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Adaptive CUSUM Location Control Charts Based on Score Functions: An Application in Semiconductor Wafer Field

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Abstract

An adaptive CUSUM (ACUSUM) control chart got special attention against classical CUSUM control chart to detect a shift of different sizes in the process location. Similarly, an ACUSUM based on classical EWMA statistic and score function, denoted as a $\mathrm{ACUSUM_E}$ control chart, is improved form of classical CUSUM control chart and can identify different sizes of shift. Classical EWMA statistic in $\mathrm{ACUSUM_E}$ control chart fails to offer clear instruction for parameter values to identify a precise shift as the classical CUSUM statistic does. To address this issue, this study proposed two ACUSUM control charts, symbolized as $\mathrm{ACUSUM_C^{(1)}}$ and $\mathrm{ACUSUM_C^{(2)}}$ to further improve detection ability of shift in the process location. Novelty of the proposed control charts is to initially adaptively renew reference parameters values based on classical CUSUM statistic and then to assign a weight on it using score functions. An algorithm is developed in MATLAB using Monte Carlo simulation method to obtain numerical results. Based on numerical results, performance measures such as average run length for a specific shift, extra quadratic loss, relative average run length, and performance comparison index for overall performance are calculated for comparison purpose. Comparison based on visual presentation and numerical results reveals the proposed control charts performed quite effective against some existing control charts. It is worthy to mention, classical CUSUM control chart is special cases of proposed $\mathrm{ACUSUM_C^{(1)}}$ control charts at specific values of parameter. Finally, proposed control charts are also implemented on real-life data to show practical significance to users and practitioners.

Keywords Adaptive · CUSUM · Performance measures · Monte Carlo simulation · Score functions

1 Introduction

Random and special causes of variations are part of every process parameter (location and /or dispersion). In more details, random causes of variations are inherited part of the process and considered harmless; therefore, a process is considered statistically in-control if it operates under random causes of variations. In contrast, special causes of variations may appear due to several reasons such as improper adjustment of tools, operator's errors, improper adjustment of machine, and defective raw material. Furthermore, a process governs under special causes of variations is stated as statistically an out-of-control. However, an effective action towards eliminating the special causes of variations results into process in-control state. The magnitude of special causes of variations occur in the process is termed as a shift. Shewhart [1] initiated the idea of quality (also called Shewhart) control charts also recognized as memory-less control charts to distinguish a large shift in the process. To handle small-to-moderate shifts, Page [2] and

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Roberts [3] offered cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) control charts, respectively; these control charts are also known as memory control charts.

An enhanced and modified forms of the classical memory control charts such as an adaptive memory control charts have been recommended by researchers to identify a broad range of shift in the process location. For example, Sparks [4] recommended an adaptive CUSUM (ACUSUM) control chart to detect a certain range of shift. The ACUSUM control chart's time-varying reference parameter is based on classical EWMA statistic. The ACUSUM control chart is more robust against the classical CUSUM control chart and as well as to other control charts when the reference parameter value is 0.5. Similarly, Capizzi and Masarotto [5] suggested an adaptive EWMA (AEWMA) control charts based on score (Huber and Bi-square) functions and the classical EWMA statistic, denoted as AEWMA_{E} control charts. The AEWMA_{E} control charts structures are based on time varying parameters and efficient to detect different sizes of shift.

