ADVANCED MONITORING TECHNIQUES OF STATISTICAL PROCESS CONTROL FOR NORMAL AND NON-NORMAL DISTRIBUTED PROCESSES

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To my family, especially my wife for the nice cuppas of latte and espressos every now and then. Forget bout putting you steady. You may marry someone else. Just a waste of space.

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ABSTRACT

This study investigates efficient computational methods for designing and evaluation phases of the Shewhart type control charts under runs rules. The efficient computational methods considered are exact equations or formulas for computing the probability of single point and run length properties of control charts. Properties of run length include average run length, variance of run length, standard deviation of run length, coefficient of variation of run length, and moments of run length. Issues in control charts can be handled by a generalized skewness correction structure that depends on the amount of skewness instead of the assumption of normality. However, one of the limitations of existing control charts is that actual false alarm rate deviates severely from intended level when probability distribution is unknowingly skewed and/or limited number of samples are used for estimation purposes. To handle the situation when inspection units are selected under ranked set schemes, multivariate control charts are proposed under bivariate ranked set schemes. In addition, a numerical technique is employed for computing power and average run length of mean control chart under ranked set schemes instead of involving Monte Carlo simulation procedures. Besides this, a comparative analysis between false alarm rate based control charts and average run length based control charts with runs rules are conducted. The proposed method is demonstrated on the following applications: generalized skewness correction structure to monitor the chemical properties, Hotelling's T^2 and cumulative sum control chart to monitor the quality of irrigation water, as well as mean control charts to monitor the quality of petrochemical process and groundwater. There are several interesting findings from this study. These include the following outcomes: the current study are interesting because exact equations for computing the probability of single point and run length properties are considered as an alternative to Markov Chain approach for similar purposes; proposed skewness correction structure of mean outperformed the existing mean control charts when process parameters and probability distribution are unknown; numerical method for computing the power and average run length of mean control chart under ranked set schemes is found more time efficient than existing methods based on Monte Carlo simulation; multivariate control charts under bivariate ranked set schemes are found more proficient than existing multivariate control charts under simple random sampling; performance order of runs rules with false alarm rate based control charts are persistent, whereas performance order of runs rules with average run length based control charts are dependent on the circumstances, that is, sample size, size of variation, type of control chart, and side of control limit (upper-sided and lower-sided). For the real data applications, cumulative sum control chart performs outstandingly in detecting small variations in calcium-magnesium and residual sodium contents of irrigation water. Likewise, skewness correction structure has been proven to be excellent in monitoring product purity.

ABSTRAK

Kajian ini menyelidiki kaedah komputasi yang cekap untuk merancang dan menilai fasa carta kawalan jenis Shewhart berdasarkan peraturan larian. Kaedah pengiraan yang cekap adalah persamaan atau formula tepat untuk menghitung kebarangkalian sifat titik tunggal dan panjang larian pada carta kawalan. Sifat panjang larian merangkumi panjang larian purata, varians panjang larian, sisihan piawai panjang larian, pekali variasi panjang larian, dan momen panjang larian. Masalah dalam carta kawalan dapat ditangani oleh struktur pembetulan kecenderungan umum yang bergantung pada jumlah kemiringan dan bukan anggapan normal. Walau bagaimanapun, salah satu batasan carta kawalan adalah kadar penggera palsu yang sebenar menyimpang teruk daripada tahap yang dimaksudkan apabila taburan kebarangkalian tidak diketahui darjah kepencongan dan/atau jumlah sampel yang terhad digunakan untuk tujuan anggaran. Untuk menangani situasi ketika unit pemeriksaan dipilih di bawah skema set peringkat, carta kawalan multivariat dicadangkan di bawah skema set peringkat bivariat. Disamping itu, teknik numerik digunakan untuk daya pengkomputeran dan panjang larian purata carta kawalan min di bawah skema set berperingkat dan bukannya melibatkan prosedur simulasi Monte Carlo. Selain itu, satu analisis perbandingan antara carta kawalan berdasarkan kadar penggera palsu dan carta kawalan berdasarkan panjang larian purata dengan peraturan larian telah dijalankan. Kaedah yang dicadangkan telah ditunjukkan untuk beberapa aplikasi: struktur pembetulan kecenderungan umum untuk memantau sifat kimia, T^2 Hotelling dan carta kawalan jumlah kumulatif untuk memantau kualiti air pengairan, serta carta kawalan min untuk memantau kualiti proses petrokimia dan air bawah tanah. Terdapat beberapa penemuan menarik daripada kajian ini. Ini yang berikut: persamaan tepat untuk mengira kebarangkalian sifat titik tunggal dan panjang larian dianggap sebagai alternatif kepada pendekatan Markov Chain untuk tujuan yang serupa; struktur pembetulan kecenderungan yang dicadangkan min melebihi carta kawalan min yang ada ketika parameter proses dan taburan kebarangkalian tidak diketahui; kaedah berangka untuk mengira daya dan panjang larian purata carta kawalan min di bawah skema set diperingkat didapati lebih berkesan berbanding kaedah yang ada berdasarkan simulasi Monte Carlo; carta kawalan multivariat di bawah skema set peringkat bivariat didapati lebih mahir daripada carta kawalan multivariat yang ada di bawah persampelan rawak mudah; urutan prestasi peraturan berjalan dengan carta kawalan berdasarkan kadar penggera adalah berterusan, sedangkan urutan prestasi peraturan berjalan dengan carta kawalan berdasarkan panjang larian purata bergantung pada keadaan, iaitu, ukuran sampel, ukuran variasi, jenis carta kawalan, dan sisi had kawalan (sisi atas dan sisi bawah). Untuk aplikasi data sebenar, carta kawalan jumlah terkumpul menunjukkan prestasi yang baik dalam mengesan variasi kecil kalsium-magnesium dan kandungan natrium sisa dalam air pengairan. Begitu juga, struktur pembetulan kecenderungan telah terbukti sangat baik dalam memantau ketulenan produk.

