A LOCALISED MULTISCALE TECHNIQUE IN BOUNDARY ELEMENT METHOD FOR ACOUSTIC WAVE MODEL

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DEDICATION

Dedicated to my beloved

father, *Zulkefli Bin Idris*

mother, *Nor Zihan Binti Jusoh*

and my siblings,

- *Nor Zulfa Irwani Binti Zulkefli*
- *Nor Atirah Izzah Binti Zulkefli*
- *Zul Hasanuddin Bin Zulkefli*
- *Zul Faziruddin Bin Zulkefli*
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for their endless love, care and support.

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ABSTRACT

The classical boundary element method (BEM) has emerged as a powerful alternative to the finite element method particularly in cases where better accuracy is required due to problems such as stress concentration or where the domain extends to infinity. In numerical calculation, the BEM has been widely used to solve acoustic problems since BEM offers excellent accuracy due to the discretization only on the structure's boundaries and easy mesh generation. However, BEM has some disadvantages. It suffers from certain drawbacks in terms of computational efficiency. Since most of the BEM leads to a linear system of equations with dense coefficient matrix, this prevents the BEM from being applied to large-scale problems or high-resolution mesh. Due to these disadvantages, according to the acknowledged literature, some researchers use hybrid BEM coupling with other methods to improve the computational efficiency or to improve the computational time. This research uses a different technique from the existing hybrid BEM which will improve both the computational efficiency and time. This study highlights that BEM is less accurate for high gradient problem and consumes more computational time. To overcome this problem, a new technique known as multiscale boundary element method (MBEM) is introduced for solving two dimensional acoustic problems. MBEM is introduced in order to reduce the computation time and improve numerical accuracy using the localised multiscale boundary element method (LMBEM) with the help of the FORTRAN language and parallel routine OpenMP. In addition, the truncated Newton method and Newton interpolation are introduced in this multiscale technique. The multiscale technique produces the results faster because of interpolation and accurate initial guess value in a linear system while the mesh refinement for particular elements based on gradient produces more accurate results. Numerical calculation is given to illustrate the efficiency of the proposed method and the solutions have been validated and compared with the BEM. The results show that the MBEM is indeed faster than BEM, with the computational time reduction is almost 33.01%. When the 38 elements are solved using LMBEM, it is more accurate as it gives an average error that is almost similar to a ratio of 38:36 with the 1024 elements using MBEM and BEM. In addition, this research is solving the problem on the boundary. It is suggested that the current study be expanded to solve the problem for the internal nodes of the domain since the internal node value is needed.

