APPLICATION OF CAPUTO FRACTIONAL DERIVATIVES TO THE THERMAL RADIATIVE CONVECTIVE CASSON FLOW IN A MICROCHANNEL

MARJAN BINTI MOHD DAUD

UNIVERSITI TEKNOLOGI MALAYSIA

APPLICATION OF CAPUTO FRACTIONAL DERIVATIVES TO THE THERMAL RADIATIVE CONVECTIVE CASSON FLOW IN A MICROCHANNEL

MARJAN BINTI MOHD DAUD

A thesis submitted in fulfilment of the
requirements for the award of the degree of
Master of Philosophy

Faculty of Science
Universiti Teknologi Malaysia

JULY 2022

ACKNOWLEDGEMENT

In the name of Allah, the Entirely Merciful, the Especially Merciful.

First and foremost, praises and thanks to Allah S.W.T, The Almighty, for His showers of blessings, the strength, and good of health upon me to complete this thesis successfully.

I would like to express my deep and sincere gratitude to my respected supervisor, Assoc. Prof. Dr. Sharidan Shafie for providing invaluable guidance throughout this research. With his constant help and advice has help me to be where am I today. Also, to my external co-supervisor, Dr Rahimah Mahat, it was a great privilege and honour to work and study with her guidance.

Not least of all, I am extremely grateful to my parents and family for their endless support and prayers. Their encouragement is indeed useful. I would also like to extend my sincere appreciation to my friends who always giving ideas, tips, and emotional support. It is a gift to have these lovely people in the research in some way, whether directly or indirectly. I will never be able to express how grateful I am for everything they have done.

ABSTRACT

The application of fractional derivative is currently convenient and anticipated in the industrial and technological fields due to its unique properties. Therefore, the goal of this research is to learn more about the characteristics of the Caputo fractional derivative, which is one of the most often used fractional derivative operators. Additionally, microchannels exist in many industries and engineering process equipment, and their geometrical structure is one of the most important factors influencing fluid flow. Therefore, in this thesis, the Casson fluid behavior flowing in three different forms of microchannel which are static, accelerated, and oscillating is investigated. The effect of thermal radiation on the Casson fluid is also considered. The formulation of the governing equation for the problems is thoroughly discussed. First, the partial differential equations and boundary conditions are transformed into dimensionless equations by using appropriate dimensionless variables. Second, the resultant dimensionless governing equations are transformed into fractional form by using Caputo fractional derivatives. The equations are then reduced to linear ordinary differential equations by using the Laplace transform technique and solved by using appropriate methods. Finally, the numerical solution is obtained by using the inverse Laplace transform technique with the help of Zakian's explicit formula approach. The result of velocity and temperature profiles are plotted by using Mathcad software. The obtained solutions are reduced to the published results for such problem for verification and accuracy, and have achieved excellent agreement. The influence of key physical parameters on the velocity and temperature profiles is analyzed and discussed in depth. The results reveal that as the fractional and radiation parameters are increased, the velocity and temperature profiles for all three geometries of the microchannel increase. On the contrary, high Prandtl numbers have increased the viscous force, resulting in a reduction in both profiles. Since the Grashof number has a positive influence on the buoyancy force, it has caused the velocity profile to increase. Meanwhile, the velocity profile reveals a contrasting pattern, with the Casson fluid parameter increasing due to increased viscous forces compared to thermal forces.

ABSTRAK

Penggunaan terbitan pecahan pada masa ini mudah dan dijangkakan pengunaannya dalam bidang perindustrian dan teknologi kerana sifatnya yang unik. Oleh itu, matlamat penyelidikan ini adalah untuk meneroka lebih lanjut tentang ciri-ciri terbitan pecahan Caputo iaitu salah satu operator terbitan pecahan yang paling banyak digunakan. Tambahan pula, mikrosalur banyak terdapat dalam peralatan proses di industri dan kejuruteraan, dan struktur geometri mikrosalur merupakan salah satu faktor paling kritikal yang mempengaruhi aliran bendalir. Oleh itu, dalam tesis ini, kelakuan bendalir Casson yang mengalir dalam tiga jenis mikrosalur iaitu statik, dipercepat, dan berayun diselidiki. Kesan sinaran terma ke atas bendalir Casson juga dipertimbangkan. Formulasi persamaan menakluk bagi semua masalah dibincangkan secara terperinci. Pertama, persamaan terbitan separa dan semua syarat sempadan diubah menjadi persamaan tak berdimensi, dengan menggunakan pembolehubah tak berdimensi yang sesuai. Kedua, persamaan menakluk tak berdimensi yang diperoleh diubah menjadi bentuk pecahan, dengan menggunakan terbitan pecahan Caputo. Persamaan tersebut kemudiannya diturunkan ke persamaan terbitan biasa linear dengan menggunakan teknik penjelmaan Laplace dan diselesaikan dengan menggunakan kaedah yang bersesuaian. Akhirnya, penyelesaian berangka diperoleh menggunakan teknik penjelmaan Laplace songsang dengan bantuan pendekatan formula eksplisit Zakian. Keputusan profil halaju dan suhu diplot dengan menggunakan perisian Mathcad. Penyelesaian yang diperoleh diturunkan kepada keputusan yang telah diterbitkan bagi setiap masalah tersebut bagi tujuan pengesahan dan ketepatan, dan telah memperoleh persetujuan yang baik. Pengaruh parameter fizikal ke atas profil halaju berserta suhu dianalisis dan dibincangkan secara terperinci. Keputusan menunjukkan bahawa dengan peningkatan parameter pecahan dan sinaran, profil halaju dan suhu meningkat untuk ketiga-tiga geometri mikrosalur. Sebaliknya, nombor Prandtl yang tinggi telah meningkatkan daya likat, mengakibatkan kedua-dua profil menurun. Oleh kerana nombor Grashof mempunyai pengaruh positif ke atas daya keapungan, maka profil halaju telah meningkat. Sementara itu, profil halaju menunjukkan keadaan yang bertentangan apabila meningkatnya parameter bendalir Casson yang disebabkan oleh peningkatan daya likat berbanding dengan daya terma.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamic
ODE	-	Ordinary Differential Equations
PDE	-	Partial Differential Equations

LIST OF SYMBOLS

A	-	Acceleration of plate
C_p	-	Specific heat at constant pressure
d	-	Distance between two plates
$D_t^{\alpha}(..)$	-	Time fractional derivative of order α
e	-	Internal energy
g	-	Gravitational acceleration
Gr	-	Grashof number
$H(t)$	-	Heaviside function
$\mathbf{i}$	-	Cartesian unit vector in $x-$ axis direction
$\mathbf{j}$	-	Cartesian unit vector in $y-$ axis direction
$\mathbf{k}$	-	Cartesian unit vector in $z-$ axis direction
k	-	Thermal conductivity
k_1	-	Mean absorption coefficient
$\mathcal{L}$	-	Laplace transform
p	-	Pressure
p_d	-	Dynamic pressure
p_h	-	Hydrostatic pressure
p_y	-	Yield stress
Pr	-	Prandtl number
q	-	Laplace transform parameter
q_r	-	Radiative heat flux
$\mathbf{q}''$	-	Heat conduction per unit area
R	-	Radiation parameter
t	-	Time
T	-	Temperature
T_w	-	Wall temperature
T_o	-	Ambient temperature
u	-	Velocity in $x-$ axis direction
U	-	Amplitude of the plate oscillations

v	-	Dimensionless velocity
V^2	-	Magnitude of velocity
x	-	Dimensional coordinate axis along the plate
y	-	Dimensional coordinate axis normal to the plate

Greek Letters

α	-	Fractional parameter
β	-	Casson fluid parameter
β_T	-	Volumetric coefficient of thermal expansion
β_o	-	Dimensionless Casson fluid parameter
∇	-	Vector operator Del
γ	-	Dimensionless acceleration of plate
μ	-	Dynamic viscosity
μ_B	-	Plastic dynamic viscosity
τ	-	Shear stress
π	-	Product of deformation rate with itself
π_c	-	Critical value of product
ρ	-	Fluid density
$\rho\mathbf{b}$	-	Body force
$\overline{\theta}$	-	Dimensionless temperature
υ	-	Kinematic viscosity
ξ	-	Dimensionless coordinate axis normal to the plate
$\Omega\tau$	-	Phase angle
σ	-	Stefan-Boltzmann constant
ω	-	Frequency of oscillation

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Introduction

In this chapter, research background, problem statement, research objectives, scope of study as well as the significance of study is presented. The research background briefly describes the study of fractional derivative which is the Caputo fractional derivative operator. The discussion is focused on the convective flow of Casson fluid in a microchannel with the presence of thermal radiation. The problem statement provides some questions on the mathematical modelling, analytical solutions using Laplace transform method and the effects of pertinent parameters. The research objectives have fulfilled the problem statement together with scope and significance of the study.