TABLE OF CONTENTS

CHAPTER	TITLE		PAGE
	DECLARATION		iii
	DEDICATION		v
	ACKNOWLEDGEMENT		vii
	ABSTRACT		ix
	ABSTRAK		xi
	TABLE OF CONTENTS		xiii
	LIST OF TABLES		xvii
	LIST OF FIGURES		xxi
	LIST OF ABBREVIATIONS		xxv
	LIST OF SYMBOLS		xxvii
	LIST OF APPENDICES		xxxi
1	**INTRODUCTION**		**1**
	1.1	Background of the Study	1
	1.2	Problem Statements	3
	1.3	Motivation of the Study	5
	1.4	Research Questions	5
	1.5	Research Hypotheses	6
	1.6	Research Objectives	7
	1.7	Study Methods	7
	1.8	Scope of the Study	8
	1.9	Limitation of the Study	9
	1.10	Significance of the Study	9
	1.11	Thesis Organization	9
2	**LITERATURE REVIEW**		**11**
	2.1	Generalized Geomtric Distribution and its Properties	11
	2.2	Control Charts Based on Runs Rules	12

	2.3	Mean Control Chart Under SRS		15
	2.4	Multivariate Control Charts Based on SRS		16
	2.5	Mean Control Chart Under Ranked Set Techniques		17
	2.6	Gap Analysis		18
3	**METHODOLOGY**			**21**
	3.1	Proposed Properties of Generalized Geometric Distribution		21
	3.2	Proposed Equations for Computing the IPSP and RLP of Shewhart Control Chart		31
		3.2.1	Implementation procedures of proposed equations in designing and evaluation phases of Shewhart control chart	37
			3.2.1.1 $\bar{X}$ control chart	38
			3.2.1.2 S^2 control chart	40
			3.2.1.3 S control chart	41
			3.2.1.4 Multivariate T^2 control chart	43
			3.2.1.5 Generalized variance $\|S\|$ control chart	45
	3.3	Proposed Generalized Skewness Correction Structure		47
	3.4	Proposed Estimator Under Bivariate Ranked Set Techniques		48
	3.5	Proposed Hotelling's T^2 Control Chart Under Bivariate Ranked Set Techniques		62
		3.5.1	Control chart statistic	63
		3.5.2	Control limit	63
	3.6	Proposed Multivariate Cumulative Sum Control Chart Under Bivariate Ranked Set Schemes		64
		3.6.1	Construction of plotting statistic	64
		3.6.2	Construction of control limit	65
	3.7	Exact Expression for Computing AFAR, Power, AIARL, and OARL through Numerical Method		68
		3.7.1	Generalized design structure of mean control chart under ranked set schemes	68
		3.7.2	Pivotal quantity	69
		3.7.3	Exact expression for computing the AFAR through numerical method	71

		3.7.4	Exact expression for computing the power through numerical method	71
		3.7.5	Exact expression for computing the AIARL and OARL through numerical method	72
	3.8	Proposed Code for Computing Constant and Power of Control Chart		72
		3.8.1	Control chart structures	72
		3.8.2	Procedure/algorithm for computing constants	74
			3.8.2.1 Unbiasing constant	74
			3.8.2.2 Corrected constants	75
		3.8.3	Development and illustration of program in R language	76
	3.9	Dual Purpose Control Charts with Runs Rules Based on the Generalized Polynomial Equation		79
		3.9.1	Generalized polynomial equation for computing the IPSP of Shewhart-type control charts	79
		3.9.2	Design structures of dual purpose Shewhart-type control charts under runs rules	81
4	**RESULTS AND DISCUSSION**			**85**
	4.1	Special Cases of the Moments of Generalized Geometric Distribution of Order k		85
	4.2	Role of the Exact Formulas for Computing Run Length Properties of Control Charts Under k/k Runs Rules		86
	4.3	Proposed Generalized Skewness Correction Structure and Existing Structures		98
	4.4	Effect of Factors on the Performance of Proposed Hotelling's T^2 Control Chart Under Bivariate Ranked Set Schemes		113
	4.5	Role of Factors on the Performance of Proposed Multivariate Cumulative Sum Control Chart Under Bivariate Ranked Set Schemes		127
		4.5.1	Comparative analysis	132

	4.6	Comparative Analysis Between Numerical Method and Monte Carlo Simulation Method for Computing Performance Measures of Mean Control Chart	148
	4.7	Interpretation of the Outputs Along with Advantages of Proposed Code in R Language	154
	4.8	Behavior of Dual Purpose Control Charts Under Runs Rules	164
		4.8.1 $\bar{X}_U$ and $\bar{X}_L$ control charts	164
		4.8.2 S_U^2, S_L^2, S_U, S_L, $\lvert S\rvert_L$ and $\lvert S\rvert_U$ control charts	170
		4.8.3 T_U^2 control chart	179
	4.9	Summary	182
5	**REAL LIFE APPLICATIONS OF CONTROL CHARTS**		**185**
	5.1	Generalized Skewness Correction Structure and Existing Structures for Monitoring Chemical Properties	185
	5.2	Hotelling's T^2 Control Chart for Monitoring Quality of Irrigation Water	189
	5.3	Multivariate Cumulative Sum Control Chart for Monitoring the Quality of Irrigation Water	190
	5.4	Mean control charts for Monitoring Quality of Petrochemical Process and Groundwater	194
		5.4.1 Monitoring of plasticizer characteristic of petrochemical process	195
		5.4.2 Monitoring of physicochemical characteristic of groundwater	197
6	**CONCLUSION AND RECOMMENDATION**		**199**
	6.1	Introduction	199
	6.2	Contributions	200
	6.3	Future Recommendations	202
REFERENCES			**205**
Appendices A – B			211 – 217

LIST OF TABLES

TABLE NO.	TITLE	PAGE
3.1	d_2^{**} at different choices of k_3, $L = 20$, n, and E	50
3.2	$C_{\alpha/2}$ and $C_{1-\alpha/2}$ for varying choices of n, $L = 20$, E, and $\alpha = 0.0027$	50
3.3	$C^{**}_{L(4)}$ and $C^{**}_{U(4)}$ for varying choices of k_3, $L = 20$, n, E, and $\alpha = 0.0027$	51
3.4	C^{**}_L and C^{**}_U for varying choices of k_3, $k = 20$, n, E, and $\alpha = 0.0027$	52
3.5	Sample correlation (ρ_D) for each choice D with varying choices of ρ_{YZ}, ρ_{XZ}, ρ_{YX} and n	62
3.6	Control limits ($LCL_D = 0$, UCL_D) of proposed control chart for each choice D with varying choices of ρ_{YZ} , ρ_{XZ}, n and $\alpha = 0.0027$	64
3.7	Control limit constant H of proposed MC_D control chart for each choice D with varying choices of ρ_{YZ}, ρ_{XZ}, n, $k^* = 0.50$, and $ARL_{True} = 200$	66
3.8	Values of $k_{2(D_I,n,\rho_{YX})}$ at a given choice of sampling strategy and some choices of sample size n	69
3.9	Arguments for the distribution along with their corresponding parameters	77
4.1	$IRLP$ and $ORLP$ of $\bar{X}_U$ control chart at different choices of k, δ, n and $ARL_{True} = 370$	87
4.2	$IRLP$ and $ORLP$ of $\bar{X}_L$ control chart at different choices of k, δ, n and $ARL_{True} = 370$	88
4.3	$IRLP$ and $ORLP$ of S^2_U control chart at different choices of k, δ, n and $ARL_{True} = 370$	89
4.4	$IRLP$ and $ORLP$ of S^2_L control chart at different choices of k, δ, n and $ARL_{True} = 370$	90
4.5	$IRLP$ and $ORLP$ of S_U control chart at different choices of k, δ, n and $ARL_{True} = 370$	91

