GRAPH OF PSEUDO DEGREE ZERO GENERATED BY SEQUENCE OF FUZZY TOPOGRAPHIC TOPOLOGICAL MAPPING

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DEDICATION

This thesis is especially dedicated to my mother and father.

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ABSTRACT

Fuzzy topological topographic mapping ($FTTM$) is a set of topological spaces and algorithms used in solving neuro magnetic inverse problem. The original $FTTM$ consists of four topological spaces and three algorithms. Previous study has led to a new version, namely $FTTM$ 2 which has been used to transform magnetic data to image data. It has been known that $FTTMs$ have special properties, namely the homeomorphism between its components as well as homeomorphism between components of different $FTTM$ versions. Through this property new $FTTM$ can be generated by combining $FTTM$ components of different versions which can be depicted in a form of graphs. A special type of $FTTM$ graph called $FTTM$ graph of pseudo degree zero can be obtained by generating an $FTTM$ where each adjacent components belong to different versions. Furthermore, $FTTM$ can be generalized to include n number of components and k number of versions in a sequence of $*FTTM_n^k$. Investigation by previous research has yielded a conjecture on $*FTTM_n^3$, named the Elsafi's Conjecture. This study focuses on the number of $FTTM$ graph of pseudo degree zero generated from the sequence $*FTTM_n^k$, specifically by $*FTTM_n^3$. This study proves the Elsafi's Conjecture analytically and it generalizes the conjecture to include k number of components. A simulation is developed to determine the number of $FTTM$ graph of pseudo degree zero. The algorithm is implemented using the C++ programming language and further improved by parallelizing the algorithm. This implementation yields two new conjectures. Further, a novel concept to depict $FTTM$ namely $FTTM$ grid is introduced for sequence of $FTTM$ and $FTTM$ path. Using this newly concept, the Elsafi's Conjecture is finally proven.

ABSTRAK

Pemetaan topografi topologi kabur ($FTTM$) adalah satu set ruang topologi dan algoritma yang digunakan untuk menyelesaikan masalah songsang neuro magnetik. $FTTM$ asal terdiri daripada empat ruang topologi dan tiga algoritma. Kajian sebelum telah menghasilkan $FTTM$ versi baru, iaitu $FTTM$ 2 yang digunakan untuk mengubah data magnetik ke data imej. Telah diketahui bahawa $FTTM$ mempunyai beberapa ciri khas, iaitu homeomorphisma antara komponennya begitu juga homeomorphisma antara komponen versi $FTTM$ yang berbeza. $FTTM$ baru boleh dihasilkan dengan menggabungkan komponen $FTTM$ versi berbeza, ianya boleh digambarkan dalam bentuk graf. Jenis graf $FTTM$ khas yang disebut graf $FTTM$ dengan darjah sifar boleh diperolehi dengan menghasilkan $FTTM$ di mana setiap komponen bersebelahan tergolong dalam versi yang berlainan. Lagi pula, $FTTM$ boleh diperluaskan untuk memasukan n jumlah komponen dan k nombor versi dalam sebuah urutan $* FTTM_n^k$. Penyiasatan oleh penyelidikan sebelumnya telah menghasilkan ramalan mengenai $* FTTM_n^3$ yang dinamakan konjektur Elsafi. Penyelidikan ini berfokus kepada jumlah graf $FTTM$ dengan darjah sifar yang boleh diperolehi daripada $* FTTM_n^k$, khususnya daripada $* FTTM_n^3$. Penyelidikan ini membuktikan konjektur Elsafi secara analitik dan meluaskan konjektur Elsafi untuk menyertakan k versi. Satu simulasi dibangunkan untuk menentukan bilangan graf $FTTM$ darjah sifar. Simulasi ini dilaksanakan dengan menggunakan bahasa pengaturcaraan C ++ dan kemudiannya dipertingkatkan dengan menyusun algoritma selari. Pelaksanaan ini menghasilkan dua konjektur baru. Kemudian konsep baru untuk menggambarkan $FTTM$ iaitu grid $FTTM$ diperkenalkan untuk urutan $FTTM$ dan laluan $FTTM$. Dengan menggunakan konsep baru ini, konjektur Elsafi's akhirnya dibuktikan.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MEG	-	Magnetoencephalography
FRG	-	Fuzzy Research Group
FTTM	-	Fuzzy Topographic Topological Mapping
MC	-	Magnetic Contour Plane
BM	-	Base Magnetic Plane
FM	-	Fuzzy Magnetic Field
TM	-	Topographic Magnetic Field
AEDs	-	Antiepileptic Drugs
UTM	-	Universiti Teknologi Malaysia
SQUID	-	Superconducting Quantum Interference Device
fMRI	-	functional magnetic resonance imaging
EEG	-	electroencephalography