ABSTRAK

Kaedah unsur sempadan klasik (BEM) telah muncul sebagai alternatif yang kuat kepada kaedah unsur terhingga terutamanya dalam kes di mana ketepatan yang lebih baik diperlukan disebabkan masalah seperti penumpuan tekanan atau di mana domain meluas sehingga tak terhingga. Dalam pengiraan berangka, BEM telah banyak digunakan untuk menyelesaikan masalah akustik memandangkan BEM mencadangkan ketepatan yang sangat baik kerana diskret hanya pada sempadan struktur dan penjanaan jaring yang mudah. Walau bagaimanapun, BEM mempunyai beberapa kekurangan. Ia mengalami kekurangan tertentu dari segi kecekapan pengiraan. Oleh kerana kebanyakan BEM menghala kepada sistem persamaan linear dengan matriks pekali yang padat, ini menghalang BEM daripada digunakan pada masalah berskala besar atau jaringan resolusi yang tinggi. Oleh kerana kelemahan ini, menurut kesusasteraan yang diakui, beberapa penyelidik menggunakan gandingan BEM hibrid dengan kaedah lain untuk meningkatkan kecekapan pengiraan atau untuk meningkatkan masa pengiraan. Penyelidikan ini menggunakan teknik yang berbeza daripada BEM hibrid sedia ada yang akan meningkatkan kecekapan pengiraan dan masa. Kajian ini menekankan bahawa BEM adalah kurang tepat untuk masalah kecerunan yang tinggi dan mengguna lebih banyak masa pengiraan. Untuk mengatasi masalah ini, teknik baharu yang dikenali sebagai kaedah pelbagai skala unsur sempadan (MBEM) diperkenalkan untuk menyelesaikan masalah akustik dua dimensi. MBEM diperkenalkan untuk mengurangkan masa pengiraan dan meningkatkan ketepatan berangka dengan menggunakan kaedah lokasikan pelbagai skala unsur sempadan (LMBEM) dengan berbantukan bahasa FORTRAN dan rutin selari OpenMP. Di samping itu, kaedah Newton terpangkas dan interpolasi Newton diperkenalkan dalam teknik pelbagai skala ini. Teknik pelbagai skala menghasilkan keputusan lebih cepat kerana interpolasi dan nilai tekaan awal yang tepat dalam sistem linear manakala penghalusan jaring untuk unsur tertentu berdasarkan kecerunan menghasilkan keputusan yang lebih tepat. Pengiraan berangka diberikan untuk menggambarkan kecekapan kaedah yang dicadangkan dan penyelesaiannya telah disahkan dan dibandingkan dengan BEM. Hasil kajian menunjukkan bahawa MBEM sememangnya lebih pantas daripada BEM, dengan pengurangan masa pengiraan adalah hampir 33.01%. Apabila 38 unsur diselesaikan menggunakan LMBEM, ianya adalah lebih tepat kerana ia memberikan ralat purata yang hampir sama dengan nisbah 38:36 dengan 1024 unsur menggunakan MBEM dan BEM. Di samping itu, penyelidikan ini menyelesaikan masalah di sempadan. Kajian semasa ini dicadangkan dapat diperluaskan bagi menyelesaikan masalah untuk nod dalaman domain memandangkan nilai nod dalaman diperlukan.

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LIST OF ABBREVIATIONS

FEM	-	Finite Element Method
BEM	-	Boundary Element Method
BIE	-	Boundary Integral Equation
CBIE	-	Conventional Boundary Integral Equation
MBEM	-	Multiscale Boundary Element Method
LMBEM	-	Localised Multiscale Boundary Element Method
PVM	-	Parallel Virtual Machine
CPU	-	Central Processing Unit
MPI	-	Message Passing Interface
OpenMP	-	Open Multi-Processing
GMRES	-	Generalized Minimal Residual
API	-	Application Program Interface
2D	-	Two-dimensional

LIST OF SYMBOLS

ϕ	-	Acoustic pressure
q	-	Velocity
∇^2	-	Laplace operator
f	-	Function
x	-	Point of x
y	-	Point of y
c	-	Constant
w	-	Weighting function
$\Re^2$	-	Two-dimensional space
t	-	Time
G	-	Fundamental solution
F	-	Normal derivative of G
H	-	Hankel function
V	-	Domain enclosed by S
Q	-	Typical point source
L	-	Infinite domain outside S
S	-	Boundary of the domain
s	-	Speed
r	-	Distance between source point x and field point y
δ	-	Dirac function
n	-	Normal vector
N	-	Size parameter
i	-	Imaginary number
A	-	Matrix of coefficient
b	-	Right-hand side vector
$\mathbf{x}$	-	Unknown vector
u	-	Solution of the problem
ω	-	Circular frequency
h	-	Increment

R	-	Residue in vector form
Q	-	Quadratic function
H^{es}	-	Hessian matrix
Superscript T	-	Matrix transpose
Superscript -1	-	Matrix inverse
ω	-	Circular frequency
g	-	Gradient
i	-	Element
E	-	Error
$\bar{E}$	-	Average Error
$\| \|$	-	Modulus
W	-	Weights of gaussian quadrature
P	-	Number of processors
$\neq$	-	Not equal to
$\leq$	-	Less than or equal to
$\geq$	-	Greater than or equal to
$\blacksquare$	-	Proven

