FRACTAL AND PROBABILISTIC ANALYSIS ON FATIGUE CRACK GROWTH RATE OF METALLIC MATERIALS

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A thesis submitted in fulfilment of the
requirements for the award of the degree of
Doctor of Philosophy

Faculty of Mechanical Engineering
Universiti Teknologi Malaysia

FEBRUARY 2023

DEDICATION

This thesis is dedicated to my parents and the family members sacrificing a lot during the completion of my studies. Without their prayers and moral support, I would have been unable to complete the thesis.

ACKNOWLEDGMENT

Besides the special help of Almighty Allah to whom I am esp. thankful, the thesis was cultivated through painstaking contributions from numerous individuals. I am indebted to my supervisors Prof. Dr. Mohd Nasir Bin Tamin and Dr. Mohd Foad Bin Abdul Hamid for their guidance, advice and encouragement. I'd like to convey my heartfelt gratitude to Prof. Dr. Mohd Nasir Bin Tamin from whom I learned analytical and comprehension skills in addition to technical support, direction, criticism, and friendship. I am grateful to Mr. Safdar Ali for his input in FE analysis using Abaqus. I appreciate the technical staff of Universiti Teknologi Malaysia (UTM) labs due to their assistance in running the testing machines and the experimental setup and aid in providing pertinent accessories. I am obliged to the CSM group who helped conduct research in a comfortable environment. I am also grateful to Higher Education Commission (HEC), Pakistan for financing my doctoral studies. I am beholden to my family who sacrificed a lot due to my focus on work.

ABSTRACT

Load-bearing and complex geometry structures such as aircraft wing spars, thick-walled chemical processing vessels, offshore platforms and jacket structures are designed based on damage-tolerant design philosophy. The design employs fracture mechanics and test data to ensure that structural cracks nucleating during the operation will not propagate before they are detected by periodic inspections. The fracture mechanics equation describing the crack tip stress field (K-field) is expressed in terms of the far-field stress and relies on the crack geometry factor. Closed-form equations for the far-field stress and the crack geometry factors have been established for standard fracture test coupons and relatively simple structures. The unavailability of the crack geometry factor for complex structures and loading renders the use of the fracture mechanics equation impractical. Inaccurate assessment of the fatigue crack and crack growth rates could jeopardize the safety and integrity of the structures. An alternative approach employing fractal analysis to quantify the fatigue crack growth rates of single-phase metallic material is proposed and examined. The fractal approach avoids the need for the crack geometry factor when calculating the crack tip driving force. The fractal analysis is carried out on digital images of the crack with a precision of 1.19 pixel/μm^2 employing the box-counting algorithm to determine the fractal dimension (d_F) along the edge of the crack length. The analysis is confined to the power law crack growth rate stage (Paris crack growth regime). Compact tension, C(T) specimens fabricated from AISI 410 martensitic stainless steel provide the reference fatigue crack growth response. Results show that the crack initially exhibits a Euclidean nature ($d_F \approx 1.0$). The fractal dimension increases steadily with increasing crack length in Paris region with $1.05 < d_F < 1.24$. The corresponding extent of disparity in the crack tip driving force range is between $18 \leq \Delta K \leq 40$ MPa$\sqrt{\text{m}}$. The fractal dimension (d_F) correlates linearly with the normalized crack tip driving force range ($\Delta K / K_{\text{IC}}$) within the Paris region. The coefficient of fractality (C_F) is identified as a characteristic material parameter. This enables the multifractal crack growth rate semi-empirical model to be established in terms of Paris coefficient and exponent, fractal characteristics, and fatigue fracture properties of the material. A significant statistical dispersion is noted which is typical of a fatigue response. Given this, a probabilistic model based on Walker's crack growth rate equation considering the variability in the crack tip driving force range, ΔK and stress ratio, R is developed. The model's validity is examined using selected sets of fatigue crack growth curves of $A\ell$-7075-$T6$, $A\ell$-2024-351 and Ti-6$A\ell$-4V alloys. A good fit of the experimental data is noted. The model variance shows a convergent trend with an increasing number of test coupons, thus providing the statistical means of establishing sample sufficiency. The probabilistic model is annexed to the fractal analysis to yield an integrated probabilistic-fractal fracture model. The application of the integrated model to the general structures that lack the crack geometry factor for fatigue crack growth analysis is demonstrated on a bell crack structure. The results are contrasted with ΔK estimate established through the contour integral (CI) approach using Abaqus software and a close resemblance is noted. Thus, the model could be employed for the prediction of the fatigue crack growth response of engineering structures where the crack geometry factor is not readily available.