LIST OF SYMBOLS

$x \oplus y$	-	Combine operation between $FTTM$ blocks
$d(x)$	-	Degree of vertex
E^2	-	Ellipsoid
$\mu_{B_{z(x,y)}}$	-	Fuzzy magnetic reading
$B_{z(x,y)}$	-	Magnetic field reading on (x, y)
$B_{Z\ Max}$	-	Maximum possible value of magnetic field reading
$B_{Z\ Min}$	-	Minimum possible value of magnetic field reading
$\lvert G_0(*FTTM_n^k)\rvert$	-	Number of pseudo degree zero generated from $FTTM_n^k$
$*FTTM_n^k$	-	Sequence of k fuzzy topographic topological mapping with n components
$E(G)$	-	Set of edges from graph G
$B\left(G(*FTTM_n^k)\right)$	-	Set of FTTM blocks
$\underset{m_j=i}{\mathrm{G}}(*FTTM_n^k)$	-	Set of FTTM with certain index
$G_d(*FTTM_n^k)$	-	Set of FTTM with diagonal paths
$G_0(*FTTM_n^k)$	-	Set of graph of pseudo degree zero generated from $FTTM_n^k$
S^2	-	Sphere

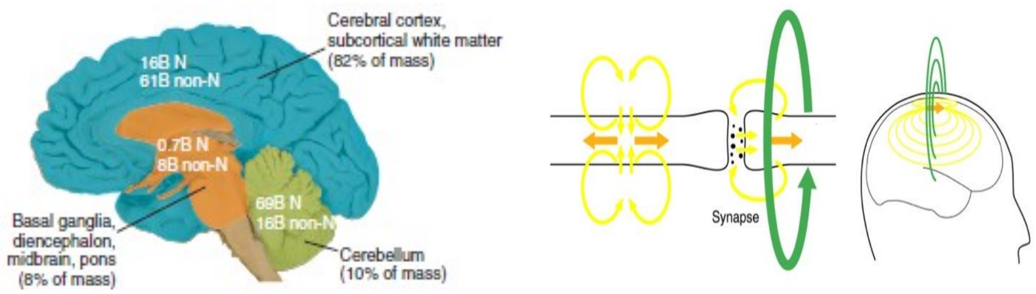
LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.0 Introduction

An average person has at least one hundred billion neurons on his/her entire body. The majority of them are located in the brain. The brain is the most essential and complex organ of the human body. The neurons communicate with other neurons by electrical and chemical signals. When a brain processes information, a small electrical current flows between neurons [1]. By the electromagnetic law, the electrical current then produces a miniscule amount of magnetic field which can be detected by Magnetoencephalography (MEG) [2].



(a) (b)

Figure 1.1: (a) The percentage of neuron cells from each region in a human brain (b) magnetic field caused by neuron activity [1]

In an epileptic patient, the brain produces an unnatural electric signal caused by an imbalance between cerebral excitability and inhibition [3]. This abnormality can alter perception, consciousness, and motor activity [4]. The brain tissue that creates this unnatural signal is called an epileptic foci. Surgical operation is one of the treatment that is opted for some certain types of epilepsy [5]. In order for this treatment to have a higher chance of success, surgeon needs to locate the exact location of the epileptic foci [6].