Likewise, Jiang et al. [6] designed an ACUSUM based on the classical EWMA statistic and Huber function, denoted as a ACUSUM_{E} control chart. The ACUSUM_{E} control chart is efficient to differentiate different sizes of shift in the process location. Equally, an ACUSUM control chart based on standardized input statistic, the classical EWMA statistic, and Huber function recommend by Wu et al. [7] to detect different sizes of shift. Recently, Zaman et al. [8] offered AEWMA control charts based on score functions and classical CUSUM statistic, denoted as AEWMA_{C} control charts. Like AEWMA_{E} control charts, the AEWMA_{C} control charts structures are based on time varying parameters and also effective to detect different sizes of shift. More insight and details on the advanced forms of memory control charts can be seen in the studies of Luceño and Puig-Pey [9], Hawkins and Zamba [10], Khoo [11], Zhao et al. [12], Jiao and Helo [13], Khoo and Teh [14], Chatterjee and Qiu [15], Maravelakis [16], Liu et al. [17], Ou et al. [18], Amiri et al. [19], Hawkins and Wu [20], Zaman et al. [21], Haq et al. [22], Hussain et al. [23], Chernoff and Zacks [24], Chen and Elsayed [25], and references therein.

The classical EWMA statistic in the ACUSUM_{E} control chart (cf. Jiang et al. [6]) does not provide explicit rules for parameter values to diagnose a specific shift (cf. Hawkins and Wu [20]). In other words, the classical EWMA statistic in ACUSUM_{E}control chart fails to offer clear instruction for parameter values to identify a precise shift as the classical CUSUM statistic does. Additionally, the ACUSUM_{E} control chart is designed only for Huber function, but for other functions such as Bi-square function may be valuable to enhance detection ability. These points are taken as an inspiration to design this study. So, this study proposes an ACUSUM control chart-based Huber function and on the classical CUSUM statistic (symbolized as a $\text{ACUSUM}_{\text{C}}^{(1)}$) instead of the classical EWMA statistic to address the issue of the ACUSUM_{E} control chart (cf. Jiang et al. [6]). Moreover, the proposed ACUSUM control chart is also designed for Bi-square function, denoted as an $\text{ACUSUM}_{\text{C}}^{(2)}$ control chart. The rationality behind the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts initially adaptively renew the reference parameters values based on the classical CUSUM statistic and then give weights using score (Huber and Bi-square) functions to distinguish specific and as well certain range of shifts.

To evaluate the performance of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts, performance measures such as average run length (ARL) for a specific shift, extra quadratic loss (EQL), relative average run length (RARL), and performance comparison index (PCI) for certain range of shift are studied. Monte Carlo simulation method is used to obtain numerical results. Comparison reveals that the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts perform quite better against some existing control charts to differentiate exact and as well broad range of shifts. The existing control charts such as classical CUSUM and EWMA, AEWMA_{E}, mixed EWMA-CUSUM, named as MEC [26], Hybrid EWMA [27], mixed CUSUM-EWMA, denoted as MCE [21], and AEWMA_{C} are considered for comparison purpose. Besides, it is worthy to mention that classical CUSUM control chart is a special case of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ control charts at specific value of parameter. Finally, the proposed control charts are also implemented on real-life data (layer thickness on semiconductor wafers) to show significance to practical users over existing control charts.

The rest of the study is organized as follows: Sect. 2 presents the research methodologies of existing control charts. Likewise, the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts methodologies are outlined in Sect. 3. Correspondingly, performance evaluation measures are given in Sect. 4. Comparative analysis of proposed control charts against other control charts is provided in Sect. 5. The implementation of proposed and other control chart with real-life data is offered in Sect. 6. Finally, summary, conclusion, and recommendation are given in Sect. 7.

2 Existing Memory Control Charts

This section contains variable of interest and basic structures of existing memory control charts. In more details, Sect. 2.1 presents variable of interest. In addition, methodologies of the classical CUSUM [2] and EWMA [3] control charts are given in Sect. 2.2 and 2.3, respectively. Similarly, the

AEWMA_E [5] and AEWMA_C [8] control charts procedures are presented in Sect. 2.4 and 2.5, respectively.

2.1 Variable of Interest

Let $x_i \sim N(\mu_0, \sigma_0^2)$ is a process characteristic of interest that follows a normal probability distribution with known in-control mean μ_0 and variance σ_0^2, whereas $i = 1,2,3 \ldots n$, and n is the total number of sample to be monitored. The x_i also presents the input statistic of the classical memory control charts to monitor a shift in the process location.