ABSTRAK

Struktur galas beban dan geometri kompleks seperti spar sayap pesawat, kapal pemprosesan kimia berdinding tebal, pelantar luar pesisir dan struktur jaket direka berdasarkan falsafah reka bentuk tahan kerosakan. Reka bentuk ini menggunakan mekanik patah dan data ujian untuk memastikan retakan struktur yang terbentuk semasa operasi tidak akan merebak sebelum ia dikesan melalui pemeriksaan berkala. Persamaan mekanik patah yang menentukan medan tegasan hujung retak (medan-K) dinyatakan berdasar kepada tegasan medan jauh dan bergantung pada faktor geometri retak. Persamaan bentuk tertutup untuk tegasan medan jauh dan faktor geometri retak telah ditetapkan untuk kupon ujian patah standard dan struktur yang agak mudah. Ketiadaan faktor geometri retak untuk struktur kompleks dan beban menjadikan penggunaan persamaan mekanik patah tidak praktikal. Penilaian yang tidak tepat terhadap kadar pertumbuhan retak dan retakan lesu boleh menjejaskan keselamatan dan keutuhan struktur. Pendekatan alternatif yang menggunakan analisis fraktal untuk mengukur kadar pertumbuhan retak lesu untuk bahan logam fasa tunggal dicadangkan dan diteliti. Pendekatan fraktal mengelakkan keperluan faktor geometri retak semasa mengira daya penggerak hujung retak. Analisis fraktal dijalankan pada imej digital retakan dengan ketepatan ketepatan 1.19 piksel/μm^2 menggunakan algoritma pengiraan kotak untuk menentukan dimensi fraktal (d_F) di sepanjang pinggir panjang retakan. Analisis terhad kepada peringkat kadar pertumbuhan retak hukum kuasa (rejim pertumbuhan retak Paris). Ketegangan padat spesimen, C(T) yang terbikin dari keluli tahan karat martensit AISI 410 memberikan rujukan kepada tindak balas pertumbuhan retak lesu. Keputusan menunjukkan bahawa retakan pada mulanya menunjukkan sifat Euclidean (d_F≈1.0). Dimensi fraktal meningkat secara berterusan dengan peningkatan panjang retak di rantau Paris dengan $1.05<d_F<1.24$. Tahap ketaksamaan yang sepadan dalam julat faktor keamatan tegasan adalah antara $18 \leq \Delta K \leq 40$ MPa√m. Dimensi fraktal (d_F) menunjukkan hubungkait secara linear dengan julat faktor keamatan tegasan ternormal ($\Delta K/K_{IC}$) dalam rantau Paris. Pekali kefraktalan (C_F) dikenal pasti sebagai parameter ciri bahan. Ini membolehkan model separa empirik dibentuk berdasar kepada kadar pertumbuhan retak berbilang fraktal yang terdiri daripada pekali dan eksponen Paris, ciri fraktal dan sifat patah lesu bahan. Sebaran statistik yang ketara dicatatkan yang merupakan tindak balas lazim lesu. Dengan ini, model kebarangkalian ini berdasarkan persamaan kadar pertumbuhan retak Walker dengan mengambil kira kebolehubahan dalam julat faktor keamatan tegasan, ΔK dan nisbah tegasan, R dibentuk. Kesahihan model diperiksa menggunakan set lengkung pertumbuhan retak lesu dipilih dari set $A\ell$-7075-T6, $A\ell$-2024-351 dan aloi Ti-6$A\ell$-4V. Kesesuaian dengan data eksperimen disahkan. Varians model menunjukkan trend penumpuan dengan peningkatan bilangan kupon ujian, sekali gus menyediakan kaedah statistik untuk mewujudkan kecukupan sampel. Model kebarangkalian ditambah kepada analisis fraktal untuk menghasilkan model retakan berkebarangkalian-fraktal bersepadu. Aplikasi model bersepadu pada struktur umum yang tidak mempunyai faktor geometri retak untuk analisis pertumbuhan retak lesu digunakan pada struktur loceng retak. Keputusan dibezakan dengan anggaran ΔK yang diwujudkan melalui pendekatan kamiran kontur (CI) menggunakan perisian Abaqus dan kemiripan rapat dicatatkan. Oleh itu, model fraktal boleh digunakan untuk meramalkan tindak balas pertumbuhan retak lesu struktur kejuruteraan di mana faktor geometri retak tidak tersedia.

TABLE OF CONTENTS

	TITLE	PAGE
	DECLARATION	**iii**
	DEDICATION	**iv**
	ACKNOWLEDGMENT	**v**
	ABSTRACT	**vi**
	ABSTRAK	**vii**
	TABLE OF CONTENTS	**viii**
	LIST OF TABLES	**xiii**
	LIST OF FIGURES	**xv**
	LIST OF ABBREVIATIONS	**xxii**
	LIST OF SYMBOLS	**xxiv**
	LIST OF APPENDICES	**xxvi**
CHAPTER 1	**INTRODUCTION**	**1**
1.1	Background of Study	1
1.2	Problem Statement	3
1.3	Research Objectives	4
1.4	Scope of the Study	4
1.5	Significance of the Study	6
1.6	Thesis Layout	6
CHAPTER 2	**LITERATURE REVIEW**	**9**
2.1	Introduction	9
2.2	Fundamentals of a Fatigue Fracture	9
	2.2.1 Approaches to Solve a Fatigue Problem	12
	2.2.2 Linear Elastic Fracture Mechanics (LEFM)	14
	2.2.3 Crack Growth Rate Equations	16
	2.2.4 Crack tip driving force Solutions	17
	2.2.5 Reference Stress	18

		2.2.6 Crack Geometry Factor	19
	2.3	Fractal Geometry and Characterization	22
		2.3.1 Coastline Paradox and Richardson Effect	23
		2.3.2 Fractal Dimension and Its Evaluation	26
	2.4	Fractal Model of a Rugged Fractal Crack	29
	2.5	Link of Fractality with Fracture Parameters	30
		2.5.1 Fractal Dimension and Surface Roughness	31
		2.5.2 Fractal Dimension and Cack Branching	32
		2.5.3 Fractal Dimension and Fracture Parameters	34
		2.5.4 Fractal Dimension and Loading	35
	2.6	Crack tip driving force using FE Analysis	37
	2.7	Probabilistic Modelling of Fatigue Crack Growth	41
		2.7.1 The Inordinate and Indispensable Scatter	42
		2.7.2 Differential Equation (DE) Models	43
		2.7.3 Auto-Covariance of the Crack Growth Rate	43
		2.7.4 Lognormality of Distribution	44
		2.7.5 Initial Flaw Size (IFS) Effect	45
		2.7.6 Model Parameter-Estimation Techniques	46
		2.7.7 Scatter Escalation due to *R*-Ratio	47
		2.7.8 Unified Parameter Models	48
		2.7.9 Scatter in Variable-Amplitude (VA) Loading	49
	2.8	Summary	51
CHAPTER 3		**RESEARCH METHODOLOGY**	**53**
	3.1	Introduction	53
	3.2	Research Framework	54
	3.3	Material Characterization	56
		3.3.1 Chemical Composition Analysis	56
		3.3.2 Microstructural Examination	57
		3.3.3 Vickers Hardness Test	57
		3.3.4 Tension Test	58
		3.3.5 Fatigue Crack Growth Rate Test	58