Most of the time, an epileptic foci is detected through the magnetic signal generated by the brain. This is called a neuro magnetic inverse problem. One of the way of solving the neuro magnetic inverse problem is by a statistical approach [7]. Another approach is to use Fuzzy Topographic Topological Mapping ($FTTM$) which is introduced by Fuzzy Research Group (FRG) at UTM in 1999.

The $FTTM$ consists of four components and three algorithms. The components are: Magnetic Contour plane (MC), Base Magnetic plane (BM), Fuzzy Magnetic field (FM) and Topographic Magnetic field (TM) [8]. The $FTTM$ version 1 was developed to present a 3-D view of an unbounded single current source [9] and $FTTM$ version 2 was developed to present a 3-D view of a bounded multi current sources in four observation angles.

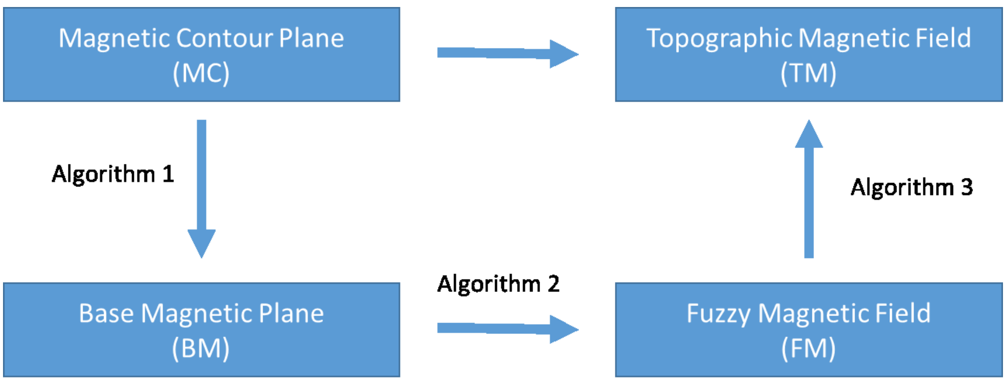


Figure 1.2: Components of $FTTM$

1.1 Background of the Research

From previous studies, it is proven that $FTTM$ contains homeomorphic components i.e. $MC \cong BM \cong FM \cong TM$ [10,11]. The homeomorphisms enable us to generate new elements of $FTTM$. These new $FTTM$s consist of components from different versions of $FTTM$.

An $FTTM$ with n number of component is denoted as $FTTM_n$, where n is the number of components. Different versions of $FTTM_n$ can be arranged into a set called a sequence of $FTTM$, denoted by $*FTTM_n^k$ where k is the number of version. It can be illustrated as a graph in the form of a sequence of n sided polygon, where each polygon represents a version of $FTTM_n$ [11]. A vertex of the polygon represents a component of $FTTM_n$, and an edge represents the homeomorphism between the components of $FTTM_n$ (Figure 1.3) [12]. A generated $FTTM$ can be represented as a graph. This representation is plotted on the skeleton of $*FTTM_n^k$, where the component of the generated $FTTM$ is represented as a vertex on the sequence of polygon. Edges are added which connect subsequent components of the same version [13].