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CHAPTER 1

INTRODUCTION

1.1 Introduction

The focus of this research is to improve the boundary element method (BEM), because BEM is less accurate for high gradient problems and consumes more computational time. Two-dimensional acoustic problem is considered and can be written as

$$\nabla^2\phi - \frac{1}{s^2}\frac{\partial^2\phi}{\partial t^2} + Q\delta(x, x_Q) = 0, \qquad \forall x \in L, \tag{1.1}$$

where $\phi = \phi(x,t)$ is the complex acoustic pressure, x is the coordinate, t is the time, s is the speed of the sound in the medium and $Q\delta(x, x_Q)$ represents a possible point source located at x_Q inside domain L. The acoustic domain L is considered to be isotropic and homogeneous and can be an infinite domain interior to a body or a finite domain interior to a closed surface. For time harmonic waves, the time dependent velocity potential ϕ can be reduced to a sum of components, the point source intensity $Q = \tilde{Q}e^{-i\omega t}$ and the solution to the governing equation can be written as:

$$\phi(x,t) = \tilde{\phi}(x,\omega)e^{-i\omega t}, \tag{1.2}$$

in which $\tilde{\phi}(x,\omega)$ is the complex acoustic pressure in the frequency domain, ω is the circular frequency, $i = \sqrt{-1}$ and for convenience, the tildes are dropped in the preceding equation. Substituting Equation (1.2) into Equation (1.1), the governing equation for acoustic wave problem is written as:

$$\nabla^2\phi + k^2\phi + Q\delta(x, x_Q) = 0, \;\; x \in L, \tag{1.3}$$

where $k = \frac{\omega}{s}$ is the wavenumber (Liu, 2009). Equation (1.3) is the physical representation of the problem. The source point will produce the vibrations. This research focuses on pressure and velocity on the boundary of the problem which is the solution of the problem on the boundary. Equation (1.1) is reduced to Equation (1.3) which is time independent and depend on frequency $\omega, \omega = ks$.

1.2 Research Background

There are various methods for solving numerical computations such as the Finite Element Method (FEM) and the BEM. The FEM and the BEM are similar in using elements and nodes, but only on the boundaries. The FEM is a method of breaking down a physical structure into smaller parts for analysis, while the BEM is obtained by discretizing an integral equation. The discretization in the BEM is only done on the boundary, which eases the computation since there are fewer elements and nodes compared to when using the FEM.

The BEM is often used to solve boundary value problems for systems of partial differential equations. Its ability to reduce the dimension of a problem by one is the principal advantage of the BEM over other numerical methods. This property is advantageous as this means that the size of the system of the problem will also be reduced, leading to improved computational efficiency (Grecu and Vladimirescu, 2009).

Boundary Integral Equations (BIE)s for partial differential equations are a classical tool for the analysis of boundary value problems. BEM denotes any method for the approximate numerical solution of these BIEs (Costabel, 1986). The dimension reduction in the BIE formulations makes the BEM mesh much easier to generate for three dimensional problems or infinite domain problems. Besides that, the BEM is a semi-analytical method that is more accurate, especially for stress concentration problems such as fracture of structures, and can be applied along with other domain-based methods to verify solutions to problems for which there is no analytical solution.

However, BEM has several disadvantages. BEM suffers from certain drawbacks in terms of computational efficiency. There is a serious issue regarding its efficiency in solutions for analyzing large-scale models since the conventional BEM, in general, produces dense and nonsymmetric matrices (Liu, 2009). Since the BEM leads to a linear system of equations with dense coefficient matrix, this prevents the BEM from being applied to high-resolution mesh or large-scale problems.

Besides that, BEM produces a dense linear system in solving inhomogeneous and non-linear problems because the resulting matrices are asymmetric and fully populated. Thus, it increases the computational time significantly. BEM also requires more knowledge about suitable fundamental solutions.

