

Figure 1.1 Research methodology

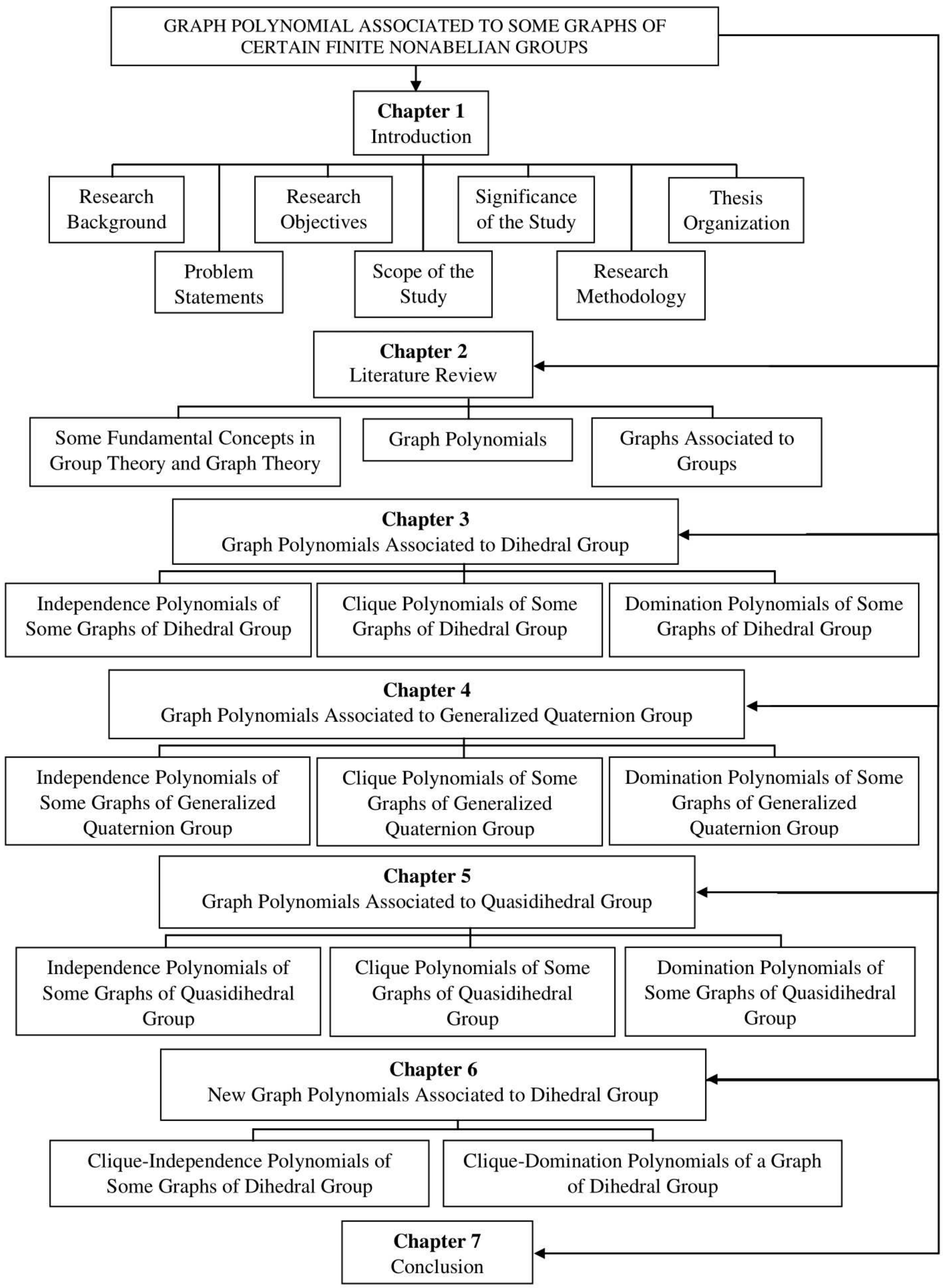

Figure 1.2 Thesis organization

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