1.2 Research Background

Generally, the motion in the nature such as fluid flow, heat transfer, wave of sound and others can be mathematically described by using the partial differential equation (PDE). Stated by Strauss (2007), PDE is an equation that consist of more than one independent variable such as x, y, and z. For example, Farlow (1993) declared that the temperature $T(x,t)$ is dependent on two variables which are x (location) and t (time). Moreover, Renardy and Rogers (2004) have mentioned that partial differential equation (PDE) is more advanced as compared to ordinary differential equations or theory of functions of a single complex variable.

PDE is widely use in various area such as engineering, physics, and stochastic problems, since natural phenomena including velocity and acceleration are normally described in derivatives (Farlow, 1993). Besides, PDEs are often used by scientist and engineers to explore a wide range of physical phenomenon including fluid dynamics, electricity, magnetic fields, and thermal transfer (Farlow, 1993). The Maxwell's equations, the Navier-Stokes equations and Newton's equations of motion are some examples of physicist's natural law that is stated in terms of PDE. There are a few PDE that are well-known and commonly used without being notice. Some of the PDE are as shown in Table 1.1.

Table 1.1: Examples of commonly used PDE

$u_t = u_{xx}$	Heat equation in one dimension
$u_t = u_{xx} + u_{yy}$	Heat equation in two dimensions
$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$	Laplace's equation in polar coordinates
$u_t = u_{xx} + u_{yy} + u_{zz}$	Wave equation in three dimensions

According to Causon and Mingham (2010), PDE can be existed in several dimensions depending to the number of independent spatial variables it contains as represented on the Table 1.1. However, the most common dimensions that PDE ever use is in 2D and 3D. They also stated that second order linear PDE are classified into 3 generic types which are elliptic, parabolic and hyperbolic. The importance of PDE can be seen to describe the conservation laws in fluid dynamic since the flow variables rely on other independent variables. The partial derivative of a multivariable function is its derivative with respect to one of the variables while the rest are held constant. The symbol of partial derivative is represented as ∂. Researchers are always curious on the usage of fluid dynamics due to its development in sciences and the utilization of it's as a critical tool in numerous industrial and technological applications for centuries. Fluid dynamics has evolved from a well-developed research field to a widely practical topic with a greater scope.

Fluids can be categorized in two main classes which are known as Newtonian fluids and non-Newtonian fluids. A Newtonian fluid is one in which the stress versus strain rate curve shows a linear trend and passes through origin. Viscosity is the result of having a constant proportionality. Hence Newtonian fluid is said to be the simplest mathematical model of fluid. Historically, Isaac Newton was the founder to the fluid hence it is named after him who first used the differential equation to hypothesize the interaction between shear stress and shear rate. This fluid has a linear relationship which obey the Newton's law of viscosity,

$$\tau_s = \mu \frac{du}{dy}, \tag{1.1}$$

where the τ_s denoted the shear stress in the fluid, the shear viscosity of the fluid, μ, is a scalar constant of proportionality and u is the velocity in the direction of $y-$ axis. There are a few examples of Newtonian fluids such as water and oil. These are the examples over the range of shear stress and shear rate that people experience in daily life. Whereas benzene, glycerine and alcohol are examples of Newtonian fluids that are commonly found in scientific lab. Stated by Khan *et. al*. (2014) many researchers have been including shear stress as the boundary related to fluid mechanics.

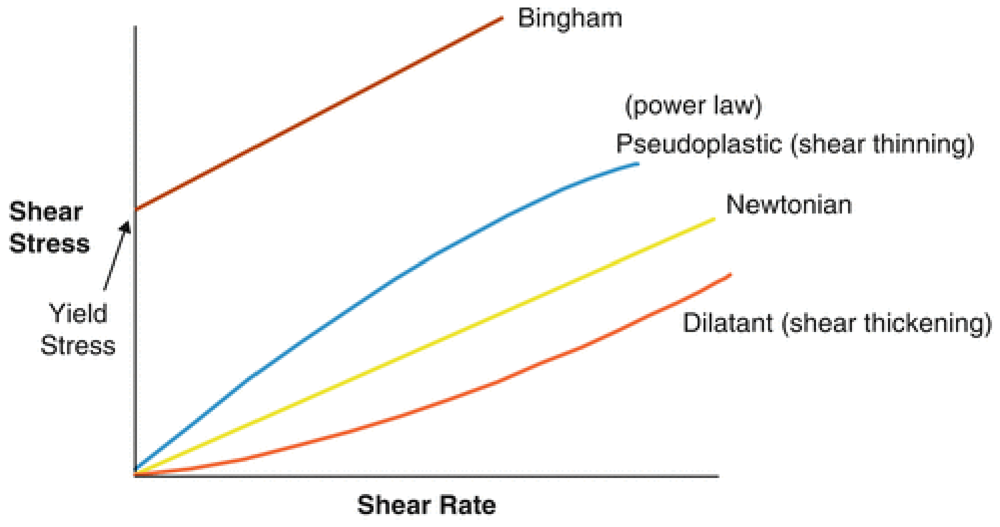


Figure 1.1: Shear stress versus shear rate of Newtonian fluids

Figure 1.1 is retrieved from George and Qureshi, 2013. Based on Figure 1, it shows the plot of shear stress versus shear rate shows Newtonian offer a linear increase in stress as shear rates increase. Meanwhile the viscosity of the fluid is represented by the slope. Hence, it means that their viscosity will remains constant all the time as no matter how quick the Newtonian fluids are pressured to flow through such a pipe or channel.

Meanwhile, non-Newtonian is a vice versa of Newtonian fluid. Non-Newtonian fluid does not obey the Newton's Law hence the shear stress is not necessarily proportional to the shear rate. Referring to Figure 1.2 (UKEssays, 2018), the viscosity of non-Newtonian fluid is depending on shear rate either the shear will be thickening or thinning. Shear thinning fluid is known as pseudoplastic fluid whereas shear thickening fluid is recognized as dilatant fluid. The contrary of non-Newtonian fluid is the fluid shows either a non-linear relation relationship between shear stress and shear rate, as well as yield stress or a viscosity that is time or deformation dependent. If the viscosity of the fluid increase, the shear rate increases hence the fluid is called as shear thickening. A mixture of a corn starch and water is the examples of shear thickening fluids whereas the example of shear thinning fluids is wall paint.