In order to achieve more successful results, various researchers have modified the BEM formulation. Guminiak (2016) investigated static and free vibration analysis of thin plates with curved edges by using the BEM with an alternative formulation of boundary conditions. The successful application of the BEM in solving the thin plate bending issue demonstrated the sufficient effectiveness and efficiency of the proposed approach. In 2019, Carrer et al. (2019) developed two different BEM formulations for the dynamic analysis of Euler-Bernoulli continuous beams. Their results showed that both BEM formulations can produce accurate results for the dynamic analysis of continuous beams. Based on their study, they did not focus on computational time, which leads to this research.

Lee and Polisoc (1990) stated that the gradients are produced by numerically differentiating the potentials. As a result, the gradients are significantly less accurate and frequently discontinuous between elements. Karlis et al. (2000) developed an advanced BEM for solving two-dimensional and three-dimensional static problems in Mindlin's strain gradient theory of elasticity. The solution of a simple two-dimensional gradient elastic problem has demonstrated the importance of paying close attention to how the problem's boundary conditions are handled, particularly when non-smooth boundaries are taken into consideration.

Other researchers investigated the high gradient problem by using other techniques such as Akeremale et al. (2021) determined that the h-adaptive FEM is preferred or superior to the classic FEM in high gradient problems in terms of accuracy and computation cost.

Acoustic waves often exist in vibration or are impinged on by incident waves, which is also known as an infinite medium outside of the structure. In acoustic wave analysis, there are two types of problems. A radiation problem occurs when a structure vibrates and causes disturbances in the acoustic field outside or inside the structure. The second type is called a scattering problem, in which an incoming disturbance interacts with the structure causing the waves to be scattered. In this study, the general form of acoustic wave problem is discussed.

Solving acoustic wave problems can also be used in other fields, such as for predicting noise control in the study of sound.

1.3 Statement of the Problem

BEM has been widely used to solve numerical problems as it offers excellent accuracy. BEM discretization only on the domain boundary, and easy for mesh generation. However, the accuracy of a results vicinity to a high gradient is low. Lee and Polisoc, 1990 stated that the gradients are produced by numerically differentiating the potentials and they observed that the computed results are significantly less accurate and frequently discontinuous between elements. For high gradient problem, the solution change drastically with small changing of coordinate. Thus, this will induce inaccurate results. In order to improve the accuracy and reduce the computation cost, some researchers had investigated the high gradient problem by using other techniques (Akeremale et al., 2021). Gao and Hu (2011) stated that BEM have the disadvantage in limiting the scope and speed. BEM also shows less accurate for high gradient problems based on current simulation results.

The acoustic problem that has been focused in the present thesis is a high gradient problem. To overcome the inaccuracy and time consuming in computing the results, the present research introduces a new multiscale technique coupled with the BEM for solving the two-dimensional acoustic problems. Some researchers have investigated the integration of the multiscale technique and BEM. However, the dimension of matrices affects the computational efficiency of the method. Thus, a longer time is taken to calculate the results when more elements that are picked up in the inlet and outlet of each substructure, (Yang and Ji, 2013). The truncated Newton method and Newton interpolation are implemented here as a new multiscale technique. These two approaches are among the methods for reduce the computation time. MBEM is introduced in order to reduce the computation time. Based on the acknowledged literature, this new multiscale technique has not yet been explored by any researchers. To improve the developed MBEM, the study designed a localised multiscale for certain elements based on the gradient for a better accuracy. Furthermore, parallel routine OpenMP with the help of FORTRAN language is introduced to further reduce the time consumption for generating the mesh and computing the solutions.

1.3.1 Research Question

To achieve the objectives, the following questions need to be answered:

(a) How to develop a numerical algorithm of BEM and MBEM using FORTRAN language for two-dimensional acoustic problems?

(b) How to improve MBEM by introducing a localised approach?

(c) What is the accuracy of LMBEM, BEM and MBEM?

(d) What is the speed of MBEM using a parallel technique?

(e) How to validate and compare LMBEM with the BEM?

