FREE VIBRATION OF LAMINATED COMPOSITE PLATE AND SHELL STRUCTURES FOR CONSTANT AND VARIABLE THICKNESS

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ABSTRACT

This study is conducted to analyse the free vibration of rectangular plates, circular plates and conical shells of anti-symmetric angle-ply laminated composite using classical theory, first order shear deformation theory and third order shear deformation theory of constant and variable thickness. The variations of thickness used are in the form of linear, exponential and sinusoidal. Free vibration of conical shells with constant thickness is investigated under classical theory. An extended study has been done by using developed shell theories where shear deformation is included. Free vibration of circular plates of variable thickness is analysed under first order shear deformation theory. Using the same theory, the free vibration analysis of conical shells for variable thickness is conducted. Third order shear deformation theory is adopted to the study of free vibration of rectangular plates for variable thickness. In this study, stress resultants and strain-displacement relations are substituted into the governing equation of structures. The solution is assumed to be separable in the form of displacement and rotational functions to obtain ordinary differential. The displacement and rotational functions are approximated using spline method. The obtained equations together with equations of boundary conditions are reduced to eigenvalue problem. The solutions of the eigenvalue problems are the frequencies of the plates and shells. The effects of boundary conditions, aspect ratio, side-to-thickness ratio, ply angle, number of layers, circumferential node number, variable thickness, cone angle, length ratio and radii ratio on the vibration of structures are investigated. The results show that the frequencies are higher for clamped-clamped boundary conditions than simply-supported and clamped-free boundary conditions. Also, it is found that the geometric parameters affect the vibration of structures.

ABSTRAK

Kajian ini dijalankan untuk menganalisa getaran bebas bagi plat segi empat tepat, plat bulat dan juga cangkerang kon dengan komposit lapisan sudut anti-simetri menggunakan teori klasik, teori deformasi ricih peringkat pertama dan teori deformasi ricih peringkat ketiga dengan ketebalan malar dan boleh ubah. Kepelbagaian ketebalan yang digunakan adalah dalam bentuk linear, eksponen dan sinusoidal. Getaran bebas bagi cangkerang kon dengan ketebalan malar dikaji menggunakan teori klasik. Kajian lanjutan telah dilakukan menggunakan teori klasik yang mengambilkira deformasi ricih. Getaran bebas bagi plat bulat dengan ketebalan boleh ubah dianalisis menggunakan teori deformasi ricih peringkat pertama. Teori yang sama digunakan untuk menganalisa getaran bebas bagi cangkerang kon dengan ketebalan boleh ubah. Teori deformasi ricih peringkat ketiga digunakan untuk mengkaji getaran bebas bagi plat segi empat tepat dengan ketebalan boleh ubah. Dalam kajian ini, hubungan hasil tegasan dan anjakan-ketegangan dimasukkan ke dalam persamaan penakluk struktur. Penyelesaian diandaikan bolehpisah dalam bentuk fungsi anjakan dan putaran untuk mendapatkan persamaan terbitan biasa. Fungsi anjakan dan putaran dianggarkan menggunakan kaedah spline. Persamaan yang diperoleh dengan persamaan syarat sempadan diturunkan menjadi masalah nilai eigen. Penyelesaian kepada masalah nilai eigen adalah frekuensi untuk plat dan cangkerang. Kesan-kesan syarat sempadan, nisbah aspek, nisbah sisi–ketebalan, sudut lapisan, bilangan lapisan, bilangan nod lilitan, ketebalan boleh ubah, sudut kon, nisbah panjang dan nisbah jejari terhadap getaran struktur dikaji. Hasil kajian menunjukkan frekuensi lebih tinggi untuk syarat sempadan yang diapit-apit berbanding syarat sempadan yang disokong-mudah dan bebas-apit. Kajian juga mendapati parameter-parameter geometri mempengaruhi getaran struktur.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

AGE	-	AS4/3501-6 Graphite/Epoxy
KGE	-	Kevler-49 Epoxy
C-C	-	Both ends are clamped
C-F	-	One end is clamped and the other end is free
CT	-	Classical theory
FSDT	-	First order shear deformation theory
HSDT		Higher order shear deformation theory
TSDT	-	Third order shear deformation theory
HSG	-	High Strength Graphite Epoxy
SGE	-	S-Glass Epoxy
S-S	-	Both ends are simply supported