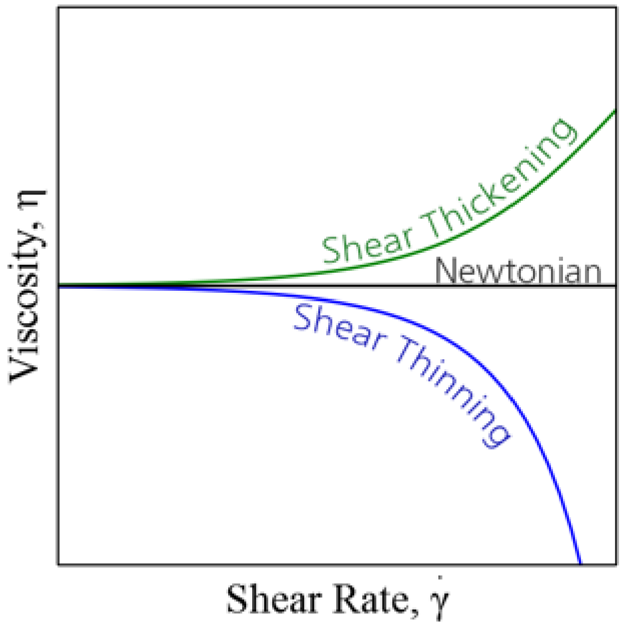


Figure 1.2: Viscosity versus shear rate of non-Newtonian fluids

Moreover, many researchers agreed that the governing equation for non-Newtonian fluid is much non-linear and complicated compared to Newtonian fluid. The statement is acknowledged by Khalid *et. al.* (2015) where the governing equations of non-Newtonian fluids manipulate highly non-linear differential equations that are normally difficult to solve. Non-Newtonian fluid are commonly used in industry such as in printing. As non-Newtonian fluid is more viscous compared to Newtonian fluid, it can enhance the sharpness and the quality of the printing. The shear thinning contributes to the properties of the printing ink. Therefore, to thicken the viscosity of the ink, this is where non-Newtonian will help as a thickening agent. Other than in printing industry, Hussanan *et. al.* (2013) also mentions that non-Newtonian fluids are used in cosmetics, pharmaceutical and food industries. Also supported by Khalid *et. al.* (2015) applications of non-Newtonian fluids are broad including in geophysics and petroleum industries.

The most common type of non-Newtonian fluid used by researchers is Casson fluid (Arthur *et. al.* (2015) and Oke *et. al.* (2020)). Basically, Casson fluid is a fluid that behaved as a shear thinning liquid. The characteristics of Casson fluid are, firstly, at zero rate of shear, it is simulated to have an infinite viscosity. Secondly, a yield stress below which there is no flow. Lastly, it has a zero viscosity at an infinite rate of shear. If lesser shear stress compared to yield stress is applied to the fluid, Casson fluid will behaves like a solid. While on the other hand, Casson fluid will be moving when greater shear stress compared to the yield stress is applied. There are several examples of Casson fluid that can be seen in real-life such as ketchup, honey, and even human blood.

Casson fluids are extensively studies theoretically employing classical or ordinary PDE. However, it is widely acknowledged that fractional PDE’s are more efficient for accurately illustrating physical phenomena. Dalir and Bashour (2010) mentioned that heat transfer, biology, physics, chemistry, quantum mechanics, viscoelastic material, rheology, fluid flow, diffusive transport, probability, electrical networks and electromagnetic theory are among the fields where time fractional model has grabbed a growing focus.

Based on Khalil *et. al.* (2014), fractional derivative has been existed a long time ago, and even as old as calculus. Fractional derivative has erupted after a conversation between L’Hospital and Leibniz take place. Since then, definition for fractional derivative has been tried to put by many researchers and most of them used an integral form to express fractional derivative. Fractional calculus is a division of mathematics that extend the concept of classical integral and derivatives of integer order to non-integer order integral and derivatives. The application of fractional derivatives has provided more general and accurate models of a system compared to the traditional calculus. Ray *et. al.* (2014) has mentioned the vast range of fields on the usage of fractional derivatives in science as well as engineering fields. In example for accurate modelling of systems that require correct damping modelling, the fractional derivative models are used. Furthermore, fractional order PDEs control most physical processes in electricity, quantum physics and other models within their scope of validity. Consequently, to solve fractional order PDEs are important for researchers to know all the traditional and the develop methods.

The most popular and frequently used of fractional derivative operators are Riemann-Liouville fractional derivative as well as Caputo fractional derivative. The Riemann-Liouville and Caputo fractional operators have been extensively employed in a variety discipline of science and technology (Metzler and Klafter, 2000). However, the development of fractional derivatives has brought it to a various definition according to its type. Each fractional derivative operator has development in its features such as the kernel features, singularity, locality, and the benefits of the implementation. Riemann-Liouville fractional derivative is said to have an unusual initial condition. In addition, the derivative of a constant is not zero. Therefore, the existence of Caputo fractional derivative operator solves the problem of this unusual initial condition that has no physical meaning and is difficult to compute.

According to Luchko and Gorenflo (1999), Caputo was the first to realise that the Laplace transform could be used to create a fractional derivative operator from the convolution product: the convolution of the function’s classical derivative with the power Law kernel, which is given by

$$D_t^{\alpha} f(y,t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \displaystyle\int_0^t \dfrac{1}{(t-\tau)^{\alpha}} \dfrac{\partial f(y,\tau)}{\partial \tau} \partial\tau; 0 < \alpha < 1, \\ \dfrac{\partial f(y,t)}{\partial t}; \alpha = 1, \end{cases} \tag{1.2}$$

where the Laplace transform of equation (1.2) is as follows

$$\mathcal{L}\left\{D_{\tau}^{\alpha} f(y,t)\right\}(q) = q^{\alpha} \overline{f}(y,q) - f(y,0). \tag{1.3}$$

From equation (1.2), $D_t^{\alpha}(.)$ represents the Caputo fractional derivative, α depicts the fractional order, q is the Laplace transform variable, $\overline{f}(y,q)$ illustrate the Laplace transform of function $f(y,t)$ and the usual initial conditions is represented by $f(y,0)$. Kernel of Caputo fractional derivative is singular. The result acquired from equation (1.2), shows that the power law kernel's singularity vanishes. With the existence of Caputo fractional derivative operator, the flaws of the Riemann-Liouville fractional derivative were fixe. It was competently implemented in a variety of fields of science and technology. As mentioned by Haque *et. al.* (2018), application of Caputo fractional derivative can be seen as such in glass blowing, biometric foods and metallic plate cooling.

Many researchers have demonstrated the importance and effectiveness of fractional derivatives in engineering applications through their research and studies. From previous research conducted by Song and Jiang (1998), they empirically confirmed that the fractional Jeffrey model is suitable for illustrating the behaviour of Sesbania gel with xanthan gum. In 2007, Jumarie has investigate PDE using fractional derivative and solve using modified Riemann-Liouville derivative. Recently, Abro *et. al.* (2019) analyses fractional derivate approach to solve PDE problem. Then, Abro and Aguilar (2019) also have discussed on fractional derivative approach to solve Walter's-B fluid. The fractional Voigt model is demonstrated by Meral *et. al.* (2010) in simulating the wave response of soft tissue, such as phantoms. Meanwhile heat transmission in biological tissues can be consistently evaluated using fractional wave models, according to Jiang and Qi (2012). According to Chen *et. al.* (2013), the data produced from fractional derivative models had a significantly better agreement with experimental data as compared to the classical model.

For fractional differential equations, a broad class of initial value problems is solved by applying the Laplace transform technique. A French mathematician, Pierre Simon de Laplace has introduced the Laplace transform. It is widely used since its main purpose is to solve differential equations easier as it provides a structured alternative approach. Moreover, Kexue and Jigen (2011) has investigate the rationality of solving fractional differential equations by using Laplace transform approach. The Laplace transform is very efficient in solving linear ODEs that usually occurs in the study of electronic circuits and control systems. While the Laplace inverse is used to transfer any variable domain back to its basic domain in a fractional differential equation.

The application of fluid problem in micro devices is currently in attention by many researchers due to its importance in real-life problems. Due to its deep mathematical significance and wide variety of applications in biological research, industries, and engineering, convection heat transfer of fluid flow in a microchannel are of immense interest in fluid dynamics. The analysis of microchannel flow problems to investigate trends and properties of embedded flow parameters in microchannel flows has mathematical impact. Khan *et. al.* (2018) agrees that many researchers has gain interest to study on the fluid flow in microchannel since its practical uses include in space technology, engineering, and material processing operations as well as in high power density processors in supercomputers and many other devices. In engineering context, microfluidics is flows of fluid and gases in single or multiple phases through microdevices fabricated by Micro Electro Mechanical Systems (MEMS) technology (Tabeling, 2001). Stated by Gad-el-Hak (1999), MEMS devices that involves fluid flows are such as microducts, micropumps, microturbines and microvalves.