4.6	$IRLP$ and $ORLP$ of S_L control chart at different choices of k, δ, n and $ARL_{True} = 370$	92
4.7	$IRLP$ and $ORLP$ of Hotelling T^2_U control chart at different choices of k, d_1, n, $\beta = 3$ and $ARL_{True} = 370$	93
4.8	$IRLP$ and $ORLP$ of $\|S\|_U$ control chart at different choices of k, d^*_1, n and $ARL_{True} = 370$	94
4.9	$IRLP$ and $ORLP$ of $\|S\|_L$ control chart at different choices of k, d^*_1, n and $ARL_{True} = 370$	95
4.10	Actual false alarm rate of different structures for $D^* =$ Lognormal, $L = 20$, $\alpha = 0.0027$, varying choices of k_3, n, and E	100
4.11	Actual false alarm rate of different structures for $D^* =$ Weibull, $L = 20$, $\alpha = 0.0027$, varying choices of k_3, n, and E	101
4.12	Percentage of absolute difference (PAD) of different structures for $D^* =$ Lognormal, $k = 20$, $\alpha = 0.0027$, varying choices of k_3, n, and E	102
4.13	Percentage of absolute difference (PAD) of different structures $D^* =$ Weibull, L = 20, $\alpha = 0.0027$, varying choices of k_3, n, and E	103
4.14	Actual false alarm rate of different structures for $D^* =$ Lognormal, $k_3 = 0$, $k = 20$, $\alpha = 0.0027$, varying choices of E, and n	103
4.15	Actual false alarm rate of different structures for $D^* =$ $Burr3$, $k_3 = 0$, $L = 20$, $\alpha = 0.0027$, varying choices of E, and n	104
4.16	ARL_1, EQL, $RARL$, and PCI values of MC_{SRS} and proposed MC_D control charts for each choice D with varying choices of ρ_{YZ}, ρ_{XZ}, $n = 3$ and $ARL_{True} = 200$	129
4.17	ARL_1, EQL, $RARL$, and PCI values of MC_{SRS} and proposed MC_D control charts for each choice D with varying choices of ρ_{YZ}, ρ_{XZ}, $n = 5$ and $ARL_{True} = 200$	131
4.18	ARL_1, EQL, $RARL$, and PCI values of MC_{SRS}, T^2_{SRS}, T^2_D, and proposed MC_D control charts for each choice D with varying choices of $\rho_{YZ} = 0.50$, $\rho_{XZ} = 0.50$, $n = 5$ and $ARL_{True} = 200$	147
4.19	Performance measures through Monte Carlo Simulation Method (MCSM) and Numerical Method (NM) for varying choices of D_1, δ, $n = 5$ and $\rho_{XY} = 1$	149

4.20 Performance measures through Monte Carlo Simulation Method (MCSM) and Numerical Method (NM) for varying choices of D_1, δ, $n = 8$ and $\rho_{XY} = 1$ 150

4.21 Performance measures through Monte Carlo Simulation Method (MCSM) and Numerical Method (NM) for varying choices of D_1, δ, $n = 10$ and $\rho_{XY} = 1$ 151

4.22 Performance measures through Monte Carlo Simulation Method (MCSM) and Numerical Method (NM) for varying choices of D_1, δ, $n = 5$ and $\rho_{XY} = 0.75$ 152

4.23 Time (expressed in Minutes) consumed by the MCSM and NM for computing power P_1 at different choices of D_1 and n 153

4.24 Unbiasing constants based on pooled sample standard deviation $C_{4(D^*,L,n)}$ at different choices of PD, L and n 155

4.25 Corrected constants when parameters are unknown $(C_{(D^*,L,n,r,k,p_0/2)}, C_{(D^*,L,n,r,k,1-p_0/2)})$ for varying choice of PD, $L = 20$, $n = 6$, r, and k at $\alpha = 0.0027$ 155

4.26 Corrected constants when parameters are unknown $(C_{(D^*,L,n,r,k,p_0/2)}, C_{(D^*,L,n,r,k,1-p_0/2)})$ for varying choice of PD, $L = 30$, $n = 6$, r, and k at $\alpha = 0.0027$ 156

4.27 Corrected constants when parameters are known $(C^*_{(D^*,n,r,k,p_0/2)}$ and $C^*_{(D^*,n,r,k,1-p_0/2)})$ for different choices of PD, r, k, $n = 6$ and $\alpha = 0.0027$ 156

4.28 False alarm rate of Eq.(3.69) and Eq.(3.70) for varying choices of $L = 20$, r, k, n and at $\alpha = 0.0027$ 157

4.29 False alarm rate and power of Eq.(3.70) based on Eqs. (3.72) and (3.73) with varying choices of r, k, $n = 6$, PD=normal and $\alpha = 0.0027$ 158

4.30 α_{act} and ARL_0 of proposed control charts at $\alpha = 0.0027$, $ARL_{True} = 370$, $\delta_1 = 0$, $\delta_2 = 0$, $\delta_3 = 1$, $\delta_4 = 1$, $d^* = 1$, $d^* = 0$ and various choices of runs rules k/k 166

4.31 Power P_1 and out-of-control average run length ARL_1 of $\bar{X}_U$ control chart at $n = 5$, $\alpha = 0.0027$, $ARL_1 = 370$ and different choices of k/k and δ_1 167

4.32 Power P_1 and out-of-control average run length ARL_1 of $\bar{X}_L$ control chart at $n = 5$, $\alpha = 0.0027$, $ARL_{True} = 370$ and different choices of k/k and δ_2 168

4.33 Power P_1 and out-of-control average run length ARL_1 of S^2_U control chart at $n = 5$, $\alpha = 0.0027$, $ARL_{True} = 370$ and different choices of k/k and δ_3 172

4.34	Power P_1 and out-of-control average run length ARL_1 of proposed S_L^2 control chart at $n = 5$, $\alpha = 0.0027$, $ARL_{True} = 370$ and different choices of k/k and δ_4	173
4.35	Power P_1 and out-of-control average run length ARL_1 of proposed S_U control chart at $n = 5$, $\alpha = 0.0027$, $ARL_{True} = 370$ and different choices of k/k and δ_3	174
4.36	Power P_1 and out-of-control average run length ARL_1 of S_L control chart at $n = 5$, $\alpha = 0.0027$, $ARL_0 = 370$ and different choices of k and δ_4	175
4.37	Power P_1 and out-of-control average run length ARL_1 of proposed $\|S\|_U$ control chart at $n = 5$, $\alpha = 0.0027$, $ARL_{True} = 370$ and different choices of k and d_2^*	176
4.38	Power P_1 and out-of-control average run length ARL_1 of $\|S\|_L$ control chart at $n = 5$, $\alpha = 0.0027$, $ARL_0 = 370$ and different choices of k and d_2^*	177
4.39	Power P_1 and out-of-control average run length ARL_1 of proposed T_U^2 control chart at $n = 5$, $\alpha = 0.0027$, $ARL_{True} = 370$ and different choices of k and d	181
5.1	Summary of out-of-control signals by USS, SCS and GSCS	188
5.2	Summary of out-of-control signals by different control charts	194