1.4 Research Objectives

This study embarks on the following objectives:

1. To introduce new multiscale technique with BEM for improvement of BEM in term of reduce the computation time.
2. To speed up the proposed method by introducing the parallel technique.
3. To improve the developed MBEM using LMBEM for higher accuracy solutions with less total mesh.
4. To introduce error analysis in order to calculate real-time error and it is useful for adaptive numerical method in the future research.

1.5 Scope of the Study

This research focuses on a method known as MBEM, a combination of a new multiscale technique and BEM. This study attempts to make use of the truncated Newton method and Newton interpolation as a new multiscale technique. In addition, LMBEM is used to improve the developed MBEM. The effectiveness of the proposed method is illustrated in the application of the numerical computation of the two-dimension acoustic wave. A program is developed with the help of the FORTRAN language and parallel routine OpenMP. Lastly, the numerical computation of the two-dimension acoustic wave problem is validated, the exact solution is computed and a comparison between the new proposed method with the BEM is conducted.

1.6 Significance of the Study

This research can enhance the understanding of the concept of a new multiscale technique in BEM. This new multiscale technique introduces the truncated Newton method and Newton interpolation. The MBEM is used to reduce the computation time and improve numerical accuracy using the localised multiscale boundary element method (LMBEM) with the help of the FORTRAN language and parallel routine OpenMP. Since a linear system uses the interpolation technique and accurate initial guess value, the multiscale approach will yield fast computational results, whereas mesh refinement for specific elements based on gradient will produce more accurate results.

The faster and more accurate numerical algorithm for the numerical computation of acoustic wave problems can be applied using the proposed method. Besides that, this study is expected to establish a numerical library for the solution of the numerical computation of acoustic wave problems. In addition, the study includes an analysis on the boundary of the acoustic wave problem. The numerical results obtained can serve as reference and be used for validation purposes against other (future) investigations and numerical results. It can also be used in other fields, such as for predicting noise control in the study of sound. The proposed method can also be used as a reference for future studies in other various fields of science and engineering.

1.7 Outline of the Thesis

This thesis consists of six chapters. Chapter 1 presents the general introduction and overview of the research, including the research background, statement of the problem, research objectives, as well as scope and significance of the study. In Chapter 2, a literature review of previous research works regarding the research area is reviewed and discussed. The brief review is divided into several sections which discuss the topics of BEM, multiscale schemes, acoustic wave problems, parallel computation and critical analysis of the literature.

Chapter 3 discusses the research methodology consisting of the overall research framework. This chapter starts with a description of the derivation of BEM for a simple two-dimensional problem. This chapter gives the research methodology, the multiscale technique and theoretical proof of the error analysis. Afterwards, Chapter 4 solves the acoustic wave problem using BEM and MBEM. The application of parallel computation and boundary condition involved is discussed.

Meanwhile, Chapter 5 discusses the localised multiscale for acoustic wave problems with BEM. The solution algorithm and fundamental results in Chapters 4 and 5 provide the answers to the research questions. The complexity of the numerical schemes is also provided in Chapter 4.

Chapter 6 summarizes the whole work and draws a conclusion on the findings. This chapter also discusses future research that may be conducted for a deeper understanding of the problems considered.

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1. **Zulkefli, N. A. H.**, Ismail, M., Zulkefli, N. A. I., Hoe, Y. S. (2016). Multiscale boundary element method for Laplace equation. *Jurnal Teknologi*, 78, 83-88. http://doi.org/10.11113/jt.v78.7818. **(Indexed by SCOPUS)**

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3. **Zulkefli, N. A. H.**, Hoe, Y. S., and Ismail, M. (2019). Multiscale Boundary Element Method for Acoustic Wave Model. *MATEMATIKA: Malaysian Journal of Industrial and Applied Mathematics*, 35(3), 387-395. https://doi.org/10.11113/matematika.v35.n3.1165. **(Indexed by WOS)**

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