Heat convection is the transport of heat from one spot to another. It can either be free, force or even mixed convection. According to Ullah *et. al.* (2017), non-

Newtonian fluids have poor thermophysical properties by nature and are incapable of transferring the desired quantity of heat in a heat transport system. In heat transport systems, the convection heat transmission has become a tricky problem for engineers and industrialist. Therefore, there was a sense of urgency in dealing with this issue. Changes in boundary conditions, flow geometry or by enhancing the thermal conductivity of the flowing fluid can all improve heat transfer in a convective flow of fluid. Thermal radiation is widely use in the application of industries. It is the emission of electromagnetic waves from any type of matter that has temperature greater than absolute zero. In other words, when there is a thermal motion of particles present in matter, the thermal radiation is produced. In addition, whenever there is particle motion also, it will activate charge-acceleration or dipole oscillation which will eventually results in an electromagnetic radiation. As an example, electric heater emits an infrared radiation which will help a person to feel a radiated heat of the fire even the surrounding air is very cold. Obeying the Kirchhoff's radiation law, objects that are good emitters are also nominated as good absorbers, such as a black surface which is an excellent emitter, hence it will absorb excellently. In contrast with black surface, silver surface is a poor emitter and poor absorber.

In the present research, the application of Caputo fractional derivative for Casson fluid is studied. The geometry of this research is a vertical microchannel. The result from this study can be used to improve industrial thermal conductivity activities. Furthermore, the application of Caputo fractional derivative itself is recognized in solving the accurate description of physical phenomena. The involution product of fractional derivative and power-law function has successfully made up a Caputo fractional derivative operator. Hence, the issue of an unusual initial condition with no physical meaning that had been difficult to compute was resolved. Bearing in mind the importance of fractional derivatives, very few studies have been reported on non-Newtonian fluid flowing in microchannel with fractional derivative approach. Khan *et. al.* (2017) discussed on the application of Caputo-Fabrizio fractional derivative to the convective flow of Casson fluid in a microchannel. Whereas the application of Caputo-Fabrizio and Atangana-Balenau fractional derivative to a rotational second grade fluid is discussed by Abro *et. al.* (2020). To fill in the research gap, this study also adds physical parameter which is thermal radiation to enhance the knowledge on thermal radiation effect on convective flow of Casson fluid with the application of Caputo fractional derivative.

1.3 Problem Statement

Non-Newtonian fluid is a type of fluid that is important for researchers and engineers. Furthermore, non-Newtonian fluid currently have gained much attention by them due to its potential in industrial and technological processes, where it can help testing on automotive heat industries as well as heat changers situation. Non-Newtonian fluids such as Casson fluid is normally used by researchers to describe the blood flow due to its characteristics. Previous researchers have discussed that fluid behave differently in micro and macro scale. Bearing in mind, that most of arteries in human body are in micro scale. So, studying the behaviour of fluid flow in microchannel should be given an attention. In addition, limited research has been noticed on usage of fractional derivative on unsteady fluid flow problem in microchannel. Furthermore, since most of real-life plate is not in static mode, oscillating movement of the plate is appropriate to consider. This situation will become more realize if heat transfer in an oscillating microchannel is studied together with the influence of thermal radiation. Thus, to fill this research gap, present study on solving equation of Casson fluid flow in an oscillating microchannel with thermal radiation approached by Caputo fractional derivative. Therefore, to understand the fluid field in this study the following questions are explored:

1. How to model mathematically the Caputo fractional Casson fluid flow over a static, an accelerating and an oscillating vertical microchannels?
2. How do the fractional mathematical models behave in the problem involving thermal radiation?
3. How does the Laplace transform technique be applied to obtain the analytical solutions of the governing equations?

4. How can approximation solutions of fractional mathematical model solutions be obtained in different geometries?

5. How does the pertinent parameter such as fractional parameter, and radiation parameter affect the velocity and temperature profiles?

1.4 Objectives

The purpose of this research is to analyse the convection flow of Casson fluid over a static, an accelerating and an oscillating vertical microchannels. Laplace transform method is applied to acquire the exact solutions. Then to validate the solution, graphical illustrations from published results are used in comparing with the graphical outcomes of the obtained solutions. Specifically, the objectives of this study are:

1. To formulate the Caputo fractional derivative governing equations for the concerned fluid problem.

2. To solve the governing equations by using the Laplace transform technique to obtain the exact solution for the velocity and temperature profiles.

3. To acquire the approximation solutions by applying Zakian explicit formula approach.

4. To analyse the effect of fractional parameter applied to the fluid flow problem with thermal radiation effect.

5. To investigate the behaviour of the velocity and temperature profiles for various important physical flow parameters.

1.5 Scope of Research

This research applying Caputo fractional derivative approach on the governing equation of unsteady flow problem. The non-Newtonian fluid which is Casson fluid is considered to flow through a vertical microchannel. Three proposed problems are highlighted in this research. The first problem is emphasis on the Casson fluid flowing through a static microchannel with the presence of thermal radiation. Caputo fractional derivative operator is used to fractionalize the governing equations of momentum as well as energy equations. Next, the second problem is focusing on an accelerated microchannel for the Casson fluid to be flowing. Whereas the third problem is intensified on the geometry of the fluid where the Casson fluid flowing in an oscillating microchannel. All the proposed problems are based on the assumptions and limitations as follow:

1. The Casson type fluid model as well as energy and momentum equations are fractionalized using Caputo fractional derivative operator.

2. The thermal radiation variables are considered and solve using Taylor series where the higher order terms are neglected.

3. The Laplace transform method is used to generate analytical solutions for each of the suggested problems.

4. The vertical microchannel has a ramped wall temperature and non-homogenous boundary conditions.

5. The inverse Laplace transform is numerically obtained by employing Zakian explicit formula approach.

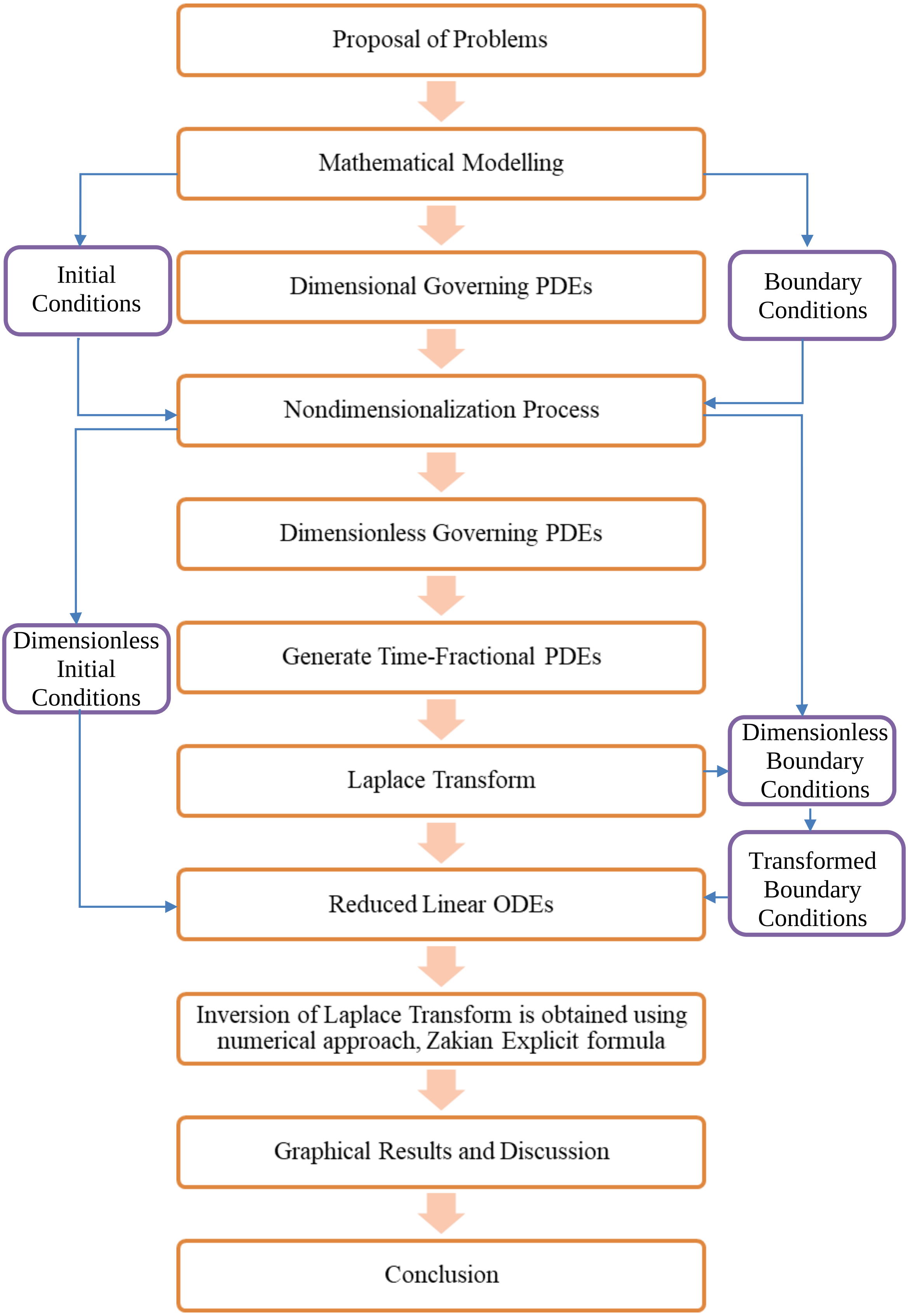
6. The approximation solutions are generated, and the results are for velocity and temperature profiles are plotted using Mathcad software.