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
3.1	Flowchart to show the relationship among the research objectives and proposed methodology	22
3.2	Procedure for the higher order moments of generalized geometric distribution of order k	24
3.3	Flowchart for the exact equations to compute the IPSP and RLP of Shewhart control charts	33
3.4	Flowchart for the generalized skewness correction structure of mean control chart	49
3.5	Procedure for collecting a sample of size 5 using BVRSS	55
3.6	Procedure for collecting a sample of size 5 using BVMRSS	56
3.7	Procedure for collecting a sample of size 5 using BVERSS	57
3.8	Kernel density plot of Z and Q at various choices of n, D_I, and $\rho_{XY} = 0.75$	70
4.1	PAD of different structures with varying choices of skewness, D^*, $E = \bar{R}$, $n = 6$, $L = 20$, and $\alpha = 0.0027$	105
4.2	PAD of different structures with varying choices of skewness, D^*, $E = \bar{S}$, $n = 6$, $L = 20$, and $\alpha = 0.0027$	106
4.3	PAD of different structures with varying choices of skewness, D^*, $E = \overline{IQR}$, $n = 6$, $L = 20$, and $\alpha = 0.0027$	107
4.4	PAD of different structures with varying choices of skewness, D^*, $E = \bar{R}$, $n = 6$, $L = 20$, and $\alpha = 0.0027$	108
4.5	In-control ($d = 0$) run length distribution of proposed control chart (legend as Actual) and probability distribution of geometric random variable (legend as True) with parameter $\alpha = 0.0027$	115
4.6	In-control ($d = 0$) run length distribution of proposed and classical Hotelling T^2 control charts for a given choice of D, ρ_{YZ}, ρ_{XZ}, n and $\alpha = 0.0027$	116

4.7	Out-of-control ($d \neq 0$) run length distribution of proposed Hotelling T^2 control chart under ranked set schemes and classical Hotelling T^2 control chart under SRS at $\alpha = 0.0027$	117
4.8	$ARL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, n and $\alpha = 0.0027$	118
4.9	$ARL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, ρ_{YZ}, ρ_{XZ} and $\alpha = 0.0027$	119
4.10	$MRL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, n and $\alpha = 0.0027$	120
4.11	$MRL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, ρ_{YZ}, ρ_{XZ} and $\alpha = 0.0027$	121
4.12	$P_{75(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, n and $\alpha = 0.0027$	122
4.13	$P_{75(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, ρ_{YZ}, ρ_{XZ} and $\alpha = 0.0027$	123
4.14	$P_{90(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, n and $\alpha = 0.0027$	135
4.15	$P_{90(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, ρ_{YZ}, ρ_{XZ} and $\alpha = 0.0027$	136
4.16	$ECDFRL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d and $\alpha = 0.0027$	137
4.17	$ECDFRL_{(1)}$ curve of proposed Hotelling T^2 control chart for varying choices of d, ρ_{YZ}, ρ_{XZ} and $\alpha = 0.0027$	138
4.18	$ECDFRL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, n and at $\alpha = 0.0027$	139
4.19	$ECDFRL_{(1)}$ curves of proposed Hotelling T^2 control chart for varying choices of d, ρ_{YZ}, ρ_{XZ} and at $\alpha = 0.0027$	140
4.20	ARL_1 curves of proposed MC_D control chart for varying choices of d, n and $ARL_{True} = 200$	141
4.21	ARL_1 curves of proposed MC_D control chart for varying choices of ρ_{YZ} and ρ_{XZ} at $ARL_{True} = 200$	142
4.22	RL curves of proposed MC_D control chart for varying choices of d at $ARL_{True} = 200$	143
4.23	RL curves of proposed MC_D control chart for varying choices of D and with specific choices of ρ_{YZ}, ρ_{XZ}, and $ARL_{True} = 200$	144
4.24	RL curves of proposed MC_D control chart for varying choices of n and $ARL_{True} = 200$	145

4.25	RL curves of proposed MC_D control chart for varying choices of ρ_{YZ} and ρ_{XZ} at $ARL_{True} = 200$	146
4.26	CR(para=“False”, simu,r,k,alpha,L=20,n=6,delta,PD= “normal”,mean=0,sd=1)	159
4.27	CR(para=“False”, simu,r,k,alpha,L,n=8,delta,PD=“logistic”,location=1,scale=1)	159
4.28	CR(para=“True”, simu,r,k,alpha,L,n=8,delta,PD=“weibull”,shape=6,scale=1)	160
4.29	CR(para=“True”, simu,r,k,alpha,L,n=8,delta,PD=“chisquare”,df=20)	160
4.30	CR(para=“False”, simu,r=1,k=1,alpha,k,n=3,delta,PD=“normal”,mean=0,sd=1)	161
4.31	CR(para=“False”, simu,r=2,k=3,alpha,k,n=3,delta,PD=“normal”,mean=0,sd=1)	161
5.1	Process flow scheme for the DOP manufacturing process	187
5.2	Different structures of $\bar{X}$ control chart for the monitoring of Product Purity	188
5.3	Real life example of proposed control chart under BVMRSS for monitoring of irrigation water	190
5.4	Probability density plot of the standardized bivariate variables (residual sodium carbonates X and calcium magnesium contents Y)	192
5.5	Implementation of MC_D and T_D^2 control charts on real life data when small shift occurs in the process characteristics (calcium-magnisum and residual sodium contents in irrigation water)	193
5.6	Monitoring of plasticizer characteristic in a petrochemical process	197
5.7	Monitoring of physicochemical characteristic of groundwater	198

LIST OF ABBREVIATIONS

AFAR	–	Actual false alarm rate
AIARL	–	Actual in-control average run length
ARL	–	Average run length
BVDERSS	–	Bivariate double extreme ranked set sampling
BVDMRSS	–	Bivariate double median ranked set sampling
BVDRSS	–	Bivariate double ranked set sampling
BVERSS	–	Bivariate extreme ranked set sampling
BVMRSS	–	Bivariate median ranked set sampling
BVRSS	–	Bivariate ranked set sampling
CSS	–	Corrected Shewhart structure
CUSUM	–	Cumulative sum
CVRL	–	Coefficent of variation of run length
ECDFRL	–	Emperical cumulative distribution function of run length
EQL	–	Extra quadratic loss
ERSS	–	Extreme ranked set sampling
EWMA	–	Exponentially weighted moving average control chart
FAR	–	False alarm rate
IARL	–	In-control average run length
IRLP	–	In-control run length properties
MCA	–	Markove chain approach
MRL	–	Median of run length
MRSS	–	Median ranked set sampling
OARL	–	Out-of-control average run length
ORLP	–	Out-of-control run length properties
PCI	–	Performance comparision index
PRL	–	Percentile of run length
RARL	–	Relative average run length
RSS	–	Ranked set sampling

SCS	–	Skewness correction structure
SDRL	–	Standard deviation of run length
SPC	–	Statistical process control
SRS	–	Simple random sampling
VRL	–	Varince of run length
	–	

