ANALYTICAL SOLUTION OF UNSTEADY MHD FREE CONVECTION FLOW OF CASSON FLUID THROUGH A VERTICAL CHANNEL

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DEDICATION

This dissertation is dedicated to my husband, who always become my backbone throughout this journey and encouraged me to believe in myself. It is also dedicated to my kids, who inspired me that success come after our efforts and patience. Lastly, I dedicated to my brother and sisters who taught me that knowledge is our dignity.

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ABSTRACT

Unsteady flow of Casson fluid past through a vertical channel has been studied by some researchers due to its importance applications in science and technology. Therefore, the main purpose of this thesis is to obtain exact solutions for unsteady free convection flows of Casson fluid with effects of magnetohydrodynamics (MHD) past through vertical channel. Dimensional governing equations are converted into dimensionless forms by using appropriate dimensionless variables. Dimensionless parameters are obtained through dimensionless process such as Casson fluid, time, Prandtl number, Grashof number and magnetic field. Laplace transform method is used to solve the dimensionless equations with associated initial and boundary conditions. Solutions for velocity and temperature profiles are obtained. Skin friction and Nusselt number are also calculated. The obtained analytical results for velocity and temperature are plotted graphically to discuss the influence of dimensionless parameters on profiles. It is observed that fluid velocity increases with increases of Grashof number, Gr and time, t whereas it decreases with increases of Casson parameter, γ, magnetic field parameter, M and Prandtl number, Pr. Besides that, it is found that temperature profiles decrease with high value of Prandtl number, Pr while increases with high value of time, t. In order to validate the results, the obtained results in limiting cases are compared with the published results and it is found to be in a mutual agreement.

ABSTRAK

Aliran tak mantap bagi bendalir Casson yang melalui saluran yang menegak telah dikaji oleh beberapa penyelidik disebabkan oleh kepentingannya dalam sains dan teknologi. Oleh sebab itu, tujuan utama tesis ini adalah untuk mendapatkan penyelesaian yang tepat bagi olakan bebas tak mantap bendalir Casson dengan kesan kehadiran hidrodinamik magnet (MHD) yang melalui saluran yang menegak. Persamaan menakluk yang berdimensi telah ditukarkan kepada bentuk tak berdimensi dengan menggunakan pembolehubah tak berdimensi yang sesuai. Parameter tak berdimensi diperolehi melalui proses tak berdimensi seperti parameter Casson, masa, nombor Prandtl, nombor Grashof dan medan magnet. Kaedah penjelmaan Laplace digunakan untuk menyelesaikan persamaan tak berdimensi bersama dengan syarat awal dan syarat sempadan. Penyelesaian bagi profil halaju dan suhu diperolehi. Geseran kulit dan nombor Nusselt juga dikira. Keputusan penyelesaian untuk halaju dan suhu dipaparkan secara graf bagi membincangkan mengenai pengaruh parameter tak berdimensi terhadap profil. Ianya dapat diperhatikan bahawa halaju bendalir meningkat dengan peningkatan nombor Grashof dan masa manakala ianya menurun dengan peningkatan parameter Casson, γ, parameter medan magnet, M and nombor Prandtl, Pr. Di samping itu, profil suhu berkurangan dengan nombor Prandtl yang tinggi manakala meningkat dengan nilai masa, t, yang tinggi. Untuk mengesahkan keputusan, keputusan yang diperolehi dibandingkan dengan keputusan yang telah diterbitkan dan didapati penyesuaian yang sangat baik diperolehi.

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LIST OF ABBREVIATIONS

MHD	-	Magnetohydrodynamics
HPM	-	Homotopy Pertubation Method
VPM	-	Variation Parameter Method
PTS	-	Prescribed Surface Temperature
PHF	-	Prescribed Heat Flux
ODE	-	Ordinary Differential Equation
PDE	-	Partial Differential Equation
Cu	-	Copper
Al_2O_3	-	Aluminium Oxide
TiO_2	-	Titanium Oxide
CuO	-	Copper Oxide
MOS_2	-	Molybdenum Disulphide
Ag	-	Silver