1.6 Significance of Research

The significance of this research is listed as follows:

1. Enhance more knowledge on formulation of Caputo fractional derivatives.
2. The result of this study will improvise the knowledge of fractional derivatives in equation of motion over a static, an accelerating and an oscillating microchannels.
3. The result will help in understanding the thermal radiation behaviour in fluid flow applications.

1.7 Research Operational Framework



1.8 Thesis Outline

There are six chapters in all organized in this thesis. The research background is described in Chapter 1, which includes the general introduction, problem statements, research objectives, scope of research, research methodology, and significance of the current research. Then, in Chapter 2 contains a thorough survey of the literature on the issues raised in the study objectives. Chapter 3 discussed the first problem to be solved regarding the unsteady free convection flow of Casson fluid generated by a static microchannel with the presence of thermal radiation. The momentum equation as well as energy equation are all deduced in detail within this chapter. The governing equations of the specified problems are formulated in terms of linear partial differential equations associated to momentum and energy transformations together with specified initial and boundary conditions. The dimensional governing equations, as well as the initial and boundary conditions, are simplified using dimensionless variables. Applying the time-fractional Caputo fractional derivative, the dimensionless equations are converted into time-fractional notation. Exact solutions of the time-fractional governing equations are obtained via Laplace transform technique. As limiting cases, the general solutions discovered in this chapter are found to reduce to several well-known solutions in the literature. Finally, graphs depict the effect of significant flow parameters on velocity and temperature equations.

Chapter 4 is studied by taking into consideration the flow of Casson fluid passing through an accelerated microchannel with thermal radiation. The velocity and temperature expressions are generated. To construct the system dimensionless, certain adequate dimensionless variables are embedded into the governing expressions and initial and boundary conditions to abolish units and reduce the number of variables. Similar process as in Chapter 3 is implemented to solve the governing equations in this chapter by transformed into a time-fractional equations using Caputo fractional derivative operator definition. Graphs are used to assess the graphical outcomes for various embedded flow parameters. The acquired solutions are compared to those already published in the literature. Moreover, the result for the addition of thermal radiation to the problem, is explained graphically.

Chapter 5 investigates the unsteady free convection flow of Casson fluid over an oscillating microchannel based on the Caputo time-fractional fractional derivative with the presence of thermal radiation effect. This chapter starts with the mathematical formulation of the problem to simulate the Casson fluid's governing equation in an oscillating movement of microchannel system. The governing equations are subjected to appropriate initial and boundary conditions which are then transformed into dimensionless partial differential equations using suitable dimensionless variables. Then, the equations are solved using the Laplace transform technique. The acquired solutions are reduced to the existing solutions in the literature. The velocity and temperature results are graphically shown and analysed.

The overview of this research, as well as ideas for further research, are offered in Chapter 6. At the end of this thesis, the references are listed.

REFERENCES

Abegunrin, O. A., Okhuevbie, S. O., and Animasaun, I. L. (2016). Comparison between the flow of two non-Newtonian fluids over an upper horizontal surface of paraboloid of revolution: Boundary layer analysis. *Alexandria Engineering Journal*. *55*(3), 1915–1929.

Abiev, R. S. (2022). Mathematical model of two-phase Taylor flow hydrodynamics for four combinations of non-Newtonian and Newtonian fluids in microchannels. *Chemical Engineering Science*, *247*, 116930.

Abro, K. A., and Gomez-Aguilar, J. F. (2019). A comparison of heat and mass transfer on a Walter ' s-B fluid via Caputo-Fabrizio versus Atangana-Baleanu fractional derivatives using the Fox-H function. *The European Physical Journal Plus*. *134*(101), 1–12.

Abro, K. A. (2021). Role of fractal – fractional derivative on ferromagnetic fluid via fractal Laplace transform: A first problem via fractal – fractional differential operator. *European Journal of Mechanics /B Fluids*. *85*, 76–81.

Abro, K. A., Siyal, A., and Atangana, A. (2020). Thermal stratification of rotational second-grade fluid through fractional differential operators. *Journal of Thermal Analysis and Calorimetry*.

Abro, K. A., and Atangana, A. (2020). A comparative study of convective fluid motion in rotating cavity via Atangana–Baleanu and Caputo–Fabrizio fractal–fractional differentiations. *European Physical Journal Plus*, *135*(2).

Ahmad, M., Imran, M. A., and Nazar, M. (2020). Mathematical modeling of (Cu-Al_2O_3) water based Maxwell hybrid nanofluids with Caputo-Fabrizio fractional derivative. *Advances in Mechanical Engineering*. *12*(9), 1–11.

Ahmad, M., Asjad, M. I., Akgül, A., and Baleanu, D. (2021). Analytical solutions for free convection flow of casson nanofluid over an infinite vertical plate. *AIMS Mathematics*. *6*(3), 2344–2358.

Ali, F., Jan, S. A. A., Khan, I., Gohar, M. and Sheikh, N. A. (2016). Solutions with special functions for time fractional free convection flow of Brinkman-type fluid. *The European Physical Journal Plus*. 131 (9), 310

Ali, F., Sheikh, N. A., Khan, I., and Saqib, M. (2017). Solutions with Wright Function for Time Fractional Free Convection Flow of Casson Fluid. *Arabian Journal for Science and Engineering*, *42*(6), 2565–2572.

Ali, F., Sheikh, N. A., Khan, I., and Saqib, M. (2017). Magnetic field effect on blood flow of Casson fluid in axisymmetric cylindrical tube: A fractional model. *Journal of Magnetism and Magnetic Materials*. *423*, 327–336.

Ali, F., Murtaza, S., Ahmad, N., & Khan, I. (2019). Heat transfer analysis of generalized Jeffery nanofluid in a rotating frame : Atangana – Balaenu and Caputo – Fabrizio fractional models. *Chaos*, *Solitons and Fractals*, *129*, 1–15.

Ali, K., Ali, A., Chandio, D., Ali, I., and Ilyas, A. (2019). Dual thermal analysis of magnetohydrodynamic flow of nanofluids via modern approaches of Caputo–Fabrizio and Atangana–Baleanu fractional derivatives embedded in porous medium. *Journal of Thermal Analysis and Calorimetry*. *135*(4), 2197–2207.