LIST OF SYMBOLS

A_{ij}	-	Elastic coefficients representing the extensional rigidity
B_{ij}	-	Elastic coefficients representing the bending-stretching coupling rigidity
D_{ij}	-	Elastic coefficients representing the bending rigidity
$E_{11}^{(k)}$	-	Young's modulus along x directions of the *k*-th layer
$E_{22}^{(k)}$	-	Young's modulus along θ directions of the *k*-th layer
$G_{13}^{(k)}, G_{12}^{(k)}, G_{23}^{(k)}$	-	Shear modulus in the respective directions of the *k*-th layer
h	-	Thickness
$H(X - X_j)$	-	The Heaviside step function
I_1	-	Normal inertia coefficient
I_3	-	Rotary inertia coefficients
K	-	Shear correction factor
l	-	Length
L_{ij}	-	Differential operator occurring in the equations of motion
$M_x, M_\theta, M_{x\theta}$	-	Moment resultants in the respective directions of the shell
$N_x, N_\theta, N_{x\theta}$	-	Stress resultants in the respective directions of the shell
N	-	Number of intervals of spline interpolation
$Q_{ij}^{(k)}$	-	Elements of the stiffness matrix for the material of *k*-th layer
$\bar{Q}_{ij}^{(k)}$	-	Elements of the transformed stiffness matrix for the material of *k*-th layer
$Q_{xz}, Q_{\theta z}$	-	Transverse shear resultants in the respective directions of the shell

U, V, W	-	Displacement functions in x, θ, z direction
$\bar{U}, \bar{V}, \bar{W}$	-	Non-dimensionalised displacement functions in x, θ, z direction
X	-	Non-dimensionalised meridional distance coordinate
X_s	-	The equally spaced knots of spline interpolation
a, b	-	Distance of the small and large ends of the conical shell from the vertex (or centre)
$\left.\begin{matrix} a_i \\ c_i \\ e_i \\ g_i \\ l_i \end{matrix}\right\}, \left.\begin{matrix} b_j \\ d_j \\ f_j \\ p_j \\ q_j \end{matrix}\right\}$	-	Spline coefficients
h	-	Thickness of the shell
h_k	-	Thickness of the *k*-th layer of the shell
i, j, k	-	Summation or general indices
ℓ	-	Length of the shell
n	-	Circumferential node number
r	-	Radius of reference surface of shell at a general point
r_a, r_b	-	The radius at the small and the large end of the cone
t	-	Time coordinate
u, v, w	-	Displacements in x, θ, z direction
u_0, v_0	-	The in-plane displacements of the reference surface of the shell
x, θ, z	-	Axial, circumferential and normal coordinates of any point on the shell
z_k	-	Distance of the top of the *k*-th layer from the reference surface of the shell
α	-	Semivertical angle of the cone shell
β	-	length ratio a/b of the conical shell
δ_k	-	Relative layer thickness of the k-th layer
$\varepsilon_x, \varepsilon_\theta$	-	Normal strain components of the reference surface of the shell

γ	-	Ratio of the constant thickness to radius of small end of the cone
γ'	-	Ratio of the constant thickness to the distance of small end from vertex of the cone
$\gamma_{x\theta}, \gamma_{xz}, \gamma_{\theta z}$	-	Shear strain of the reference surface
$\kappa_x, \kappa_\theta, \kappa_{x\theta}$	-	Change in curvature on the reference surface of the shell
λ	-	Non-dimensional frequency parameter
$\upsilon_{12}, \upsilon_{21}$	-	Poison's ratio
ω	-	Angular frequency
Ψ_x, Ψ_θ	-	Shear rotational functions of the shell
ψ_x, ψ_θ	-	Shear rotations of any point on the middle surface of the shell
$\bar{\Psi}_X, \bar{\Psi}_\theta$	-	Non-dimensionalised shear rotational functions of the shell
ρ	-	Mass density of the material of the shell
σ_x, σ_θ	-	Normal stress in the respective directions of shells
$\tau_{x\theta}, \tau_{\theta z}, \tau_{xz}$	-	Shear stress in the respective directions of shells
θ	-	Ply orientation angle

LIST OF APPENDICES

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

Thin plate and shell structures function as a structural support where the thickness is much smaller than the other dimensions that makes an important contribution to the development of several branches of engineering such as civil, mechanical, marine and aeronautical engineering (Ugural, 2010). The generally high strength-to-weight ratio of the shell form combined with its inherent stiffness has formed the basis of applications of shell structures. Plate and shell structures also have the efficiency of load-carrying behaviour and high degree of reserved strength and structural integrity which is among the properties needed for structural frame in engineering industries (Ventsel, 2001). Example of plates and shells structures can be seen in wings and rocket fins in aerospace engineering, automotive body panels and disk wheels in mechanical engineering, off-shore platforms in ocean, large-span roofs, water tanks, turbine disks, aircrafts, missiles, submarines and more.