LIST OF SYMBOLS

Roman Letter

A	-	Surface area
B_0	-	applied magnetics field
B_0	-	magnitude of applied magnetics field
c_p	-	specific heat at constant pressure
d	-	Distance between two vertical plates
erf	-	error function
$erfc$	-	complementary error function
exp	-	exponential function
g	-	gravitational acceleration
Gr	-	thermal Grashof number
h	-	Convective heat transfer coefficient
k	-	thermal conductivity
L	-	Laplace transform -
L^{-1}	-	Inverse Laplace transform
M	-	dimensionless magnetic parameter
Nu	-	Nusselt number
Pr	-	Prandtl number
s	-	Laplace transform parameter
T	-	temperature of the fluid near the plate
t	-	dimensionless time
t_0	-	characteristic time
u	-	velociy in x-direction
v	-	velociy in y-direction
x	-	dimensionless coordinate axis of the plate
y	-	dimensionless coordinate axis of the plate
z	-	dimensionless coordinate axis normal to the plate

Greek Letters

γ	-	dimensionless non-Newtonian Casson parameter
β	-	Heat transfer coefficient
ρ	-	density
σ	-	electrical conductivity
ν	-	kinematic viscosity
μ	-	dynamic viscosity
τ	-	dimensionless skin friction

Subscripts

w	-	conditions on the wall at $y=0$
d	-	conditions on the wall at $y=d$
c	-	complementary solution
p	-	particular solution

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CHAPTER 1

INTRODUCTION

1.0 Introduction

This chapter contains introduction of fluid mechanics for heat transfer of Newtonian and non-Newtonian fluids in the research background, statement of problem, objectives of study, scope of study and significance of study.

1.1 Background Study

Fluids are liquids and gases which are deformed with existing of shear stress. On the other hand, fluid is at rest when no shear resistance. Fluid flow behaviours are influenced by two properties of fluid which are density and viscosity. In the real life, fluid has viscosity and no such ideal fluid (inviscid fluid) exists. Viscous fluid can be categorized into two types: Newtonian and Non-Newtonian fluids.

Gases, gaseous mixture, low-molecular-weight liquids and their mixtures exhibit Newtonian behaviour known as simple fluid or Newtonian fluids. Founder of Newtonian fluid, Sir Issac Newton (1642 - 1726), described the fluid flow behaviour as shear stress and shear rate having linearly relationship with a zero intercept and a slope of constant velocity which obeys the Newton's law of viscosity (Alderman and Pipelines, 1977). Newtonian fluid definition fulfill the Navier-stokes equation rather than exhibiting shear viscosity constantly. For the past few decades, researches about Newtonian fluid has been widely conducted since it is a simple fluid and convenient solutions have been obtained. However, its applications are restricted in real life, industrial and other technologies.

Currently, complex fluids such as molten polymers, synthetic lattices, high polymer solutions, and protein solutions (blood, paints, printing ink, starch suspension, toothpaste, sauce, and others) are broadly used in the industry and technology applications (Lu, 1977). Viscous complex fluid is also known as non-Newtonian fluid. This type of fluid behaviour does not follow Newton's law of viscosity which means it has nonlinear relationship or consists of yield stress between shear stress and strain rate. It can be categorized into three groups which are time independent fluid, time dependent fluid and viscoelastic fluid. Time independent fluid is a memoryless fluid in which shear rate of fluid at a point depending only on current shear stress (shear rate dependence) but not depending on past shear stress. It is variously known as purely viscous, inelastic, time independent or generalized Newtonian fluids (Chhabra, 2010). Due to that, most of researchers interested to study time independent fluids as a fluid model since time dependent and viscoelastic fluids are not well-developed due to their complex behaviour. Other than that, shear rate dependence is one of the important behaviour characteristics for non-Newtonian fluid. Besides that, time independent fluid based on shear flow also can be classified into shear-thinning behaviour, shear-thickening or dilatant behaviour and viscoplastic with or without shear-thinning behaviour. Present work will be focused on viscoplastic fluid behaviour due to their importance in engineering practice. It is described that non-zero shear stress which is required to initiate flow or deform (shear). Fluid will behave like an elastic solid if yield stress is greater than external applied shear stress. Meanwhile, fluid will start to flow or deform if external applied shear stress is larger than existing yield stress.