LIST OF SYMBOLS

ARL_{True}	–	True or prefixed in-control average run length
ARL_0	–	Actual in-control average run length
ARL_{1}	–	Out-of-control average run length
$AADM^*$	–	Average absolute deviation from median
$\overline{AADM^*}$	–	Mean of average absolute deviation from median
C_U	–	Normality based upper-sided corrected multiplier
C_L	–	Normality based lower-sided corrected multiplier
$C_{4(L(n-1)+1)}$	–	Normality based unbiasing constant of $\widetilde{S}$
$C_{4(n)}$	–	Normality based unbiasing constant of $\bar{S}$
C_L^*	–	Skewness correction based lower-sided control limit multiplier
C_U^*	–	Skewness correction based upper-sided control limit multiplier
$C_{4(L)}^*$	–	Lower-sided skewness correction factor
$C_{4(U)}^*$	–	Upper-sided skewness correction factor
$CVRL_0$	–	Actual in-control coefficent of variation of run length
$CVRL_{\mathit{1}}$	–	Out-of-control coeffcent of variation of run length
C_L^{**}	–	Skewness based lower-sided corrected control limit multiplier
C_U^{**}	–	Skewness based upper-sided corrected control limit multiplier
$C_{\alpha/2}$	–	Normality based lower-sided corrected control limit multiplier
$C_{1-\alpha/2}$	–	Normality based upper-sided corrected control limit multiplier
d_2^*	–	Skewness correction constant
d_2^{**}	–	Generalized skewness correction constant
D	–	Bivariate ranked set schemes
DN	–	Downton statistic
$\overline{DN}$	–	Mean of downton statistic
D^*	–	Skewed distribution
$d_{2(n)}$	–	Normality based unbiasing constant of $\bar{R}$
$d_{2(2)}$	–	Normality based unbiasing constant of $\bar{G}$
d_2	–	Mean of the relative dispersion statistic E/σ_0

E	–	Dispersion statistic
$ECDFRL_{(1)}$	–	Out-of-control emperical cumulative distribution function of run length
e_1	–	Optimize constant of lower-sided skewness correction constant
e_2	–	Optimize constant of upper-sided skewness correction constant
$F_{1[k]}$	–	First factorial moment of order k
$F_{2[k]}$	–	Second factorial moment of order k
$F_{3[k]}$	–	Third factorial moment of order k
$F_{4[k]}$	–	Fourth factorial moment of order k
G	–	GINI's mean sample differences
$\overline{G}$	–	Mean of GINI's mean sample differences
IQR	–	Sample interquartile range
$\overline{IQR}$	–	Mean of sample interquartile range
k_3	–	Skewness
$k_3(\overline{X})$	–	Skewness of sample mean
k	–	Number of concecutive successes
k/k	–	k statistic out-of-control out of k concecutive statistic
L	–	Number of samples
$\widehat{LCL}$	–	Estimated lower control limit
$\widetilde{M}$	–	Sample median
$MRL_{(0)}$	–	Actual in-control median of run length distribution
$MRL_{(1)}$	–	Out-of-control median of run length distribution
M_3RL_0	–	Actual in-control third moment about mean of run length distribution
$M_3RL_{\boldsymbol{1}}$	–	Out-of-control third moment about mean of run length distribution
M_4RL_0	–	Actual in-control fourth moment about mean of run length distribution
$M_4RL_{\boldsymbol{1}}$	–	Out-of-control fourth moment about mean of run length distribution
M^*ADM	–	Median absolute deviation from mean
$\overline{M^*ADM}$	–	Mean of median absolute deviation from mean
M^*ADM^*	–	Median absolute deviation from median
$\overline{M^*ADM^*}$	–	Mean of median absolute deviation from median
MD	–	Mean deviation
$\overline{MD}$	–	Sample mean deviation

n	–	Sample size
p_0	–	In-control probability of single point
$P_{75(0)}$	–	Actual in-control 75th percentile of run length distribution
$P_{75(1)}$	–	Out-of-control 75th percentile of run length distribution
PWM	–	Probability weighted moment
$\overline{PWM}$	–	Mean of probability weighted moment
$q(n)$	–	Normality based unbiasing constant of $I\bar{Q}R$
Q_n	–	$Q - n$ statistic
$\overline{Q_n}$	–	Mean of $Q - n$ statistic
R	–	Sample range
$\overline{R}$	–	Mean of sample range
R_3RL_0	–	Actual in-control third moment about origion of the run length distribution
$R_3RL_{\boldsymbol{1}}$	–	Out-of-control third moment about origion of run length distribution
R_4RL_0	–	Actual in-control fourth moment about origion of run length distribution
$R_4RL_{\boldsymbol{1}}$	–	Out-of-control fourth moment about origion of run length distribution
S	–	Sample standard deviation
S_U^2	–	Upper-sided variance control chart
S_L^2	–	Lower-sided variance control chart
S_U	–	Upper-sided standard deviation control chart
S_L	–	Lower-sided standard deviation control chart
$SDRL_0$	–	Actual in-control standard deviation of run length distribution
$SDRL_{\boldsymbol{1}}$	–	Out-of-control standard deviation of run length distribution
$\bar{S}$	–	Mean of sample standard deviation
$\widetilde{S}$	–	Pooled sample standard deviation
$\|S\|_U$	–	Upper-sided generalized variance control chart
$\|S\|_L$	–	Lower-sided generalized variance control chart
T_U^2	–	Upper-sided Hotelling's T-squared control chart
$\widehat{UCL}$	–	Estimated upper control limit
VRL_0	–	Actual in-control variance of run length distribution
$VRL_{\boldsymbol{1}}$	–	Out-of-control variance of run length distribution
$\overline{X}$	–	Mean
$\overline{\overline{X}}$	–	Combined sample means

$\overline{X}_L$	–	Lower-sided mean control chart
$\overline{X}_U$	–	Upper-sided mean control chart
α	–	True or prefixed false alarm rate
α_{act}	–	Actual false alarm rate
σ	–	In-control standard deviation
$\widehat{\sigma}$	–	Normality based unbiased dispersion estimator
$\widehat{\sigma}^*$	–	Skewness correction based dispersion estimator of σ_0
μ	–	In-control mean
	–	

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
A	Terminologies	211
B	List of Publications	217

CHAPTER 1

INTRODUCTION

1.1 Background of the Study

A production process consists of many characteristics or variables of interest. Each characteristic has parameters (location and/or dispersion). The stability of a parameter is desired for producing the product at specified standard. It is a fact that the parameter is affected by two sources of variations (Montgomery, 2009). The two sources of variations are listed as natural (common) and un-natural (special). Furthermore, the common variations are permanent and always be a part of running process. However, it is small in magnitude and not easy to remove. On the other hand, special variations are considered as a special cause of variations, and harmful for a process. The magnitude of special variations is called as shift in statistical process control (SPC). In many practices, the interest is to search out the special cause of variations by using SPC. The SPC is a method to find out the special cause of variations that occurs in any production or manufacturing process. The SPC represents a collection of tools which is constituted of Pareto diagram, check sheets, Histogram, and control chart. In aforementioned tools, most popular one is quality control chart.