The application of laminated composite shell extends the characteristics of shells since it provides higher strength-to-weight ratios, better corrosion resistance, longer fatigue life and also one can design the directional properties. A composite material is a mixture of two or more materials with properties superior to the materials of which it is made. The materials work together to give the composite unique properties (Gibson, 1994). The biggest advantage of modern composite materials is that they are light as well as strong. By choosing an appropriate combination of two or more materials, a new material can be made that exactly meets the requirements of a particular application. Composites also provide design flexibility because many of them can be moulded into complex shapes. Some common composite materials include concrete, fiberglass, mud bricks, and natural composites such as rock and wood. For example, wood is an example of a composite because cellulose fibers are held together by a substance called lignin. These fibers can be found in cotton and thread, but it’s the bonding power of lignin in wood that makes it much tougher (Staab, 1999).

Lamina is described as a thin sheet or plate of material. Laminae are stacked together to form a laminate with the required thickness and stiffness (Ye, 2003). Laminated structure gives the better properties to the whole structure since it provides the advantage of combining appropriate constituent layers. The lamination of structure can be seen in Figure 1.1.

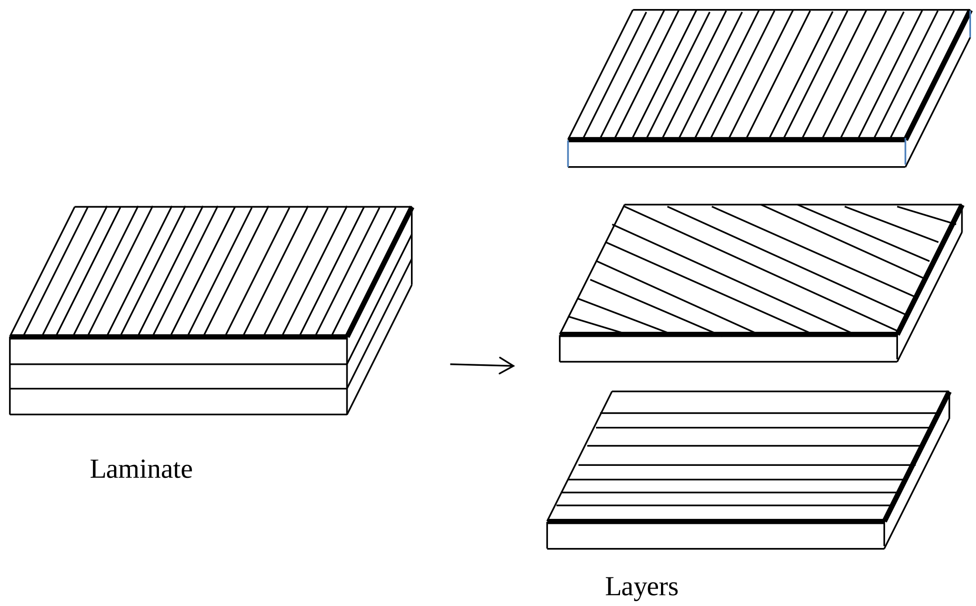

Figure 1.1 Lamination of structure

Laminates can be divided into three types which are angle-ply, cross-ply and unidirectional. For unidirectional laminate, each ply of laminae is oriented in the same direction. Other than that, a laminate is called cross-ply laminate if all the plies used to fabricate the laminate are only 0° and 90° whereas angle-ply laminate has plies oriented at $-\theta$ and $+\theta$ between 0° and 90° (Reddy, 2007). In this research, anti-symmetric laminates are considered. A laminate is called symmetric when the material, angle and thickness of the layers are the same above and below the mid-plane. For anti-symmetric laminate, the material and thickness of the plies are the same above and below the mid-plane but the ply orientation at the same distance above and below the mid-plane are negative of each other. Figure 1.2 shows the example of cross-ply, angle-ply, symmetry and anti-symmetric laminates.