One of the most regularly used viscosity models for viscoplastic fluid is Casson fluid model. In 1959, Casson fluid was introduced by Casson to estimate the pigment-oil suspensions flow behaviour (Khalid, Khan and Shafie, 2015). Casson fluid is a shear thinning liquid which can be assumed that no flow occurs if yield stress is greater than applied shear stress and have an infinite viscosity at zero rate of shear. Its behaviour can be referred in Figure 1.1. Some of famous Casson fluid examples are honey, jelly, blood, tomato sauce, concentrated fruit juices and others.

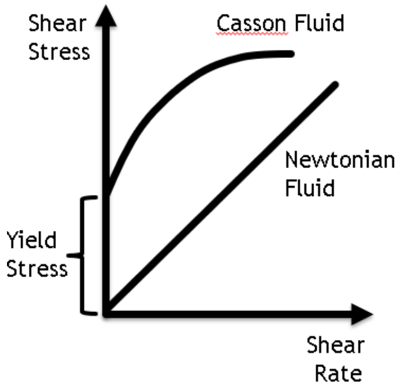

Figure 1.1 Rheogram graph of Casson Fluid

The rheological equation, τ_{ij} of the Casson fluid is given by Mabood and Das (2019) as

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + P_y / \sqrt{2\pi}\right) e_{ij}, & \pi > \pi_c, \\ 2\left(\mu_B + P_y / \sqrt{2\pi_c}\right) e_{ij}, & \pi > \pi_c, \end{cases} \tag{1.1}$$

where μ_B plastic dynamic viscosity, $\pi = e_{ij} e_{ij}$ denotes deformation rate, e_{ij} is the $(i,j)^{th}$ component of deformation rate, π_c is a critical value of non-Newtonian model and P_y is the yield stress of fluid.

Besides that, the effect of free convection flow in Casson fluid flow problem is main operational requirements in engineering systems and technology such as flow through boilers, condensers and others. Free convection or Natural convection can be measured based on the temperature distribution in the fluid flow. It is also one type of heat transferring processes which involve the movement of fluid masses, buoyancy and gravity forces from regions of one temperature to another temperature. In other word, fluid motion occurs when density in fluid is different due to temperature gradient. For example, warm fluid is less dense compared to cold fluid. It will cause buoyancy force to induce a motion of warm fluid to rise and cold fluid to go down by gravity force. Having such motivation in mind, this has motivated many researchers to investigate the behaviour of free convection flow in Casson fluid problem.

One of the sources of the heat transfer for free convection flow comes from the magnetohydrodynamics (henceforth MHD) effect. Magnetohydrodynamics simple means; magneto as in magnetic, hydro as in water or liquid and dynamics as in an object that moves by forces which literally means motion of electrically conducting fluid with interaction of magnetic field. In 1970, Nobel Prize Physics winner, Hannes Alfvén (1908-1995) introduced MHD. Besides that, MHD equation is a combination of the Navier–Stokes equation and Maxwell's equation. MHD concepts are utilized by the engineers in metallurgical industries, heat exchangers design application, electromagnetic stirring in induction motor, casting operation, electrolysis for energy intensive, in space propulsion, military area for propulsion mechanism in submarine and latest in medical science for disease treatment which is related with blood flow. Due to its broad application, MHD effect in Casson fluid mechanic problems have gained interest of researchers to do deeper study about it in current time.

Furthermore, the study of the fluid in vertical channel or between two vertical plates has gained significant interest in recent years due to its application in biological and engineering fields. Fluid in biological organism that flows through blood vessel is one of the most common examples of application as stated above. The vertical channel can be considered as cross-sectional of tube to simplify boundary conditions of complex fluids dynamic problems. Earlier works had solved problems related to this boundary condition is on Newtonian fluid only but recently, latest problems have been extended to non-Newtonian fluid model. This problems have been discussed by Khan, Saqib and Ali, (2017) and Ahmad Qushairi *et al.,* (2018).

Based on the above discussion, it is apt to study the behaviour of the fluid motion influenced by free convection flow and MHD effect past through a vertical channel. Therefore, the present study aims to investigate the unsteady free convection flow of MHD Casson fluid through a vertical channel.