Quality control charts are classified into two categories on the basis of detection ability, that is, memory and memory less control charts. In details, memory less techniques have significant role for diagnosing larger shifts in a parameter, whereas memory control techniques are appropriate for handling smaller variations (Montgomery, 2009). Most popular used memory less chart for the location parameter is mean, whereas for dispersion parameter are variance, standard deviation and range. Likewise, exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) control chart belong to memory control charts. The mean, EWMA, and CUSUM control charts are utilized to monitor one or more related characteristic independently, and therefore termed as univariate control charts. Also, in daily life practices, joint monitoring of related variables is required. For these situations, practitioners prefer multivariate schemes of charts such as multivariate Hotelling's T^2, EWMA, and CUSUM as an alternative to univariate mean, CUSUM and EWMA, respectively.

The computation of in-control probability of single point (IPSP) and run length properties (RLP) are considered important in designing and evaluation phases of control charts under runs rules. Most commonly used method for computing the IPSP is Markove chain approach (MCA). The MCA deals with recursive system of linear equations with additional constraints. To compute the correct value of IPSP and average run length (ARL), one of the most frequent used computational techniques is known as MCA. The implementation of MCS can be seen in several studies such as Adeoti and Malela-Majika (2020), Jungtaek and Weiß (2020), Rakitzis and Antzoulakos (2014), Koutras *et al.* (2007), Khoo (2003), Antzoulakos and Rakitzis (2010), Antzoulakos and Rakitzis (2007), Acosta-Mejia and Pignatiello Jr (2009) and Acosta-Mejia (2007). Moreover, based on MCA, many authors have provided formula for computing ARL generally. Here, note that the solution of recursive equations become tedious as number of decision points increases. For instance, two sided control chart under two out-of-two runs rules requires three recursive system of linear equation with constraints. Likewise, three-out-of-three runs rules requires seven recursive system of linear equation with constraint. In this practice, one may think exact equations for obtaining the IPSP and RLP of control charts under k out of k runs rules to reduce computation burden instead of involving the MCA.

There are several practical situations in which either process parameters or probability distribution are unknown. To cover scenarios when process parameters are unknown, advanced mean control charts were developed. Schoonhoven *et al.* (2009) offered $\bar{X}$ control chart structure for unknown parameters under the assumption of known normal distribution. Furthermore, they compensated effect of parameters estimations by replacing the usual control limits multipliers with corrected multipliers. The main ideology behind the mean control chart for unknown parameter is to consider the effect of parameters estimation in the control limits multipliers. Also, the mean control charts are depended on the restricted assumption of normality. To cover the situation when both process parameters and probability distribution are unknown, skewness correction mean control charts were introduced. Chan and Cui (2003) and Riaz *et al.* (2016b) designed skewness correction structure of $\bar{X}$ control chart for unknown parameters and unknown skewed distribution. The skewness correction mean control chart are considered proficient to control the actual false alarm rate

(AFAR) close to true level. Furthermore, existing skewness correction structure of mean control chart take into account the effect of skewness but discard the effect of parameter estimation. The performance of the skewness correction mean control chart can be increased by adjusting both effects of skewness and parameters estimation.

The structure of aforementioned mean control charts are confined to simple random sampling (SRS). In this research direction, various types of mean control charts were developed with ranked set technique. The ranked set schemes is used in the situation when a process units (items) can be ranked by personal judgment or an associated concomitant variable prior to actual measurement of the study variables. Salazar and Sinha (1997), Muttlak and Al-Sabah (2003), Abujiya and Muttlak (2004), and Mehmood *et al.* (2013a) proposed mean control chart under ranked set schemes. The performance of mean control chart with ranked set techniques were evaluated through Monte Carlo simulation (MCS) procedures. Besides, towards detection of small shifts in a process parameters, Al-Sabah (2010) proposed CUSUM control chart with ranked set schemes. The mean control charts with different ranked set techniques are used to monitor he multiple characteristics in an independently manner (known as univariate control charts). To monitor the several related characteristics simultaneously, Hotelling (1947) formulated multivariate Hotelling's T^2 control chart with SRS. The Hotelling's T^2 control chart is considered an advanced form of univariate mean control chart under SRS. Likewise, Pignatiello Jr and Runger (1990) designed multivariate CUSUM control chart under SRS as a substitute of univariate CUSUM control chart under SRS. The aforesaid multivariate control charts are limited to SRS (Hotelling, 1947 and Pignatiello Jr and Runger, 1990) which can be extended towards bivariate ranked set schemes.

Philippou *et al.* (1983) derived some properties of generalized geometric distribution (GGD) of order k which include probability generating function, mean and variance. It is well known that mean is used to measure the central tendency, whereas variance, standard deviation and coefficient of variation are utilized to measure the spread in a probability distribution. Furthermore, higher order moments are employed to describe the shape of probability or frequency distribution.

1.2 Problem Statements

The main goal of study is to enhance the current schemes of control charts and to propose solutions to contribute to the body of knowledge. Now the problem statements are elaborated in details.

(a) Existing properties of GGD of order k are available for describing the central tendency and spread of a data set (Philippou *et al.*, 1983). Regarding the shape of probability or frequency distribution, it is hard to find exact formulas for computing the higher order moments.

(b) MCA for computing the IPSP and RLP of control charts under k out of k runs rules deals with recursive system of linear equations (Rakitzis and Antzoulakos, 2014; Koutras *et al.*, 2007; Khoo, 2003). The linear equations are dependent on the choice of decision rule k/k. Furthermore, MCA needs extensive computational knowledge, and becomes tedious as value of k increases.

(c) Corrected Shewhart structures for unknown parameters are based on the assumption of known probability distributions (Schoonhoven *et al.*, 2009; Schoonhoven and Does, 2010). These structures becomes less efficient when process distribution is unknown and more specifically unknown skewed.

(d) Skewness correction structure does not fulfil certain statistical properties when limited number of samples are available for estimation of control limits or when data is moderate to highly skewed (Chan and Cui, 2003).

(e) Existing multivariate control charts are based on SRS (Hotelling, 1947 and Pignatiello Jr and Runger, 1990). These control charts remain meaningful as long as the inspection items from an ongoing process are selected under SRS.

(f) Univariate control chart under ranked set schemes deals with two or more related characteristics independently (Salazar and Sinha, 1997; Muttlak and Al-Sabah, 2003; Abujiya and Muttlak, 2004; Mehmood *et al.*, 2013a). In many circumstances, simultaneous or joint monitoring of the characteristics are required. For such situations, univariate control charts under ranked set schemes do not maintain important statistical properties.

(g) Power and ARL of mean control chart have been computed through MCS procedure (Salazar and Sinha, 1997; Muttlak and Al-Sabah, 2003; Abujiya and Muttlak, 2004; Mehmood *et al.*, 2013a). The computation of power and ARL requires a lot of time and good computational machine, especially when double ranked set schemes are considered.