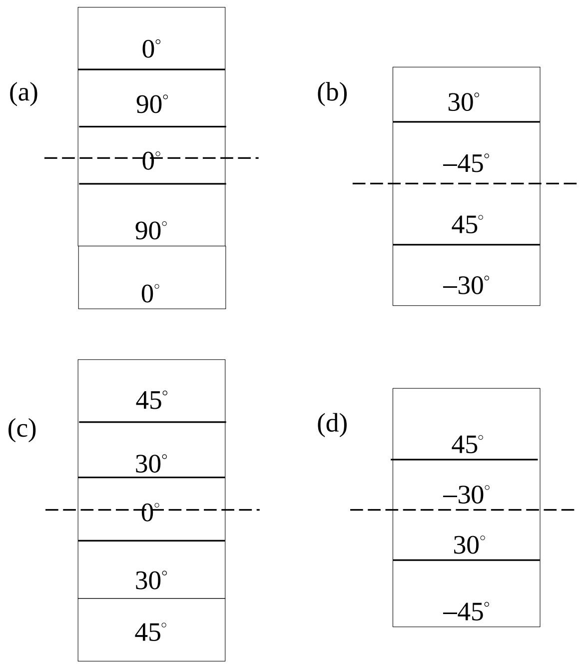

Figure 1.2 Classification of laminates examples (a) Cross-ply laminate (b) Angle-ply laminate (c) Symmetric laminate and (d) Anti-symmetric laminate

In order to design buildings (and other human-made structures) to be able to withstand the forces of earthquakes and windstorms, structural engineers compute the natural frequency of buildings. Natural frequency is the frequency at which a system tends to oscillate in the absence of any driving or damping force. When an object vibrates at a frequency equivalents to its natural frequency, its vibration amplitude increases significantly which could lead to irreparable damage. Engineers conduct research and field studies to learn how various structural designs and materials perform under anticipated hazardous conditions, so they can design the safest structures possible. That is why the study of vibration of structures such plates and shells have gain much interest for many researchers.

1.2 Problem Statement

Designer always aim at achieving economy by minimizing costs without neglecting the constraints of functional and aesthetic requirements for any structure whether it could be a building, pressure vessel or an aerospace structure. When suitably designed even thin plates and shells can support large loads and therefore they are always use in structures where light weight is essential. Composite, which is a new form of structure that resists the loads more efficiently than when the structure is designed in a conventional form is developed with the available material. Also, the structures should be designed by considering the factors such as circumferential node number, number of layers, length ratio, cone angle, materials and ply angle in order to obtain optimal design of structures in engineering industries. The vibration analysis of these structures is conducted where natural frequencies are obtained. Knowing more about vibration of structures helps us to control damage and take preventive measures from damages to occur.

1.3 Objectives

The main objective of this research is to study the free vibration of anti-symmetric angle-ply laminated composite plate and shell structures. This involves the transformation of governing equations to a system of homogeneous simultaneous algebraic equations which eventually becomes eigenvalue problem. The eigenvalue problem is solved to obtain frequencies. The details on the objectives are:

1. To obtain the frequencies for various fixed parameters of anti-symmetric angle-ply laminated composite conical shell for constant thickness based on classical theory.
2. To determine the frequencies for various fixed parameters of anti-symmetric angle-ply laminated composite circular plate and conical shell with variable thickness based on first order shear deformation theory.
3. To generate the frequencies for various fixed parameters of free vibration of anti-symmetric angle-ply laminated composite rectangular plate with variable thickness based on third order shear deformation theory.
4. To approximate displacement functions and rotational functions using spline method.

1.4 Scope of the Study

This study is focused on the free vibration analysis of plate and shell structures of circular plates, rectangular plates and conical shells. The governing differential equation are based on classical theory, first order shear deformation theory and third order shear deformation theory. Constant and variable thickness in which the variable thickness is in the form of linear, exponential and sinusoidal variation in thickness are considered. The displacement functions and rotational functions are approximated using spline method. Clamped-clamped, clamped-free, and simply-supported boundary conditions are used to analyse the problems in this study. The study is conducted for two- and four-layered structures using materials S-Glass Epoxy (SGE), High Strength Graphite Epoxy (HSG), Kevler-49 epoxy (KGE) and AS4/3501-6 Graphite/epoxy (AGE).