1.2 Statement of Problem

Previous study showed that non-Newtonian fluid flow plays an important role in real life and industrial applications compared to Newtonian fluid. Non- Newtonian fluids such as Casson fluid has attracted many researchers to study about it due to its unique behaviour which involves shear thinning liquid and yield stress behaviour. Most of the researchers' studies involved with heat transfer analysis and MHD effect. They obtained an exact solution by using the Laplace transform technique for the case of Cassson fluid flows through an infinite vertical plate. However, only a few researchers studied the problem of free convection flow of Casson fluid through an infinite vertical channel and without considering the MHD effect. This research is focused on the investigation of the behaviour of unsteady free convection flow of Casson fluid. The fluid flow with the effect of MHD in vertical channel is considered. This research explores the following questions:

(i) How does mathematical model behave in the problem of unsteady free convection flow of Casson fluid in vertical channel with effect of MHD?

(ii) How can analytical solutions for complicated proposed fluids model be obtained for velocity and temperature profile?

(iii) How do the involved physical parameters affect the behaviour of velocity and temperature profiles?

1.3 Objectives of Study

The objectives of this study are:

i) Transform the dimensional governing equation, together with initial and boundary conditions into dimensionless equation by using appropriate dimensionless variables.

ii) Obtain analytical solutions for velocity and temperature profiles by using the Laplace Transform method and compute the skin friction and Nusselt number.

iii) Analyse the obtained analytical solutions for velocity profile, temperature profile, skin friction and Nusselt number with the involvement of physical parameters graphically and tabularly.

1.4 Scope of study

The current study concentrates on the unsteady free convection flow of incompressible Casson fluid in vertical channel specifically. The vertical channel will be considered as fixed (not moving) with constant temperature as its boundary condition. The effect of related parameters to the boundary layer thickness in fluid flow also includes in this study. The presence of MHD flow effect is also considered in this problem. To solve the problem analytically in this study, technique of Laplace transform is used. Lastly, Mathcad software is used to analyse the graphical results.

1.5 Significance of Study

The results obtained from this study will be significant to engineering and mathematics community. Due to some reasons, this study aims to obtain the analytical solutions for the mathematical models with fixed and constant temperature of vertical channel. The obtained analytical results can be used to check the accuracy of the results in numerical study. Besides that, it will enhance the knowledge of fluid velocity and heat transfer analysis in free convection flow of Casson fluid with the presence of MHD through a vertical channel.

1.6 Research Methodology

This section discusses the current research development consists of the research design and procedures of the study. The unsteady dimensional governing equations of momentum and energy of incompressible Casson fluid are obtained from previous work in the form of partial differential equation with initial and boundary conditions. These governing equations are transformed into dimensionless form of governing equations by using appropriate dimensionless variables. Then, Laplace transform method with associated dimensionless initial and boundary conditions is applied into dimensionless governing equations to obtain analytical solutions of velocity and temperature profiles. Besides that, results of skin friction and Nusselt number of the fluid flow are computed.

Analytical results are plotted graphically for velocity and temperature profiles with related parameters such as Casson fluid γ, magnetic field M, Grashof number Gr, and Prandtl number Pr for deeper understanding. MATHCAD software is the main tool to solve inverse Laplace transform and plot all graphs. Additionally, to ensure the solutions are fulfilled the initial and boundary conditions, the graphical results are used. Besides that, the accuracy of the result is validated by comparing the existing solutions available in the previous publications with limited cases of the present work.

1.7 Thesis Organization

This thesis contains five chapters. The first chapter discusses research background which explains the definitions of problem, followed by the statement of problem, the objectives of study, the scope of study, the significance of study, the research methodology and the thesis organization. The second chapter presents the details review on some published researchers which is related to proposed problems as identified in the objectives.

Third chapter discusses on the problem of unsteady free convection flow of MHD Casson fluid past through vertical channel. To simplify the dimensional governing equations with associated initial and boundary conditions, the dimensionless variables are introduced. The method used to obtain the analytical solutions of velocity and temperature profiles is Laplace transform method.

Chapter four presents the validation results by comparing limited case with previous publication. Then, obtained results are plotted graphically for velocity and temperature profiles with related parameters and depicts tabularly for skin friction and Nusselt number. Next, the results are analysed and discussed in detail based on the obtained solutions. Finally, chapter five summarizes this research and includes suggestions for future research. References and appendixes are listed at the end of thesis.

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