(h) Control charts with runs rules based on exiting polynomial equations (Riaz *et al.*, 2011; Mehmood *et al.*, 2018) serve the single purpose, that is, maintain the AFAR equal to prefix false alarm rate. These control charts do not serve the purpose to sustain the actual in-control average run length (AIARL) at prefix in-control average run length.

1.3 Motivation of the Study

Various research articles are available on the topic of GGD, univariate and multivariate quality control charts (see Sec.1.1). However, limited formulas are available for computing the properties of GGD. Besides, existing control charts performed efficient when certain conditions are fulfilled (details are given in Sec.1.2). The motivation of current study is to propose methods by considering the problem statements. Some brief points regarding the motivation of present study on the basis of problem statements (see Sec.1.2) are as follows:

(a) Derivation of recursive and non-recursive formulas for computing the higher order moments of GGD of order k to describe the shape of probability or frequency distribution (see Sec. 1.2, (a)).

(b) Derivation of exact equations for computing the IPSP and RLP of control charts under k out of k runs rules to reduce computation burden instead of involving the MCA (see Sec. 1.2, (b)).

(c) Designing of generalized skewness correction structure to overcome the limitations of usual Shewhart structure, corrected Shewhart structure, and skewness correction structure (see Sec. 1.2, (c)–(d)).

(d) Constructing the multivariate control charts under bivariate ranked set schemes to resolve the issue of multivariate control chart under SRS (see Sec. 1.2, (e)–(f)).

(e) Derivation of the exact expressions for computing the power and ARL of control chart under ranked set schemes through numerical technique instead of using MCS procedure. This practice results into reducing the computational burden and processing time (see Sec. 1.2, (g)).

(f) Introducing the false alarm rate and average run length based control charts (named as dual purpose control charts) with runs rules to maintain the AFAR and AIARL (see Sec. 1.2, (h)).

1.4 Research Questions

Based on the Sec.1.3, research questions of the study are as follow:

(i) What are the recursive and non-recursive formulas for computing the higher order moments of GGD of order k tend to the formulas of classical geometric distribution?

(ii) What are the exact equations for computing the probability of single point and run length properties of control chart under k out of k runs rules provide the required results?

(iii) What are the generalized skewness correction structure perform efficient than usual Shewhart structure, corrected Shewhart structure, and skewness correction structure in terms of controlling the actual false alarm rate close to true false alarm rate?

(iv) What are the multivariate control charts under bivariate ranked set schemes perform outstandingly relative to multivariate control chart under simple random sampling?

(v) What are the numerical method for computing the power and average run length of control chart under ranked set schemes is resulted time efficient than methodologies based on Monte Carlo simulation procedure?

(vi) What are the proposed code can be executed at any choice of false alarm rate, sample size, amount of shift, parameter is known, parameter is unknown, and probability distribution?

(vii) What are the dual purpose control charts with runs rules based on the generalized polynomial equation serve the dual purpose (maintaining the actual false alarm rate and actual average run length of control charts at required level)?

1.5 Research Hypotheses

The recursive and non-recursive formulas of higher-order moments of GGD are expected to be a generalized form of the classical geometric distribution. The exact equations for IPSP and RLP of control chart with runs rules are assumed to produce the results as expected in case of MCA. In addition, it is presumed that skewness correction structure of mean control chart produces the AFAR close to true level relative to the exiting control charts. Also, multivariate control charts with bivariate ranked set schemes are expected to detect early out-of-control signals relative to the existing multivariate control charts with SRS. The numerical methodology for determining ARL and power of control chart with ranked set schemes proves time efficient than MCS.

1.6 Research Objectives

The overall aims of study are to improve the existing control charts. This covers control charts for the processes where parameters and probability distribution are unknown, introducing the bivariate ranked set schemes based multivariate control charts, and exact equations for the IPSP, RLP, power and other performance measures. Now a list of independent objectives are as follows:

(a) To construct the recursive and non-recursive formulas for higher order moments of generalized geometric distribution of order k.

(b) To design the exact formulas for in-control probability of single point and run length properties of control chart under k out of k runs rules.

(c) To formulate the generalized skewness correction structure at various choices of dispersion statistic.

(d) To develop the bivariate sampling schemes based multivariate Hotelling's T^2 and CUSUM control charts.

(e) To derive the exact expressions for power and average run length through numerical method (Simpson Rule).

(f) To develop the dual purpose control charts with runs rules based on the generalized polynomial equations which serve the dual purpose in forms of maintaining the actual in-control average run length and actual false alarm rate independently.

1.7 Study Methods

In this study we involve various method which are actually dependent on the objectives of study. For the first objective, we consider the probability generating function of GGD of order k (Philippou *et al.*, 1983). After that we utilize the *m*th order quotient rule of differentiation for differentiating the probability generating function. This results into factorial moments. Based on the factorial moments, we derive the moments about origin and mean. For the second objective, we utilize the moments of GGD of order k for formulating the exact formulas of RLP. In third objective, we consider the Cornish-Fisher expansion to design the skewness correction structure of mean control chart (Chan and Cui, 2003). Also, we develop skewness correction based constant and factors. For the fourth objective, we provide the bivariate ranked set schemes procedures for designing the Hotelling T^2 and CUSUM control charts. For the fifth objective, we define a pivotal quantity to establish the exact expression for power and ARL. Next, we solve the exact expression using Simpson rule as numerical technique. For the last objective, we consider a generalized polynomial equation and then design the dual purpose control charts under runs rules. For evaluating the performance of control charts, we plug the Monte Carlo simulation approach to calculate the values of performance measures (Riaz *et al.*, 2016a).

1.8 Scope of the Study

The scope of this study is categorized into three main aspects such as theoretical, computational, and practical that are defined below as:

(a) Theoretical aspect

In theoretical aspect, mth order quotient rule of differentiation is considered for differentiating the probability generating function. Furthermore, skewness correction models are introduced by considering the Cornish-Fisher expansion. After that bivariate ranked set schemes are defined. Also, sample mean vector and variance-covariance matrix are defined for estimation purposes. In addition, pivotal quantity is considered for power and ARL of mean control chart with ranked set schemes. The mathematical forms of various performance measures along with purposes and interpretation are also discussed.

(b) Computational aspect

To compare the performance of all proposed control charts against some existing control charts, numerical results are required. Therefore, to carry out the computational procedure, the MCS and numerical method is used to calculate the different performance measures. The algorithms for constructing the control charts and evaluating the performance measures are designed in R language. The numerical values of the performance measures are presented through line plot and histogram.

(c) Practical aspect

Besides the numerical and graphical comparison of the control charts against some of the existing counterparts, the proposed control charts are also implemented on real-life data to illustrate procedural details to quality engineers, researchers, and practitioners. This research considered real-data set from manufacturing industries and agricultural side.