	3.3.6	Fractographic Analysis	60
	3.4	Fractal Analysis of Fatigue Crack Images	61
	3.4.1	Acquisition of Crack Image and Conversion to Greyscale Image	61
	3.4.2	Greyscale Thresholding and Segmentation	63
	3.4.3	Image Scanning Scheme	63
	3.4.4	Box-Counting Method	64
	3.4.5	Scanning Window and Increment Size	68
	3.4.6	Correlating the Fractal Dimension with Crack tip driving force	68
	3.5	Methodology for Probabilistic Analysis	69
	3.5.1	Method of Obtaining Reliable Fatigue Life	70
	3.5.2	Method of Testing Normality	71
	3.5.3	Validating the Probabilistic Model	73
	3.6	Application and Validation of Multifractal Fatigue Crack Growth Model	75
	3.7	Integrating the Probabilistic and Fractal Models	80
	3.8	Summary	82
CHAPTER 4		**EXPLOITING FRACTAL FEATURES TO DETERMINE THE CRACK-TIP DRIVING FORCE IN METALLIC MATERIALS**	**83**
	4.1	Introduction	83
	4.2	Properties of AISI 410 Stainless Steel	83
	4.3	Fatigue Crack Growth Response	84
	4.4	Fractographic Analysis	86
	4.5	Results of the Box-Counting Algorithm on Standard Fractals	89
	4.6	Establishing the Crack Edge Fractality	90
	4.7	Optimum Scanning Window and Increment Size	92
	4.8	Thresholded and Segmented Crack Image Analysis	93
	4.9	Fractal Dimensions of the Fatigue Cracks	97
	4.10	Correlation between the Fractal Dimension and Crack Growth Rate	99
	4.11	Summary	101

CHAPTER 5 A ROBUST PROBABILISTIC FRAMEWORK TO ESTABLISH RELIABLE FATIGUE-LIFE ESTIMATES IN METALLIC MATERIALS 103

5.1 Introduction 103

5.2 Deterministic Crack Growth Rate Model 103

5.3 Mathematical Formulation of the Probabilistic Model 104

5.3.1 Solution of the Likelihood Equations 109

5.3.2 Optimal Estimate of Standard Deviation 110

5.3.3 Supplementary Corollary: The Two-Parameter Probabilistic Model 110

5.3.4 Crack Growth Rate Distribution 111

5.3.5 Random Time (Cycles) Distribution 112

5.3.6 Crack Exceedance Probability 114

5.4 Predictive Performance of the Model 115

5.4.1 Constant Amplitude Data with Fixed R-Ratio (Replicate Data) 116

5.4.2 Constant Amplitude Data with Variable *R* Across Specimens 119

5.4.3 Verification of the Normality Assumption 123

5.4.4 Specimen Sufficiency Test for a Fatigue Problem 125

5.4.5 Verifying the Congruity of Curve-Fitting Parameters 126

5.4.6 Model Validation for Random Loading 128

5.5 Summary 129

CHAPTER 6 VALIDATION OF THE INTEGRATED PROBABILISTIC-FRACTAL FATIGUE CRACK GROWTH MODEL USING STRUCTURES WITH COMPLEX GEOMETRY 131

6.1 Introduction 131

6.2 Fracture Response of the Bell Crank in Relation to the C(T) Specimens 132

6.3 Fractal Dimensions of the Crack 133

6.4 Multifractal fatigue crack growth rate 134

6.5 Loci of ΔK on Paris Plot 135

6.6 FE Analysis Results of the C(T) Specimen: 136

6.7 FE Analysis Results of the Bell Crank Structure 139

6.8 Comparison of the Fractal and FE Results 145

6.9 Summary 146

CHAPTER 7 CONCLUSION AND RECOMMENDATIONS 147

7.1 Conclusion 147

7.2 Recommendations 149

REFERENCES 151

LIST OF PUBLICATIONS 172

LIST OF TABLES

TABLE NO.	TITLE	PAGE
Table 3.1	Nominal chemical composition of AISI 410 Stainless Steel per ASME/ASTM A276 specifications [142]	56
Table 3.2	The parameters of the fatigue crack growth tests. Load ratio, R = 0.1	59
Table 3.3	Fractal Dimension of Computer-Generated Fractals	66
Table 3.4	Box Counting Dimension of the fractals from random resources	67
Table 3.5	Summary of Probabilistic Model Tests and Databases	74
Table 4.1	Threshold crack tip driving force range, the Paris coefficient, and the exponent for the six C(T) specimens.	86
Table 4.2	Fractal dimension (dF) of selected crack-like photographed fractal geometries calculated using the modified box-counting method	90
Table 5.1	Model parameter estimates for [126] replicate testing data	116
Table 5.2	Safe life limits for crack lengths of 11.5 mm, 17 mm, 23.7 mm corresponding to failure probabilities at significance levels of 1%, 5% and 10%.	118
Table 5.3	Result of parameters and variance using the two- and three-parameter estimation approaches for the variable R-ratio data by Noroozi *et al*. [139]	121
Table 5.4	Values of the curve-fitting parameters and variance resulting from the three-parameter estimation employed on Ghoenm and Dore [157–160] Data.	121
Table 5.5	Statistics of the error distribution resulting from the three-parameter estimation model for variable R-ratio data by Noroozi *et al*. [139]	124
Table 5.6	Statistics resulting from three-parameter estimation approach for variable R-ratio data by Ghonem and Dore [157–160]	124
Table 5.7	Selected grades of aluminum from the Forman *et al*. [135] fatigue crack growth rate database for validation of Walker equation fit parameters	127

Table 5.8 Comparison of the Walker fit parameters obtained from the proposed three-parameter estimation model with the reported values of [135] 127

Table 6.1 FE simulation results for K-solutions and direction of crack propagation for C(T) specimen established values as per directives of ASTM E647 standard showing pct. error. 140

Table 6.2 FE simulation results for K-solutions, provisional crack propagation direction, and percentage of mode-mixity in the bell crank. 141

Table 6.3 FE simulation results for $K\mathrm{I}k$-solutions for the bell crank in the direction of θk. 144