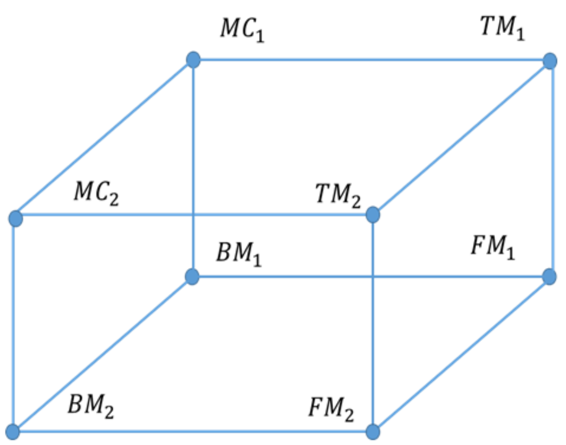


Figure 1.3: Graph of $FTTM_1$ and $FTTM_2$ as a sequence of polygon

This study focuses on the properties of these generated $FTTM$ graphs, specifically generated $FTTM$ that have a graph with pseudo degree zero. A graph of pseudo degree zero is a graph with isolated vertices without any edge that connects its components [14]. The set of graphs of pseudo degree zero generated by $*FTTM_n^k$ is denoted by $G_0(*FTTM_n^k)$.

1.2 Problem Statement

Previous study examined the number of graphs of pseudo degree zero generated from $*FTTM_n^3$ [14]. From the result of the previous study, a conjecture has been deduced. The conjecture is named as Elsafi's conjecture. The conjecture states that the number of graph of pseudo degree zero generated by $*FTTM_n^3$ is given as follows:

Conjecture 1.1 (Elsafi's Conjecture)

$$|G_0(*FTTM_n^3)| = \begin{cases} 4|G_0(*FTTM_{n-2}^3)| + 12\ , \text{ where } n \text{ is even} \\ 4|G_0(*FTTM_{n-2}^3)| + \quad 6, \text{ where } n \text{ is odd} \end{cases}$$

where $n \geq 3$, $|G_0(*FTTM_3^3)| = 6, |G_0(*FTTM_3^3)| = 12$

Since the calculation of $|G_0(*FTTM_n^3)|$ in the previous study is done manually, there are rooms for error and the conjecture may not hold in larger number. Thus an algorithm needs to be developed to produce $|G_0(*FTTM_n^3)|$ systematically. The result of the algorithm is then compared to Conjecture 1.1. If the conjecture holds then an analytical proof needs to be produce.

1.3 Research Questions

Here are some of the questions that needs to be answered.

(i) Is there a way to determine the number of graphs of pseudo degree zero?

(ii) How many graphs of pseudo degree zero that can be generated from $*FTTM_n^3$?

(iii) Can the conjecture be proven analytically?

(iv) How to describe the number of graphs of pseudo degree zero in general?

1.4 Research Objectives

The objectives of the research are to:

(i) Develop a programing code to calculate the number of graph of pseudo degree zero.

(ii) Prove the conjecture as stated in the problem statement analytically.

1.5 Scope of Research

This research focuses on the structure of graph that arises from the homeomorphisms of the $FTTM$. The developed procedure leads to $*FTTM_n^k$ in general. However, the analytical proof is only supplied for relation between the number of graph of pseudo degree zero generated for $*FTTM_n^3$.

1.6 Importance of Research

This research investigates the mathematical properties of graph of pseudo degree zero generated from $*FTTM_n^3$. An algorithm which can compute the number of $FTTM$ is coded. The result of the algorithm can gave an insight to the relations and mathematical characteristics of $FTTM$ graph of pseudo degree zero. Furthermore, in the process of investigating the structure of graph of pseudo degree zero some new definitions on the structure of $FTTM$ is introduced. Lastly, a new method of proving the conjecture is introduced. The novel method incorporates paths in $FTTM$ graph.

1.7 Thesis Outline

This thesis consists of six chapters. Chapter 1 is the introduction of the research. It discusses the background, problem statement, research questions, objectives of the research, scope of the research and the importance of the research. Next, in Chapter 2 the literature review on epilepsy, MEG, and $FTTM$ along with its generalization, namely $FTTM$ as a sequence of polygon, and some basic concepts of graph is included in this chapter. Chapter 3 is on the development of algorithm for $FTTM$ program and its results. Chapter 4 introduces some new definitions which relate to $FTTM$, namely representation of $FTTM$ as a grid, subsets of $FTTM$ and operations between these subsets. The proof to the conjecture is presented in Chapter 5. Finally, Chapter 6 concludes the research and suggestion for future works.

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