Ali, R., Asjad, M. I., and Akgül, A. (2021). An analysis of a mathematical fractional model of hybrid viscous nanofluids and its application in heat and mass transfer. *Journal of Computational and Applied Mathematics*. *383*, 113096.

Anderson, J. D. (1992). Governing Equations of Fluid Dynamics. In *Computational Fluid Dynamics*. 15–51.

Angulo, J. M., Ruiz-Medina, M. D., Anh, V. V., and Grecksch, W. (2000). Fractional diffusion and fractional heat equation. *Advances in Applied Probability*, *32*(4), 1077–1099.

Anwar, T., Kumam, P., and Thounthong, P. (2021). Fractional Modeling and Exact Solutions to Analyze Thermal Performance of Fe_3O_4-MoS_2-Water Hybrid Nanofluid Flow Over an Inclined Surface with Ramped Heating and Ramped Boundary Motion. *IEEE Access*. *9*, 12389–12404.

Arafa, A. A. M., Rashed, Z. Z., and Ahmed, S. E. (2021). Radiative flow of non Newtonian nanofluids within inclined porous enclosures with time fractional derivative. *Scientific Reports*, *11*(1), 1–18.

Arthur, E. M., Seini, I. Y., and Bortteir, L. B. (2015). Analysis of Casson Fluid Flow over a Vertical Porous Surface with Chemical Reaction in the Presence of Magnetic Field. *Journal of Applied Mathematics and Physics*, *03*(06), 713–723.

Awan, A. U., Ali, Q., Riaz, S., Shah, N. A., and Chung, J. D. (2021). A Thermal Optimization throughan Innovative Mechanism of Free Convection Flow of Jeffrey Fluid using Non-Local Kernel. *Case Studies in Thermal Engineering*. *24*(9), 100851.

Al-Mdallal, Q., Abro, K. A. and Khan, I. (2018) Analytical Solutions of Fractional Walter's B Fluid with Applications, *Complexity*. 2018.

Borah, G., Dutta, P. and Hazarika, G. C. (2021) Analysis of fractional derivatives in thermal and solutal transfer equations of second-grade non-Newtonian fluids: A numerical study, *Heat Transfer.* 2021. 1-15.

Borah, G., Dutta, P., and Hazarika, G. C. (2021). Numerical Study on Second-Grade Fluid Flow Problems Using Analysis of Fractional Derivatives Under Fuzzy Environment. *Soft Computing Techniques and Applications.* 1248.

Brewster, M. Q. (1952). *Thermal Radiative Transfer Properties.* John Wiley and Sons.

Casson, N. (1959). A flow equation for the pigment oil suspensions of the printing ink type, in rheology of disperse systems. Pergamon, New York.

Celata, G. P., Saha, S. K., Zummo, G., and Dossevi, D. (2010). Heat transfer characteristics of flow boiling in a single horizontal microchannel. *International Journal of Thermal Sciences*, *49*(7), 1086–1094.

Cengel, Y. (2004). *Heat Transfer: A Practical Approach.* 2nd Edition. New York: McGraw-Hill.

Çetinkaya, A., and Kiymaz, O. (2013). The solution of the time-fractional diffusion equation by the generalized differential transform method. *Mathematical and Computer Modelling*. *57*(9–10), 2349–2354.

Chen, W., Zhang, J., and Zhang, J. (2013). A variable-order time-fractional derivative model for chloride ions sub-diffusion in concrete structures. *Fractional Calculus and Applied Analysis*, *16*(1), 76–92.

Cieslinski, D. A., and Humes, H. D. (1996). Tissue engineering of a bioartificial kidney: A universal donor organ. *Transplantation Proceedings*, *28*(4), 2032–2035.

Failla, G., and Zingales, M. (2020). Advanced materials modelling via fractional calculus: Challenges and perspectives. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*. *378*(2172).

Gad-El-Hak, M. (1999). The fluid mechanics of microdevices—the freeman scholar lecture. *Journal of Fluids Engineering, Transactions of the ASME*. *121*(1), 5–33.

George, H. F., and Qureshi, F. (2013). Newton's Law of Viscosity. In *Encyclopedia of Tribology*, 2362–2465.

Ghalib, M. M., Zafar, A. A., Farman, M., Akgu, A., Ahmad, M. O., and Ahmad, A. (2021). Unsteady MHD flow of Maxwell fluid with Caputo – Fabrizio non-integer derivative model having slip/non-slip fluid flow and Newtonian heating at the boundary. *Indian J Phys*., *96*, 127-136

Gireesha, B. J., Srinivasa, C. T., Shashikumar, N. S., Macha, M., Singh, J. K., and Mahanthesh, B. (2019). Entropy generation and heat transport analysis of Casson fluid flow with viscous and Joule heating in an inclined porous microchannel. *Proceedings of the Institution of Mechanical Engineers, Part E: Journal of Process Mechanical Engineering*, *233*(5), 1173–1184.

Gireesha, B. J., and Roja, A. (2020). Second law analysis of MHD natural convection slip flow of Casson fluid through an inclined microchannel. *Multidiscipline Modeling in Materials and Structures*, *16*(6), 1435–1455.

Gireesha, B. J., Sindhu, S., Sowmya, G., and Felicita, A. (2021). Magnetohydrodynamic flow of Williamson fluid in a microchannel for both horizontal and inclined loci with wall shear properties. *Heat Transfer*, *50*(2), 1428–1442.

Halsted, D. J., and Brown, D. E. (1972). Zakian's technique for inverting Laplace transforms. *The Chemical Engineering Journal*. *3,* 312–313.

Haseli, Y. (2020) Entropy Analysis in Thermal Engineering Systems. Academic Press.

Hayat, T., Ullah, I., Waqas, M., and Alsaedi, A. (2019). Simulation of nanofluid thermal radiation in Marangoni convection flow of non-Newtonian fluid: A revised model. *International Journal of Numerical Methods for Heat and Fluid Flow*, *29*(8), 2840–2853.

Haque, E. U., Awan, A. U., Raza, N., Abdullah, M., and Chaudhry, M. A. (2018). A computational approach for the unsteady flow of maxwell fluid with Caputo fractional derivatives. *Alexandria Engineering Journal*, *57*(4), 2601–2608.

Herbert, O. (2004). *Prandtl's Essential of Fluid Mechanics*. Springer, New York.

Hussanan, A., Khan, I., and Shafie, S. (2013). An Exact Analysis of Heat and Mass Transfer Past a Vertical Plate with Newtonian Heating. *Journal of Applied Mathematics, 2013*.

Iftikhar, N., Saeed, S. T., and Riaz, M. B. (2021). Fractional study on heat and mass transfer of MHD Oldroyd-B fluid with ramped velocity and temperature. *Computational Method for Differential Equations*. *5*, 1–28.

Imran, M. A., Miraj, F., Khan, I., and Tlili, I. (2018). MHD fractional Jeffrey's fluid flow in the presence of thermo diffusion, thermal radiation effects with first order chemical reaction and uniform heat flux. *Results in Physics*. *10*(2), 10–17.

Jaluria, Y. (1980). Natural Convection Heat and Mass Transfer. Pergamon Press Oxford.

Jamaludin, A., Nazar, R., and Pop, I. (2022). Mixed convection stagnation-point flow of Cross fluid over a shrinking sheet with suction and thermal radiation. *Physica A: Statistical Mechanics and Its Applications*, *585*, 126398.

Jauregui, H. O., Chowdhury, N. R., and Chowdhury, J. R. (1996). Use of mammalian liver cells for artificial liver support. *Cell Transplantation*, *5*(3), 353–367.

Jiang, X., and Qi, H. (2012). Thermal wave model of bioheat transfer with modified Riemann-Liouville fractional derivative. *Journal of Physics A: Mathematical and Theoretical*, *45*(48).

Jumarie, G. (2007) MODIFIED Riemann-Liouville Derivative New Methods for Solution. *Applied Mathematics and Computation*, *24*(1), 31–48.