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
Figure 2.1	Variation in inter-atomic force and the bond strength as a function of the distance between the atoms	10
Figure 2.2	Micro mechanism of fatigue crack propagation consequent upon the successive loading and unloading stress cycles	11
Figure 2.3	Fundamental modes of fracture; (**a**) Opening mode, (**b**) Sliding mode, (**c**) Tearing mode	11
Figure 2.4	Illustration of the *stress-life approach* applicable to the *infinite-life* (fatigue cycles ≥ 10^6) designs	12
Figure 2.5	Illustration of *strain-life approach* applicable for the *finite-life* low cycle fatigue (LCF) regime (cycles ≤ 10^3)	13
Figure 2.6	Illustration of the fracture mechanics approach	14
Figure 2.7	Planar coordinate axis and the stress field ahead of the crack tip	15
Figure 2.8	Stress redistribution to accommodate the stresses above yield stress represented by the hatched area leading to an increased plastic zone size	15
Figure 2.9	Illustration of various modes of fracture	18
Figure 2.10	Examples of commonly used test specimen configurations and the geometry factors ($\alpha = aW$, unless otherwise specified).	20
Figure 2.11	Dimensions of a standard compact tension, C(T), specimen according to ASTM E647 standard [50].	22
Figure 2.12	ASTM E647 requirements for minimum pre-crack length and machining of the C(T) specimen notch [50]	22
Figure 2.13	Middle-third Cantor set reflecting self-similarity and scale-invariance characteristics of a fractal geometry [51]	23
Figure 2.14	Visual resemblance of the crack profile with invasive fractals [51]	23
Figure 2.15	Variation of the measured length of Britain's coastline through using the different measurement scales [54]	24
Figure 2.16	Richardson plots; (**a**) for experimental data on the coastlines/frontiers of five different countries [55], (**b**) for strict fractals, (**c**) for statistical fractals	25

Figure 2.17	Illustration of how the fractal dimension (non-integer) lies between the Euclidean (integer) dimension	27
Figure 2.18	Variation of $\mathcal{H}sF$ with increasing s illustrating a jumps in the value of $\mathcal{H}sF$ from ∞ to 0 at the critical value of $s =$ dimHF [51].	28
Figure 2.19	Slit-Island Analysis for metallic surfaces, adopted from [58]	28
Figure 2.20	Difference between the classical and the fractal fracture mechanics approach; (**a**) Euclidean crack, (**b**) & (**c**) Fractal crack of differing fractality [68]	29
Figure 2.21	Exemplary fractures with marked spots of crack propagation of three distinct grades of specimens [69]	30
Figure 2.22	Stereoptical (**a**) and confocal microscope (**b**) images of a glass rod. Height profiles for tracks 1 and 2 are shown in (**c**) and (**d**) [70].	31
Figure 2.23	Demonstration of how the fractal dimension increases with increasing the fracture surface roughness [74]	32
Figure 2.24	Comparison of the measured and calculated CBC values for Borosilicate [26].	33
Figure 2.25	Variation of fractal dimension with the fracture toughness in samples of AISI 4340 steel, adopted from [25].	34
Figure 2.26	Plot of the fractal dimension and loading parameter showing the existence of a relationship between fractality and loading [22,69]	35
Figure 2.27	The 3D surface view of fractured samples showing the variation of texture under changing loading conditions [69].	36
Figure 2.28	Illustration of the quarter point elements (QPE) at the crack-tip with distorted shape functions	37
Figure 2.29	Γ-contour for evaluation of the J-integral	38
Figure 2.30	A curved crack trajectory in a bell crank structure due to the presence of multiple modes of fracture.	39
Figure 2.31	Kinking angle of the crack under mixed-mode loading	40
Figure 2.32	Virkler *et al*. [117] experimental crack growth (a-N) curves	43
Figure 2.33	Equivalent initial flaw size distribution (EIFSD)	46
Figure 2.34	Noroozi [138,139] compiled data for selected materials	48

Figure 2.35	Results of the unified-parameter model proposed by [138,139]	49
Figure 2.36	a-N behavior of 7075-T6 Aℓ-alloy with variable overloads [140]	50
Figure 2.37	a-N behavior of 7075-T6 Aℓ-alloy with single repeated overload [140]	50
Figure 3.1	Breakdown of the research activities	55
Figure 3.2	Schematic of the Vickers hardness (H_V) measurement process	57
Figure 3.3	Dimensions of the quasi-static tension test specimen in mm (B=5 mm)	58
Figure 3.4	The dimensions and tolerances of the C(T)-25 specimen.	59
Figure 3.5	Fatigue test setup with a microscopic camera mounted on an XYZ-stage to precisely reveal the location of the crack-tip; a) Test setup, b) Detail-A	60
Figure 3.6	A systematic methodology for image analysis of the crack profile to establish the variation in local fractal dimension	62
Figure 3.7	Image partitioning scheme for establishing the local variation in the fractal dimension	64
Figure 3.8	Image analysis methodology for calculation of fractal dimension of the crack profile at various crack lengths	64
Figure 3.9	Description Box Count Algorithm adopted to Image Analysis of the Crack Profile	66
Figure 3.10	Fractals collected from random sources [51,52,152,153]	67
Figure 3.11	Methodology to establish reliability and probabilities of interest — crack exceedance probability and the service time to failure.	71
Figure 3.12	Interpretation of Q-Q plots in respect of a normal distribution; (**a**) Left-skewed, (**b**) Right-skewed, (**c**) Fat-tailed, (**d**) Thin-tailed	72
Figure 3.13	Interpretation of a distribution based on skewness.	73
Figure 3.14	Illustration of kurtosis as a measure of normality at tails.	73
Figure 3.15	Types of load cases employed for model validation	74
Figure 3.16	Dimensions (mm) of the bell crank (thickness, B=20mm)	75
Figure 3.17	Methodology for calculating the length of the crinkly crack generated due to mixed mode loading, n=102 points	76