1.9 Limitation of the Study

Various formulas of RLP of Shewhart control chart with runs rules are derived in this study but percentile of run length distribution needs to be explored. The multivariate control charts are derived for the process characteristics having normal probability distribution. For non-normal process characterestics, multivariate control charts are not much useful. Furthermore, skewness correction methodology with mean control chart take into account skewness level but discarded the level of kurtosis.

1.10 Significance of the Study

The skewness correction control chart provides the relaxations from restricted assumption of process parameters and probability distribution. It only requires information of skewness level instead of testing the restricted assumptions. The bivariate ranked set schemes is a cost efficient and precise sampling procedure as compared to SRS when ordering of the units can be done easily but the actual measurement of the items is difficult and expensive. Also, bivariate ranked set schemes based multivariate control charts are efficient to detect the assignable causes of variations earlier relative to SRS based multivariate control charts. In addition, exact equations for IPSP and RLP reduce the computational burden relative to the MCA. Further, numerical technique requires less time for attaining the power and ARL of mean control chart with ranked set schemes relative to MCS.

1.11 Thesis Organization

The chapter 2 contains the comprehensively illustration of relevant literature to highlight the research gaps.

In Chapter 3, proposed methodologies are elaborated. In more details, recursive and non-recursive formulas for computing the moments of GGD of order k are derived in Sec. 3.1 to meet the objective (a). In Sec. 3.2, equations for calculating the IPSP and RLP of Shewhart control chart are proposed to accomplish the objective (b). Also, computational procedures of proposed equations in designing and evaluation phases of Shewhart control under k/k runs rules are provided. In Sec. 3.3, generalized skewness correction structure are proposed to fulfill the objective (c). In Sec. 3.4, sample mean vector under bivariate ranked set schemes is defined. In Secs. 3.5 – 3.6 conjunction with Sec. 3.4, multivariate Hotelling's T^2 and CUSUM control charts with bivariate ranked set schemes are proposed as per the objective (d). In Sec. 3.7, exact expressions for computing power and ARL through numerical method are derived to complete the objective (e). In Sec. 3.8, code in R language for computing constant and power of $\bar{X}$ control chart. In Sec. 3.9, dual purpose control charts with runs rules based on the generalized polynomial equation are designed.

The Chapter 4 involves the results and discussion of present study. In Sec. 4.2, role of exact formulas for computing the RLP of univariate and multivariate control charts under k/k runs rules are elaborated. Here main theme is to show whether results obtained through exact formulas are similar as one expect by MCA. In Sec. 4.3, results of proposed generalized skewness correction structure and existing structures are explained in detail. Also, comparative analysis between proposed and existing structures is conducted by taking into account various choices of the factors. In Sec. 4.4, comprehensively discussion about the effect of each factor on the performance of proposed Hotelling's T^2 control chart under bivariate ranked set schemes is presented. In Sec. 4.5, role of factors on the performance of proposed multivariate CUSUM control chart under bivariate ranked set schemes is explained comprehensively. In addition, a comparative analysis among proposed multivariate CUSUM control chart with ranked set techniques is conducted. In Sec. 4.6, comparative analysis between numerical method and MCS method for computing the performance measures of $\bar{X}$ control chart with ranked set techniques is involved. In Sec. 4.7, interpretation of the outputs along with advantages of proposed code are elaborated. In Sec. 4.8, behavior of dual purpose control charts under runs rules are clarified. In simple words, difference between FAR and ARL based control charts under runs rules are discussed.

In Chapter 5, example of proposed as well as existing methodologies are presented. In Sec. 5.1, real life example of proposed generalized skewness correction and existing structures are provided by considering petrochemical processes and more specifically Dioctyl phathalate (DOP). In Sec. 5.2 – 5.3, real life examples of proposed multivariate Hotelling's T^2 and CUSUM control charts under bivariate ranked set schemes are provided by considering the physiochemical parameters of irrigation water. The Sec. 5.4 contains real life example and advantages of proposed code to monitor the plasticizer characteristic with parameters are unknown and physiochemical characteristic of groundwater with parameters are known.

In Chapter 6, summary followed by conclusion of present study, limitations and future recommendation of the proposed methodologies are included.

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APPENDIX B

LIST OF PUBLICATIONS

1. Mehmood, R., Lee, M. H., Hussain, S., and Riaz, M. (2019). On efficient construction and evaluation of runs rules–based control chart for known and unknown parameters under different distributions. *Quality and Reliability Engineering International*, 35(2), 582-599 (impact 1.718).

2. Mehmood, R., Lee, M. H., Riaz, M., and Ali, I. (2019). Generalized Performance Measures of Control Charts Based on Different Sampling Schemes. *Journal of Probability and Statistics*, 2019, Doi.org/10.1155/2019/5269357.

3. Mehmood, R., Lee, M. H., Bakir, M., amd Riaz, M. (2020). Application of quality control charts for early detection of flood hazards. *Polish journal of environmental sciences*, 2020, Doi.org/10.1155/2019/5269357 (impact 1.383).

4. Mehmood, R., Lee, M. H., Riaz, M., and Ali, I. (2019). Hotelling T^2 control chart under bivariate ranked set schemes, *Communications in Statistics - Simulation and Computation* (impact 0.651).

5. Mehmood, R., Lee, M. H., Afzal, M., Bashir, S., amd Riaz, M. (2020). Generalized skewness correction structure of mean control chart for unknown process parameters and skewed probability distributions, *Journal of Statistical Computation and Simulation*, 88(4), 675-711 (impact 0.918).

6. Mehmood, R., Lee, M. H., Ali, I., Riaz, M., and Hussain, S. (2020). Multivariate cumulative sum control chart and measure of process capability based on bivariate ranked set schemes. *Computers & Industrial Engineering*, 150, 106891

(impact 4.13).

7. Mehmood, R., Lee, M. H., Iftikhar, Ali, and Muhammad, Riaz (2021). Comparative analysis between FAR and ARL based control charts with runs rules. *Hacettepe Journal of Mathematics and Statistics*, 1-14.

8. Mehmood, R., Muhammad, Riaz, Iftikhar, Ali and Lee, M. H. (2021). Generalized Hotelling T^2 control chart based on bivariate ranked set techniques with runs rules. *Transactions of the Institute of Measurement and Control* (Accepted) (impact 1.649).

9. Mehmood, R., Lee, M. H., Rehman, Madooda and Iftikhar, Ali (2021). Mean control chart based on ranked set schemes for unknown skewed probability distribution and parameters. *Quality and Reliability Engineering International*, Submitted (impact 1.718).

10. Mehmood, R., Lee, M. H., Iqbal, M. R, and Hassan, S (2021). New robust location control charts for unknown process distribution with practical significance. *Soft Computing*, Submitted (impact 3.050).

11. Mehmood, R., Riaz, M., Lee, M. H., Ali, I., and Gharib, M (2021). Exact computational methods for univariate and multivariate control charts under runs rules. *Computers & Industrial Engineering*, Submitted (impact 4.13).