Kant, K., Singh, S., and Dhiman, P. (2018). Fluid flow and heat transfer characteristics within a rectangular microchannel array of different manifold shapes - Modelisation and optimisation using CFD and response surface methodology. *Progress in Computational Fluid Dynamics*, *18*(1), 19–32.

Kexue, L. and Jigen, P. (2011) Laplace transform and fractional differential equations, *Applied Mathematics Letters*. Elsevier Ltd, *24*(12), 2019–2023.

Khalid, A., Khan, I., Khan, A., and Shafie, S. (2015). Unsteady MHD free convection flow of Casson fluid past over an oscillating vertical plate embedded in a porous medium. *Engineering Science and Technology, an International Journal*. *18*(3), 309–317.

Khalid, A., Khan, I. and Shafie, S. (2015) Exact solutions for unsteady free convection flow of casson fluid over an oscillating vertical plate with constant wall temperature, *Abstract and Applied Analysis*, 2015.

Khalil, R., Horani, M. Al, Yousef, A., and Sababheh, M. (2014). Journal of Computational and Applied A new definition of fractional derivative. *Journal of Computational and Applied Mathematics*. *264*, 65–70.

Khan, M., Ali, S. H., and Qi, H. (2009). Nonlinear Analysis: Real World Applications On accelerated flows of a viscoelastic fluid with the fractional Burgers ' model. *Nonlinear Analysis: Real World Applications*, *10*(4), 2286–2296.

Khan, Y., Wu, Q., Faraz, N., Yildirim, A., and Mohyud-Din, S. T. (2012). Heat transfer analysis on the magnetohydrodynamic flow of a Non-Newtonian fluid in the presence of thermal radiation: An analytic solution. *Zeitschrift Fur Naturforschung - Section A Journal of Physical Sciences*, *67*(3–4), 147–152.

Khan, I., Karim, N., Ahmad, W., Abdelhalim, A., and Chebib, M. (2016). GABA-A Receptor Modulation and Anticonvulsant, Anxiolytic, and Antidepressant Activities of Constituents from Artemisia indica Linn. *Evidence-Based Complementary and Alternative Medicine*, *2016*.

Khan, A., Khan, D., Khan, I., Taj, M., Ullah, I., Aldawsari, A. M., Thounthong, P. and Nisar, K. S. (2019). MHD Flow and Heat Transfer in Sodium Alginate Fluid with Thermal Radiation and Porosity Effects: Fractional Model of Atangana – Baleanu Derivative of Non-Local and Non-Singular Kernel. *SS Symmetry*, *11*(1295), 1–17.

Khan, I., Saqib, M., and Ali, F. (2018). Application of time-fractional derivatives with non-singular kernel to the generalized convective flow of Casson fluid in a microchannel with constant walls temperature. *European Physical Journal*, *3802*(2017), 3791–3802.

Khan, I., Shah, N. A., and Dennis, L. C. C. (2017). A scientific report on heat transfer analysis in mixed convection flow of Maxwell fluid over an oscillating vertical plate. *Nature Publishing Group*, ((9)2016), 1–12.

Khan, U., Zaib, A., Ishak, A., Bakar, S. A., Animasaun, I. L., and Yook, S. J. (2022). Insights into the dynamics of blood conveying gold nanoparticles on a curved surface when suction, thermal radiation, and Lorentz force are significant:

The case of Non-Newtonian Williamson fluid. *Mathematics and Computers in Simulation*, *193*, 250–268.

Kundu, P., and Cohen, I. (2004). *Fluid Mechanics*. Boston.

Latif, M. (2006). *Heat Convection.* Berlin: Springer-Verlag.

Leung, S. S. Y., Gupta, R., Fletcher, D. F., and Haynes, B. S. (2012). Gravitational effect on Taylor flow in horizontal microchannels. *Chemical Engineering Science*, *69*(1), 553–564.

Luchko, Y., and Gorenflo, R. (1999). An operational method for solving fractional differential equations with the Caputo derivatives. *Acta Math. Vietnam.*, *24*(2), 207–233.

Mainardi, F. (1996). The fundamental solutions for the fractional diffusion-wave equation. *Applied Mathematics Letters*, *9*(6), 23–28.

Maiti, S., Shaw, S., and Shit, G. C. (2019). Caputo – Fabrizio fractional order model on MHD blood flow with heat and mass transfer through a porous vessel in the presence of thermal radiation. *Physica A*, 123149.

Maiti, S., Shaw, S., and Shit, G. C. (2021). Fractional order model for thermochemical flow of blood with Dufour and Soret effects under magnetic and vibration environment. *Colloids and Surfaces B: Biointerfaces*. *197*, 111395.

Makris, N., Dargush, G. F. and Constantinou, M. C. (1993). Dynamic analysis of generalized viscoelastic fluids. *Journal of Engineering Mechanics*. 119 (8), 1663-1679.

Meral, F. C., Royston, T. J., and Magin, R. (2010). Fractional calculus in viscoelasticity: An experimental study. *Communications in Nonlinear Science and Numerical Simulation*, *15*(4), 939–945.

Mills, A. F. (2018). Thermal Radiation. *Heat and Mass Transfer*, 487–594.

Momenzadeh, M., and Norozpour, S. (2021). Alternative fractional derivative operator on non-newtonian calculus and its approaches. *Nexo Revista Científica*, *34*(02), 906–915.

Moosavi, R., Moltafet, R., Lin, C. X., and Abel Chuang, P. Y. (2021). Numerical modeling of fractional viscoelastic non-Newtonian fluids over a backward facing step – Buoyancy driven flow and heat transfer. *Thermal Science and Engineering Progress*. *21*(10), 100767.

Na, W., Shah, N. A., Tlili, I. and Siddique, I. (2020). Maxwell fluid flow between vertical plates with damped shear and thermal flux: free convection. *Chinese Journal of Physics*. 65 (3), 367–337.

Nadeem, S., Haq, R. U., Akbar, N. S., and Khan, Z. H. (2013). MHD three-dimensional Casson fluid flow past a porous linearly stretching sheet. *Alexandria Engineering Journal*. *52*(4), 577–582.

Narahari, M. (2012). Transient free convection flow between long vertical parallel plates with ramped wall temperature at one boundary in the presence of thermal radiation and constant mass diffusion. *Meccanica*, *47*(8), 1961–1976.

Oke, A. S., Mutuku, W. N., Kimathi, M., and Animasaun, I. L. (2020). Insight into the dynamics of non-Newtonian Casson fluid over a rotating non-uniform surface subject to Coriolis force. *Nonlinear Engineering*, *9*(1), 398–411.

Pal, D., Das, B. C., and Vajravelu, K. (2016). Influence of Non-linear Thermal Radiation on MHD Double-Diffusive Convection Heat and Mass Transfer of a Non-Newtonian Fluid in a Porous Medium. *International Journal of Applied and Computational Mathematics*, *3*(4), 3105–3129.

Pop, I., and Sheremet, M. (2014). Free convection in a square cavity filled with a Casson fluid under the effects of thermal radiation and viscous dissipation. *International Journal of Numerical Methods for Heat and Fluid Flow*, *24*(4).

Qushairi, M. A., Yeou, J. L., Shafie, S., Khan, I., & Ismail, Z. (2018). Exact solution for unsteady free convection flow of Casson fluid in vertical channel. *MATEC Web of Conferences*, *189*, 1–6.

Ray, S. S., Atangana, A., Noutchie, S. C. O., Kurulay, M., Bildik, N., and Kilicman, A. (2014). Fractional calculus and its applications in applied mathematics and other sciences. *Mathematical Problems in Engineering*, *2014*, 2–4.

Razzaq, A. and Raza, N. (2019). Heat and mass transfer analysis of Brinkman type fractional nanofluid over a vertical porous plate with velocity slip and Newtonian heating. *Journal of Mathematics (ISSN 1016-2526)*. 51 (9), 45-69.

Renardy, M., and Rogers, R. C. (2004). An Introduction to Partial Differential Equations. In *An Introduction to Partial Differential Equations*.