Figure 3.18	Bimodal distribution of greyscale values of the crack image based on OTSU thresholding algorithm.	77
Figure 3.19	Plot of $\log_2 NB$ versus $\log_2 \epsilon$ defining the fractality of the fatigue crack.	77
Figure 3.20	The local coordinate system (LCS) superimposed at s distance along the crack front; **a**) $2D$ view on the bell crank, **b**) $3D$ view on the crack surfaces	79
Figure 3.21	Meshing attributes at different location of the bell crank structure	79
Figure 3.22	Boundary conditions and the mesh convergence studies	80
Figure 3.23	Integration of fractal and probabilistic models (Scheme-I)	81
Figure 4.1	Microstructure of AISI 410 stainless steel reflecting needle-like martensitic phase grain boundaries.	84
Figure 4.2	The crack propagation in the specimen of AISI 410 under varying loading conditions (ΔP) and a fixed stress ratio (R = 0.1).	85
Figure 4.3	Fatigue behavior of AISI 410 stainless steel illustrating threshold, Paris and fast-fracture stages of crack propagation.	86
Figure 4.4	The crack trajectory at the edge for sample CT-2 showing fracture features' intensification as the crack length increases.	87
Figure 4.5	Scanning electron microscope (SEM) images of the fracture surface at different locations along the crack path of sample CT-3: (**a**) Threshold crack growth region; **(b)** and **(c)** Paris crack growth regions; (**d**) Dimple-type ductile fracture mechanism at the fast fracture zone; (**e**) Plan view of the exposed fracture surface of the specimen.	88
Figure 4.6	Illustration of counted and uncounted boxes in the box-counting algorithm; (**a**) Uncounted box, (**b)** Counted box, (**c**) Uncounted box	89
Figure 4.7	Richardson plot for the sample CT-2 with a straight line fit demonstrating the crack edge fractality.	91
Figure 4.8	Local gradient plot for the sample CT-2 shows an approximately straight-line region for box sizes 2^3 to 2^8.	91
Figure 4.9	Establishing the minimum window size for the box count algorithm using the finite interval partitioning scheme	92
Figure 4.10	Establishing the maximum increment size for using the incremental partitioning scheme	93

Figure 4.11 Crack profiles of fatigue tested specimens imaged at 1090 pixels/mm resolution and 100X magnification; (**a**) CT-1, (**b**) CT-2, (**c**) CT-3, (**d**) CT-4, (**e**) CT-5, (**f**) CT-6. 93

Figure 4.12 Illustration of the greyscale thresholding process for CT-2; (**a**) The grayscale intensity histogram with indicated threshold values as computed from the available global thresholding algorithms; (**b**) Fractal dimension, dF values determined by the different thresholding algorithms. 95

Figure 4.13 Bimodal pixel density plots of the greyscale images of the crack before the greyscale thresholding process. 95

Figure 4.14 Segmented binary image of the crack in specimen CT-2 based on: (**a**) OTSU & ISODATA algorithm; (**b**) MEAN algorithm; (**c**) MINIMUM algorithm; (**d**) LI algorithm. 96

Figure 4.15 OTSU algorithm thresholded crack binary images for the fatigue tested specimen; (**a**) CT-1, (**b**) CT-2, (**c**) CT-3, (**d**) CT-4, (**e**) CT-5, (**f**) CT-6. 96

Figure 4.16 Local variation of the fractal dimension for C(T) specimen of AISI 410 showing distinct fracture feature regions resembling regions defined by fracture mechanics theory. 98

Figure 4.17 Local variation of the fractal dimension for specimens CT-1, CT-2 and CT-3 of AISI 410 showing distinct regions similar to CT-3, CT-4 and CT-5. 98

Figure 4.18 Linearity of correlation between crack tip driving force range (ΔK) and the fractional fractal dimension (dFF) in Paris region (18 < ΔK < 40 MPa√m). 100

Figure 4.19 Existence of the power-law correlation between the crack growth rate ($\mathrm{d}a/\mathrm{d}N$) and the fractional fractal dimension (dFF), specifically in the Paris region. 100

Figure 5.1 Basic skeleton of the proposed probabilistic framework illustrating that the total variation in crack growth rate ($\mathrm{d}a/\mathrm{d}N$) is indicted on its contributing factors (ΔK and R). 105

Figure 5.2 A step-by-step process for application of the probabilistic model 115

Figure 5.3 Results of the three-parameter probabilistic model applied to Xing and Hong data [126] in the Paris region showing 1st and 99th percentile lines. 117

Figure 5.4 The median, mean and 1st, 10th & 99th percentile curves overlayed on the original a-N data by [126] and the PDFs at selected crack lengths. 118

Figure 5.5	Results of probabilistic analysis on 2024-T351, Aℓ-alloy; (**a**) Two-parameter estimation model, (**b**) Three-parameter estimation model	120
Figure 5.6	Results of probabilistic analysis on Ti-6Al-4V; (**a**) Two-parameter estimation model, (**b**) Three-parameter estimation model	120
Figure 5.7	Results of the probabilistic model with replicate testing experimental data under three R-ratio conditions by Ghonem and Dore [157–160]; (**a**) R=0.6, (**b**) R=0.5, (**c**) R=0.4	122
Figure 5.8	Verification of the normality in error distribution for the positive R-ratio data of 2024-T351; (**a**) Histogram, (**b**) Q-Q plot.	123
Figure 5.9	Verification of the normality in error distribution for the positive R-ratio data of Ti-6Al-4V; (**a**) Histogram, (**b**) Q-Q plot.	123
Figure 5.10	Verification of the normality for variable R-ratio data by Ghonem and Dore [157–160]; (**a**) Histogram of the error, (**b**) Q-Q plot of the error.	124
Figure 5.11	Verification of sample sufficiency for Wu and Ni [125,127,161] data showing the convergence of the standard deviation value after 23 specimens.	125
Figure 5.12	Verification of sample sufficiency for [139] showing the standard deviation did not convergence with available samples	126
Figure 5.13	Results of applying the three-parameter estimation model on selected grades of aluminum from [135]; a) 2024-T351, (b) 2024-T3	128
Figure 5.14	Application of the probabilistic model on random loading data by Wu and Ni [161] showing quantiles and PDF of Nx at for ax (mm) = 20, 20, 27, 32	129
Figure 6.1	Composite image of the fatigue crack in the bell crank structure	132
Figure 6.2	Fatigue crack growth curves of AISI 410 stainless steel measured using C(T) specimen and L-shaped bell crank structure.	133
Figure 6.3	Local variation of the fractal dimension along the edge of a propagating fatigue crack in the bell crank.	134
Figure 6.4	Comparison of fatigue crack growth rate data determined using the multifractal crack growth model with that obtained using Paris crack growth equation.	135

Figure 6.5 The characteristic fatigue response of AISI 410 steel C(T) specimen using fracture mechanics approach overlayed with results of the fractal approach. 136

Figure 6.6 The von Mises stress in C(T) specimen at distinct crack-front locations reflecting symmetry across the crack front. 137