1.5 Significance of the Study

Plate and shell structures are thin, light, can span over large areas and have the advantage to carry applied loads effectively by means of their curvature. The study of vibration of plates and shells is important to determine the natural frequencies of the structures in order to avoid the destructive effect of weather and resonance with adjacent rotating or oscillating equipment. The efficiency of the structure can be increased if the structures are designed with the knowledge of the material properties. Laminated composite structures oriented at different angles added flexibility to the engineers to tailor the stiffness and strength of the laminate to match the structural requirements. Composite materials have very high ratios of in-plane Young's moduli to transverse shear moduli (Reddy, 1979). Hence, the inclusion of shear deformation may give better results since the kinematics of the shells are more accurately represented. The spline approximation is chosen as the solution method due to its fast convergence and better accuracy for lower-order approximation as compared to a global higher order approximation (Bickley, 1968). The vibration analysis of shells included all these conditions are implemented to find the natural frequencies of the structures in which benefits the engineers to design better construction in terms of strength and stability and to avoid any defects.

1.6 Thesis Outline

The study on the vibration of anti-symmetric angle-ply laminated composite plates and shells for classical theory, first order shear deformation theory and third order shear deformation theory is presented in this thesis. Seven chapters are included where introduction, literature review, problems and discussion as well as conclusion from this study are explained thoroughly in every chapter. Chapter 1 gives an overview on the study of thin plates and shells which includes the background of the study, problem statement, objectives of the study, scope of the study, significance of the study and outline of the thesis. In Chapter 2, review of previous work done by different researchers on vibration of plates and shells for classical theory, first order shear deformation theory and higher order theory are presented.

The free vibration of anti-symmetric angle-ply laminated composite conical shell using classical theory for constant thickness under C-C and S-S boundary conditions is investigated in Chapter 3. Two- and four-layered shells with combination of materials S-Glass Epoxy (SGE) and High Strength Graphite Epoxy (HSG) are considered. Convergence and comparison studies are carried out to validate the effectiveness of present method Parametric studies on the effect of number of layers, ply angle, circumferential node number, cone angle, length ratio and boundary conditions on the vibration of laminated conical shells are discussed.

Chapter 4 covers the free vibration of anti-symmetric angle-ply laminated composite circular plate for variable thickness based on the improved version of classical theory with the inclusion of transverse shear deformation which is the first order shear deformation theory. Two materials which are Kevler-49 epoxy (KGE) and AS4/3501-6 Graphite/epoxy (AGE) are used. The shells are constrained with C-C and C-F boundary conditions. The parametric studies are performed to illustrate the effects of radii ratio, number of layers, ply angle and boundary conditions on the frequency of circular plates.

Chapter 5 explains the free vibration of anti-symmetric angle-ply laminated composite conical shell based on first order shear deformation theory with variable thickness under C-C and S-S boundary conditions. Two- and four-layered shells composed of two types of material; KGE and AGE materials are used. The influence of length ratio, boundary conditions, cone angle, number of layers and ply angle on the vibration of conical shell is explored.

Free vibration for third order shear deformation theory of anti-symmetric angle-ply laminated composite rectangular plate with variable thickness under S-S boundary conditions is presented in Chapter 6. KGE and AGE materials are considered in the study. The influence of different parameters such as aspect ratio, side-to-thickness ratio, number of layers, and ply angle on the frequency of rectangular plate are demonstrated. The results obtained from previous chapters are concluded in Chapter 7. The last section of Chapter 7 suggests the future studies that can be conducted for vibration of plates and shells.

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APPENDIX A

PUBLICATION

Based on this thesis, the following papers were published in Journals, submitted for publications and presented in Conferences.

Papers published/submitted in Journals

1. Nor Hafizah, A. K., Viswanathan, K. K., Aziz, Z. A. (2016). Free Vibration of Antisymmetric Angle-Ply Laminated Conical Shell. Proceeding of the 5th International Conference on Computer Engineering and Mathematical Sciences (ICCEMS 2016).

2. Nor Hafizah, A. K., Viswanathan, K. K., Aziz, Z. A., and Lee, J. H. (2017). Free Vibration of Antisymmetric Angle-Ply Composite Laminated Conical Shell under Classical Theory. *International Journal of Applied Engineering Research*. 12 (15), 4928-4937.

3. Nor Hafizah, A. K., Viswanathan, K. K., Aziz, Z. A., and Lee, J. H. (2018). Vibration of Antisymmetric Angle-Ply Composite Annular Plates of Variable Thickness. *Journal of Mechanical Science and Technology*. 32 (5), 2155-2162.

4. Nor Hafizah, A. K., Lee, J. H., Aziz, Z. A., and Viswanathan, K. K. (2018). Vibration of Antisymmetric Angle-Ply Laminated Plates of Higher-Order Theory with Variable Thickness. *Mathematical Problems in Engineering*. 2018, 14 pages.