Sadd. M. H. (2020) *Elasticity*. Academic Press

Salim, A., Fourar, M., Pironon, J., and Sausse, J. (2008). Oil-water two-phase flow in microchannels: FLow patterns and pressure drop measurements. *Canadian Journal of Chemical Engineering*, *86*(6), 978–988.

Saqib, M., Khan, I., and Shafie, S. (2018). Application of Atangana–Baleanu fractional derivative to MHD channel flow of CMC-based-CNT's nanofluid through a porous medium. *Chaos, Solitons and Fractals*, *116,* 79–85.

Saqib, M., Khan, I., and Shafie, S. (2019). New Direction of Atangana – Baleanu Fractional Derivative with Mittag-Leffler Kernel for Non-Newtonian Channel Flow. In *Trends and Applications in Science and Engineering*, 253–268.

Saqib, M., Hanif, H., Abdeljawad, T., Khan, I., Shafie, S., and Nisar, K. S. (2020). Heat transfer in MHD flow of maxwell fluid via fractional Cattaneo-Friedrich model: A finite difference approach. *Computers, Materials and Continua*, *65*(3), 1959–1973.

Saxena, R., Kumar, S., and Gupta, A. (2014). Study of fluid flow in microchannel based devices. *Enviromental Science and Engineering*, 405–407.

Shahrim, M. N., Mohamad, A. Q., Jiann, L. Y., Zakaria, M. N., Shafie, S., Ismail, Z., and Kasim, A. R. M. (2021). Exact solution of fractional convective casson fluid through an accelerated plate. *CFD Letters*, *13*(6), 15–25.

Shaw, S., Samantaray, S. S., Misra, A., Nayak, M. K., and Makinde, O. D. (2022). Hydromagnetic flow and thermal interpretations of Cross hybrid nanofluid influenced by linear, nonlinear and quadratic thermal radiations for any Prandtl number. *International Communications in Heat and Mass Transfer*, *130*, 105816.

Shehzad, S. A., Hayat, T., and Alsaedi, A. (2016). Three-Dimensional MHD Flow of Casson Fluid in Porous Medium with Heat Generation. *Journal of Applied Fluid Mechanics*, *9*(1), 215–223.

Sheikh, N. A., Ali, F., Saqib, M., Khan, I., Jan, S. A. A., Alshomrani, A. S., and Alghamdi, M. S. (2017). Comparison and analysis of the Atangana–Baleanu and Caputo–Fabrizio fractional derivatives for generalized Casson fluid model with heat generation and chemical reaction. *Results in Physics*, *7*, 789–800.

Sheikh, N. A., Ling, D., Ching, C., Khan, I., Kumar, D., and Nisar, K. S. (2020). A new model of fractional Casson fluid based on generalized Fick ' s and Fourier ' s laws together with heat and mass transfer. *Alexandria Engineering Journal*, *59*(5), 2865–2876.

Sheremet, M. A., Pop, I., and Rosca, A. V. (2018). The influence of thermal radiation on unsteady free convection in inclined enclosures filled by a nanofluid with sinusoidal boundary conditions. *International Journal of Numerical Methods for Heat and Fluid Flow*, *28*(8), 1738–1753.

Siegel, R., and Howell, J. R. (2002). *Thermal Radiation Heat Transfer.* 4th Edition.

Song, D. Y., and Jiang, T. Q. (1998). Study on the constitutive equation with fractional derivative for the viscoelastic fluids - modified Jeffreys model and its application. *Rheologica Acta*, *37*(5), 512–517.

Song, Y. Q., Raza, A., Al-Khaled, K., Farid, S., Khan, M. I., Khan, S. U., and Khan, M. I. (2021). Significances of exponential heating and Darcy's law for second grade fluid flow over oscillating plate by using Atangana-Baleanu fractional derivatives. *Case Studies in Thermal Engineering*, *27*, 101266.

Sullivan, S. J., Maki, T., Borland, K. M., Mahoney, M. D., Solomon, B. A., Muller, T. E., and Chick, W. L. (1991). Biohybrid artificial pancreas: Long-term implantation studies in diabetic, pancreatectomized dogs. *Science*, *252*(5006), 718–721.

Sulochana, C., Payad, S. S., and Sandeep, N. (2016). Non-uniform heat source or sink effect on the flow of 3D Casson fluid in the presence of Soret and thermal radiation. *International Journal of Engineering Research in Africa*, *20*, 112–129.

Tassaddiq, A., Khan, I., and Nisar, K. S. (2020). Heat transfer analysis in sodium alginate based nanofluid using MoS2 nanoparticles: Atangana–Baleanu fractional model. *Chaos, Solitons and Fractals*, *130*, 109445.

Tabeling, P. (2001). Some basic problems of microfluidics. *Physical Review*, (12), 57–62.

Teng, J., Chu, J.-C., Liu, C., Xu, T., Lien, Y.-F., Cheng, J.-H., and Greif, R. (2012). Fluid Dynamics in Microchannels. In *Fluid Dynamics, Computational Modeling and Applications*, 403–436.

Thriveni, K., and Mahanthesh, B. (2020). Optimization and sensitivity analysis of heat transport of hybrid nanoliquid in an annulus with quadratic Boussinesq approximation and quadratic thermal radiation. *European Physical Journal Plus*, *135*(6), 1–22.

UKEssays. (2018). Measuring Fluid Viscosity and Effects of Shear Rate. Retrieved from https://www.ukessays.com/essays/chemistry/measuring-fluid-viscosity-and-effects-of-shear-rate.php?vref=1

Ullah, I., Shafie, S., and Khan, I. (2017). Effects of slip condition and Newtonian heating on MHD flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium, Effects of slip condition and Newtonian heating. *Journal of King Saud University - Science*, *29*(2), 250–259.

Umamaheswar, M. (2018). Unsteady MHD convective Casson fluid flow past a vertical porous plate in the presence of radiation and thermal diffusion. *International Journal of Advanced Scientific Research and Management*, *3*(12), 117–126.

Wakif, A., Chamkha, A., Thumma, T., Animasaun, I. L., and Sehaqui, R. (2021). Thermal radiation and surface roughness effects on the thermo-magneto-hydrodynamic stability of alumina–copper oxide hybrid nanofluids utilizing the generalized Buongiorno's nanofluid model. *Journal of Thermal Analysis and Calorimetry*, *143*(2), 1201–1220.

Wang, Q., and Zhan, H. (2015). On different numerical inverse Laplace methods for solute transport problems. *Advances in Water Resources*, *75*, 80–92.

Wang, B., Tahir, M., Imran, M., Javaid, M., and Jung, C. Y. (2019). Semi analytical solutions for fractional oldroyd-B fluid through rotating annulus. *IEEE Access*, *7*, 72482–72491.

White, F. (2006). *Viscous Fluid Flow*. McGraw-Hill, New York.

White, F. M. (1999). *Fluid Mechanics.* McGraw-Hill, Boston.

Zakian, V., and Littlewood, R. K. (1972). Numerical Inversion of Laplace Transforms by Weighted Least-Squares Approximation. *The Computer Journal*, *16*, 66–68.

Zhang, C., Yu, M. J., Pan, X. Y., Wu, G., Jin, L., Gao, W. D., and Zhang, J. C. (2014). Regioselective mononitration of chlorobenzene using caprolactam-based Brønsted acidic ionic liquids. *Journal of Molecular Catalysis A: Chemical*, *383–384*, 101–105.

LIST OF PUBLICATIONS

1. Marjan Mohd Daud, Lim Yeou Jiann, Rahimah Mahat, and Sharidan Shafie. (2022). Application of Caputo Fractional Derivatives to the Convective Flow of Casson Fluids in a Microchannel with Thermal Radiation. *Journal of Advanced Research in Fluid Mechanics and Thermal Sciences*, *93*(1), 50–63. (Published)
2. Daud, M. M., Jiann, L. Y., Shafie, S., and Mahat, R. (2022) Casson Fluid Convective Flow in an Accelerated Microchannel with Thermal Radiation using the Caputo Fractional Derivative. *CFD Letters*. (Accepted)