Figure 6.7 Stress distributions at a=18 mm; (**a**) Opening stress at the traction-free surface, (**b**) Opening stress at mid-plane, (**c**) Principal stress distribution. 138

Figure 6.8 Comparison of through-thickness FE solutions for ΔK at the crack lengths of 11, 13 and 15mm. 139

Figure 6.9 Comparison of the K-solutions obtained from ASTM E647 standard with FE results showing a harmony specifically at lower crack lengths. 139

Figure 6.10 Comparison of the K-solutions obtained from ASTM E647 standard with FE results showing a harmony specifically at lower crack lengths. 141

Figure 6.11 Distribution of the von Mises stress in the bell crank indicating complexity and asymmetricity of the stress field. 142

Figure 6.12 Mises stress distribution in the bell crank indicating complexity and asymmetricity of the stress field. 144

Figure 6.13 Opening stress distribution in the bell crank indicating complexity and asymmetricity of the stress field. 145

Figure 6.14 Comparison of multifractal fatigue crack growth model with finite element (FE) simulation results. 146

LIST OF ABBREVIATIONS

AI	-	Artificial Intelligence
AISI	-	American Iron and Steel Institute
ASME	-	American Society of Mechanical Engineers
ASTM	-	American Society of Testing and Materials
BC	-	Box Counting
C(T)	-	Compact Tension
CA	-	Constant Amplitude
CBC	-	Crack Branching Coefficient
CFD	-	Cumulative Density Function
CI	-	Contour Integral
CTE	-	Crack Tip Elements
CTOD	-	Crack Tip Opening Displacement
DCT	-	Disc-Shaped Compact Tension
DE	-	Differential Equation
DTA	-	Damage Tolerance Assessment
EIFSD	-	Equivalent Initial Flaw Size Distribution
EPFM	-	Elastic-Plastic Fracture Mechanics
FCG	-	Fatigue Crack Growth
FE	-	Finite Element
GDS	-	Glow Discharge Spectrometer
HCF	-	High Cycle Fatigue
IDE	-	Integrated Development Environment
KS	-	Kolmogorov-Smirnov
LCF	-	Low Cycle Fatigue
LEFM	-	Linear Elastic Fracture Mechanics
M(T)	-	Middle Tension
MC	-	Monte Carlo
MERR	-	Maximum Energy Release Rate
MLE	-	Maximum Likelihood Estimation
MPS	-	Maximum Principal Stress

MSE	-	Mean Square Error
MTS	-	Maximum Tangential Stress
OCR	-	Occurrence Ratio
PDF	-	Probability Density Function
QPE	-	Quarter Point Elements
Q-Q	-	Quantile-Quantile
RGB	-	Red Green Blue
SEM	-	Scanning Electron Microscope
SENB	-	Single Edge Notch Bend
SIA	-	Slit Island Analysis
SW	-	Shapiro-Wilk
TSM	-	*Tandem Scanning Confocal Microscope*
TTCI	-	Time To Initial Crack Initiation
VA	-	Variable Amplitude
WGN	-	White Gaussian Noise

LIST OF SYMBOLS

a	-	Crack Length
B	-	Thickness of Specimen
c	-	Log of Paris Coefficient
C	-	Paris Coefficient
C_F	-	Coefficient of Fractality
d_E	-	Euclidean Dimension
d_F	-	Fractal Dimension
d_{FF}	-	Fractional Fractal Dimension
$\mathrm{erf}^{-1}(.)$	-	Inverse Error Function
$\mathrm{erf}(.)$	-	Error Function
E	-	Modulus Of Elasticity
f	-	Frequency
$f_X(x)$	-	Probability Density Function
$F_X(x)$	-	Cumulative Distribution Function
g_1	-	Skewness
g_2	-	Excess Kurtosis
G	-	Gamma Variation Factor
H_v	-	Vickers Hardness
J	-	Rice's Path Integral
K	-	Crack tip driving force
K_{IC}	-	Fracture Toughness
K_{th}	-	Threshold Crack tip driving force
m	-	Paris Exponent
M	-	Mean Coefficients
N	-	Number of Cycles
N_b	-	Number of Boxes
p_F	-	Probability of Failure
p_s	-	Probability of Survival
R	-	Stress Ratio

S	-	Stress
W	-	Width of Specimen to Load Line
W_i	-	Random Variable for Total Variability
Y_i	-	Random Variable for $\mathrm{d}a/\mathrm{d}N$
$\mathrm{Y}(a/W)$ or $\mathrm{Y}(\alpha)$	-	Geometry Factor
γ_{vv}	-	Van-Valen's Coefficient
θ_k	-	Kinking Angle
σ^2	-	Variance
σ_∞	-	Far-Field Stress
σ_F	-	Fracture Strength
$\prod$	-	Product
$\mathcal{L}$	-	Likelihood Function
ℓ	-	Log-Likelihood Function
$\mathbb{P}$	-	Probability
$\mathbb{R}$	-	Real Space
$\mathcal{R}$	-	Random Variable for R-Variation
RV	-	Random Variable
var	-	Variance Operator
Δ	-	Increment Size
ΔK	-	Crack tip driving force Range
ΔP	-	Range of Applied Load
Ψ	-	Scanning Window Size
$\mathcal{K}$	-	Random Variable for K-Variation
$\boldsymbol{\Theta}$	-	Parameter Space
$\boldsymbol{\Sigma}$	-	Covariance Matrix
γ	-	Walker Exponent
κ	-	Expected Value of $\ell\mathrm{n}\ \Delta K$
ρ	-	Expected Value of $\ell\mathrm{n}$ (1-R)
σ	-	Standard Deviation
ϵ	-	Box Size
$\boldsymbol{\mu}$	-	Mean Vector

LIST OF APPENDICES

APPENDIX	TITLE	PAGE
Appendix A	Techniques for Calculating the Fractal Dimension	165
Appendix B	Image Partitioning Algorithm	167
Appendix C	Box Counting Algorithm	168
Appendix D	Solution of Likelihood Equations	169
Appendix E	Python Script for the Probabilistic Model	171

CHAPTER 1

INTRODUCTION

1.1 Background of Study

Numerous critical structures such as the aircraft wing spars, the compressor blades for aero engines, and the components of a marine vessel are designed to tolerate a propagating crack within an inspection interval. The rate, at which a crack advances, determines the time to failure. At the material coupon level, fracture mechanics tests using standard specimen geometry and test setup provide crack growth data as a function of the applied fatigue loading. The phenomenological fatigue crack growth rates, $\frac{\mathrm{d}a}{\mathrm{d}N}$ within the range that exhibits the power-law response (Stage 2) could be expressed as functions of the crack tip driving force range, ΔK as [1]:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C[\Delta K]^n \tag{1.1}$$

where
$$\Delta K = \Delta\sigma\,\sqrt{\pi a}\;\mathrm{Y}\left(\frac{a}{W}\right) \tag{1.2}$$

The crack tip driving force, ΔK assumes the value of ΔK_I or ΔK_{II} under the Mode I (opening) or Mode II (shearing) crack tip loading, respectively. The term, $\Delta\sigma$ is the remotely applied stress range, a is the crack length, and $\mathrm{Y}\left(\frac{a}{W}\right)$ is the crack geometry factor of the test specimen. The coefficient, C and exponent, n are curve fitting parameters. Variations of Eqn (1.1) and (1.2) to account for the mean stress effect have been established [2–12]. These models could also represent the threshold crack growth rate (Stage 1) and the fast fracture regime (Stage 3). However, the unavailability of the crack geometry factor for calculating the crack tip driving force range, ΔK of Eqn (1.2) for numerous structural members poses the greatest challenge in establishing the crack growth rate response of the material. In this respect, several numerical approaches employing the FE method has been examined in quantifying ΔK

of a structural crack. Commercial FE packages like FRANC3D, NASGRO, and ADAPCRACK3D have been developed. FE solutions are viable when failure recurrence is high and only one type of crack is critical. The inaccuracies resulting from ill-defined loading conditions, specifically in multiple crack systems, are well-discussed and documented [13].

Fortunately, a propagating fatigue crack inherits signature fractal features along the crack length and surfaces. A crack in a continuum is created through the breaking of the bonds between atoms in the atomic structure under the imposed stress field. A tortuous topology of various degrees is exhibited in the wake of the crack. The different observed morphology of the crack surface and along the edge is manifested by the different intensities of the crack tip stress field. Studies have demonstrated that the crack edges along the crack length could be described as a fractal continuum exhibiting multifractal characteristics at the mesoscale. The fractal modeling of the physical systems encompasses an enormous diversity of intractable phenomena including structural cracking. The real cracks embodying a rugged trajectory are unfathomable through Euclidean settings [14–17]. The homoeomorphic deformations applied to the cracking process, generating twain detached fracture surfaces, spark severe concerns about the Euclidean assumption of the classical Griffith-Irwin-Orowan fracture theory [18]. The rifts in the classical theory appear as geometry correction factors in mathematical formulations of the cracking process, vide eq. (1.2). Conversely, the fractals provide a more realistic mathematical description of the cracking process and the rugged topology [19–21]. The chain of events that the crack experiences lodge on the fracture surface as obscurely arranged micro features. Current research banks on the conjecture that the fractal analysis can recuperate the defunct information by quantifying the fractality of crack micro-features, thereby facilitating the backtracking of the crack-tip variables, particularly the crack tip driving force. Thus, the fractal analysis can potentially weed out the requirement on the crack geometry factors. A significant statistical dispersion is expected due to the inherent random nature of fatigue crack growth rate, which demands annexation to a stochastic analysis for reliable fatigue life estimates.

Thus, a gap in the current knowledge exists due to reliance of fracture mechanics equations on the crack geometry factor, $\mathrm{Y}\left(\frac{a}{W}\right)$. This research develops an integrated probabilistic-fractal fracture model to estimate fatigue crack growth rate ($\mathrm{d}a/\mathrm{d}N$) in structures independent of the crack geometry. The material used is AISI 410 martensitic stainless steel. C(T) specimens provide baseline fatigue crack growth response of the material. The box-counting method is adopted for quantification of piecewise fractality at the crack edge. The model is validated on a complex structure of practical importance. The probabilistic analysis is based on Walker crack growth rate equation. The experimental data in the literature is digitized and used to validate the probabilistic model.

1.2 Problem Statement

This research hypothesizes that fracture resistance, being a material phenomenon, should not depend on the geometry of the cracked structure. The geometry factor $\mathrm{Y}\left(\frac{a}{W}\right)$ appearing in mathematical formulations indicate flaws in the classical theory due to Euclidean approximation and homeomorphic deformation assumption. The fractal features of a fatigue fracture surface are indicative of the fracture mechanism [22] and could be exploited to obtain the crack tip driving force, ΔK. While the fractal features have been evaluated and linked to material properties, in literature, correlation with the fracture parameters, dependence and correlation with the applied stress field has been widely ignored. To the author's knowledge, no work exists that could correlate fractal features in the crack wake with the crack tip crack tip driving force range, ΔK. The existence of a correlation between the crack tip driving force and the fractality at the crack edge could assist in eliminating the geometry factors dependence of fracture mechanics equations and answer the following research problem: "*How to reliably quantify the fatigue crack growth rate of a structure where the crack geometry factor is unavailable*?"

1.3 Research Objectives

The objectives of the research are:

1. To establish relevant material properties and benchmark fatigue crack growth behavior of AISI 410 martensitic stainless steel.

2. To develop a multifractal fatigue crack growth model based on crack edge fractal features.

3. To develop a robust probabilistic model of the fatigue crack growth rate of metallic materials.

4. To validate the multifractal fatigue crack growth model for intricate structures where the crack geometry factor is not available.

1.4 Scope of the Study

This research develops an integrated probabilistic-fractal fatigue crack growth model of metallic materials and covers the following:

1. AISI 410 martensitic stainless steel is employed as the case study material. The mechanical behavior of the material is established through mechanical tests and metallurgical examinations.

2. Mechanical Tests

    a) The tension test is carried out on a dog-bone-shaped specimen of overall dimensions: 100×32×5 mm^3 wire-cut from the stock material using Sodick AQ900L Electrical Discharge Machine (EDM). The test is conducted on an Instron Universal Testing Machine (Model 5982).

    b) The hardness testing is performed on a Vicker Hardness Testing using a square-base right-pyramid diamond indenter having an angle of 136° between the opposite faces.

c) The fatigue testing is carried out on six (06) compact tension specimens of AISI 410 stainless steel labeled CT1, CT2,…, and CT6. The specimens are prepared per ASTM E647 standard. The tests are performed using ±100 kN servo-hydraulic closed-loop Shimadzu Fatigue Testing Machine under load-controlled mode. IMT Solutions® microscopic camera mounted on a traveling platform with magnification up to 50X is used to precisely locate the crack tip.

3. Metallurgical and Fractographic Analysis

a) The chemical analysis is conducted on a LEGO Glow Discharge Spectrometer (GDS) machine using a specimen of size 20×20×2 mm^3. The composition is reported as weight percentage (wt%).

b) The microstructural examination of the material is performed using Nikon Microphot-FXL Optical Microscope equipped with Image Analyzer Software. The etching solution consists of 5 ml HCl, 100 ml Ethyl Alcohol and 2gr Picric acid.

c) The fractographic studies are done using a variable-pressure Scanning Electron Microscope (VP-SEM) at magnifications up to 5000X.

4. The fractal analysis is carried out on the crack edge imaged at a magnification of 100X and a spatial resolution of 1090 pixel/mm using Olympus BX51M metallurgical microscope. The box-counting algorithm is coded in Python programming language (v3.10.7)

5. The probabilistic model is structured per Walker crack growth rate equation. The maximum likelihood technique is used to obtain parameter estimates.

6. The finite element analysis of the compact tension specimen and the bell crank structure is performed using Abaqus commercial software.

7. The research is limited to the Paris region within the linear elastic fracture mechanics (LEFM) regime.

1.5 Significance of the Study

The study enables the crack tip driving force be quantified in the absence of the crack geometry factor. Thus, it extends the applicability of fracture mechanics equations to determine fatigue crack growth rate. Knowing the crack growth rate is imperative to damage tolerance analysis (DTA). Leaning on the crack geometry factors curtails the capability of fracture mechanics equations to precisely quantify the crack tip driving force (ΔK), a parameter of vital importance to crack growth rate evaluation. The unreliable estimate of ΔK risks the structure's integrity, jeopardizing its survival during the intended service life or inspection period. The current research enacts an alternate route to evaluate ΔK. The fracture features in the crack wake allow retrieving the load information using fractal analysis. Optical measurements could capture the hi-res digital mosaics of the crack microfeatures. Therefore ΔK could be determined by optical measurements without cognizance of the geometry factor for intricate geometries where fracture mechanics equations alone succumb to despair. Once ΔK is established through fractal analysis, it is possible to assess the crack's onrush and criticality using the Paris or equivalent law to appraise if an inspection interval is of appropriate length and extent of damage is sustainable until the next inspection. Therefore, the existing fracture mechanics equations could be explored for larger applications, specifically the intricate structures, not possible before. In addition, the suggested methodology for getting ΔK inherits high fidelity, being an outcome of the direct experimental measurements on the actual crack which encountered field loadings. For cases where FE solutions are practical, the fractal analysis could be deployed, in concert, to validate FE results

1.6 Thesis Layout

The thesis consists of seven chapters arranged to establish and validate an integrated probabilistic-fractal fatigue crack growth model. The contents of each chapter are summarized here to aid in linking them with the specific objectives and scope of the research.

Chapter 1 starts with the research background. The factors hindering the determination of the crack growth rate for intricate designs are highlighted. It is discussed how the fractal features of a propagating fatigue crack could help decipher the crack tip driving force. The main problem is divided into four specific research objectives, which interlink to eliminate dependence on crack geometry factors.

Chapter 2 provides the theoretical foundations of the work based on current literature. An exhaustive review of existing techniques for quantifying fractal features and the probabilistic analysis methodologies is presented. The predictive performance of thresholding algorithms to quantify the crack's fractality is surveyed and examined. The finite element (FE) practices used to establish K-solutions are discussed.

Chapter 3 portrays the research framework and methodology adopted for the probabilistic and fractal analysis to achieve the specific objectives presented in Chapter 1. Steps assumed for crack image analysis are enlisted. The method for obtaining the probabilistic model parameters is discussed. The procedure for correlating the crack tip driving force and the fractality is adequately detailed. The integration of the fractal and probabilistic models is outlined.

Chapter 4 discusses the outcomes of the crack's fractal analysis. The fatigue response of AISI 410 martensitic stainless steel is established. The correlation of the crack's fractal features with the crack tip driving force is explicated.

Chapter 5 provides the probabilistic analysis results of the fatigue crack growth rate. The model formulation and the background mathematics are outlined. The model's validation in the context of literature data is presented for numerous load cases.

Chapter 6 validates and illustrates the application of the established model on a bell crank structure. The fractal and probabilistic analysis results are compared with their experimental and FE counterpart.

Chapter 7 summarizes the findings of the research work and the conclusion drawn to assess how effectively the research objectives are achieved. The main contributions anent to the work are described in adequate detail. The recommendations for future work are provided for expanding the knowledge base in the field.

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LIST OF PUBLICATIONS

Journal Papers

1. MH Hashmi, SSR Koloor, MF Abdul-Hamid, MN Tamin. Fractal Analysis for Fatigue Crack Growth Rate Response of Engineering Structures with Complex Geometry. *Fractal and Fractional. 6(11),* 635, (2022). **(WOS/ISI Idx. Q1, IF=3.577)**
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2. MH Hashmi, SSR Koloor, MF Abdul-Hamid, MN Tamin. Exploiting fractal features to determine fatigue crack growth rates of metallic materials. *Engineering Fracture Mechanics. 270*, p.108589 (2022). **(WOS/ISI Idx. Q1, IF=4.898)**
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3. MH Hashmi, MF Abdul-Hamid, AA Latif, MN Tamin, MA Khattak. Fractal Dimensions of a Propagating Fatigue Crack in Metallic Materials. *Journal of Failure Analysis and Prevention. 21(5)*, pp.1644-1651 (2021). **(Scopus Idx. Q3, IF=0.29)**
   doi: https://doi.org/10.1007/s11668-021-01219-2

Conference Papers

N/A