2.2 Classical CUSUM Control Chart

The classical CUSUM control chart recommended by Page [2] is famous to detect a small-to-moderate shift in the process location. The plotting statistics of the classical CUSUM control chart can be offered as follows:

$$C_i^+ = \max\left[0,\ x_i - (\mu_0 + K) + C_{i-1}^+\right], \tag{1}$$

$$C_i^- = \max\left[0,\ \left(\mu_0 - K\right) - x_i + C_{i-1}^- +\right], \tag{2}$$

where $C_0^\pm = 0$ are initial values (zero state) and K is a reference constant. Further, K can be defined as: $K = k\sigma_0$ [20, 28] and k known as a constant which usually chosen equal to half of the shift (in standard units). The upper control limit (UCL) is defined below

$$H = h\sigma_0, \tag{3}$$

where h is a control limit coefficient depends on k, n, and σ_0 for pre-fixed value of in-control average run length (ARL_0). If $C_i^+ > \text{H}$ or $C_i^- > \text{H}$, the process is considered out-of-control; otherwise, in-control.

2.3 Classical EWMA Control Chart

The classical EWMA control chart proposed by Roberts [3] is also famous to monitor a small-to-moderate shift in the process location. The plotting statistic of the classical EWMA control chart can be defined as follows:

$$E_i = (1-\lambda)E_{i-1} + \lambda x_i, \tag{4}$$

where $\lambda \in (0, 1]$ is a smoothing constant and $E_0 = \mu_0$ is initial value of the E_i statistic [28]. The E_i statistic is the weighted average of all previous samples. The lower control limit (LCL), centre line (LC), and UCL are defined as follows:

$$\text{LCL}_i = \mu_0 - \text{L}\sigma_0\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}, \tag{5}$$

$$CL_i = \mu_0, \tag{6}$$

$$\text{UCL}_i = \mu_0 + \text{L}\sigma_0\sqrt{\frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]}, \tag{7}$$

respectively. The L represents control limits coefficient depends on smoothing constant λ, μ_0, n, and σ_0 for the pre-fixed value of ARL_0. If the plotting E_i statistic falls beyond the control limits (i.e., $E_i > \text{UCL}_i$ or $E_i < \text{LCL}_i$), the process is stated out-of-control, otherwise in-control.

2.4 AEWMA_E Control Charts

This section contains the methodologies of the AEWMA_E [5] control charts to monitor different sizes of a shift in the process location. If the design structure of the AEWMA_E depends on the classical EWMA statistic and Huber function, it is symbolized as an $\text{AEWMA}_\text{E}^{(1)}$ control chart. Similarly, if the methodology of the AEWMA_E is designed using the classical EWMA statistic and Bi-square-function, it is represented as an $\text{AEWMA}_\text{E}^{(2)}$ control chart. More details on the design structures of the $\text{AEWMA}_\text{E}^{(1)}$ and $\text{AEWMA}_\text{E}^{(2)}$ control charts are provided in the following subsections.

2.4.1 $\text{AEWMA}_E^{(1)}$ Control Chart

Plotting statistic of the $\text{AEWMA}_\text{E}^{(1)}$ control chart can be designed as follows:

$$\text{AEWMA}_{Ei}^{(1)} = \left(1 - w_E^{(1)}\left(e_i\right)\right)\text{AEWMA}_{E(i-1)}^{(1)} + w_E^{(1)}\left(e_i\right)x_i, \tag{8}$$

where $w_E^{(1)}\left(e_i\right) = \emptyset_1\left(e_i\right)/e_i$ is the time-varying parameter, $e_i = x_i - E_{i-1}$ is a prediction error, the $\text{AEWMA}_{E0}^{(1)} = 0$, and $\emptyset_1\left(e_i\right)$ is the Huber function that is defined as follows:

$$\emptyset_1\left(e_i\right) = \begin{cases} e_i + (1-\lambda)\gamma & \text{if } e_i < -\gamma \\ \lambda e_i & \text{if } \left|e_i\right| \le \gamma \\ e_i - (1-\lambda)\gamma & \text{if } e_i > \gamma, \end{cases} \tag{9}$$

where $\gamma > 0$ is a constant which helps to define the range of e_i. It is important to mention that control limits constants or quantities of classical EWMA control chart are similar to $\text{AEWMA}_\text{E}^{(1)}$ control chart except different control limits coefficient ($\text{L}_{\text{AEWMA}_\text{E}^{(1)}}$). In short, replace control limits coefficient $\text{L}_{\text{AEWMA}_\text{E}^{(1)}}$ with L in Eqs. (5) and (7) to obtain control limits of $\text{AEWMA}_\text{E}^{(1)}$ control chart. The control limits coefficient $\text{L}_{\text{AEWMA}_\text{E}^{(1)}}$ depends on $\lambda, \gamma, \mu_0, \sigma_0^2$, and n for pre-fixed value of ARL_0. The LCL, CL, and UCL of the $\text{AEWMA}_\text{C}^{(1)}$ control chart are denoted as $\text{LCL}_{\text{AEWMA}_{Ei}^{(1)}}$, $CL_{\text{AEWMA}_{Ei}^{(1)}}$, and $\text{UCL}_{\text{AEWMA}_{Ei}^{(1)}}$, respectively. A process is said to be out-of-control if the $\text{AEWMA}_{Ei}^{(1)} > \text{UCL}_{\text{AEWMA}_{Ei}^{(1)}}$ or $\text{AEWMA}_{Ei}^{(1)} < \text{LCL}_{\text{AEWMA}_{Ei}^{(1)}}$, otherwise, in-control.

2.4.2 $\text{AEWMA}_E^{(2)}$ Control Chart

The $\text{AEWMA}_{\text{E}}^{(2)}$ control chart plotting statistic is given below:

$$\text{AEWMA}_{Ei}^{(2)} = \left(1 - w_E^{(2)}(e_i)\right)\text{AEWMA}_{E(i-1)}^{(2)} + w_E^{(2)}(e_i)x_i, \tag{10}$$

where $w_E^{(2)}(e_i) = \emptyset_2(e_i)/e_i$ is the varying weight function, and $\emptyset_2(e_i)$ is the Bi-square function that is defined as follows:

$$\emptyset_2(e_i) = \begin{cases} e_i\left(1-(1-\lambda)\left[1-(e_i/\gamma)^2\right]^2\right) & if \quad |e_i| \le \gamma \\ e_i & otherwise \end{cases}. \tag{11}$$

Like Sect. 2.4.1, replace control limits coefficient $\text{L}_{\text{AEWMA}_{\text{E}}^{(2)}}$ of the $\text{AEWMA}_{\text{E}}^{(2)}$ control chart with L in Eqs. (5) and (7) to obtain control limits of $\text{AEWMA}_{\text{E}}^{(2)}$ control chart. The $\text{L}_{\text{AEWMA}_{\text{E}}^{(2)}}$ depends λ, γ, μ_0, σ_0^2, and n for pre-fixed value of ARL_0. The LCL, CL, and UCL of the $\text{AEWMA}_{\text{E}}^{(2)}$control chart are symbolized as $LCL_{AEWMA_{Ei}^{(2)}}$, $CL_{AEWMA_{Ei}^{(2)}}$, and $UCL_{AEWMA_{Ei}^{(2)}}$, respectively. A process issues out-of-control signal if the statistic $AEWMA_{Ei}^{(2)} > UCL_{AEWMA_{Ei}^{(2)}}$ or $AEWMA_{Ei}^{(2)} < LCL_{AEWMA_{Ei}^{(2)}}$; otherwise, in-control.

2.5 AEWMA_C Control Charts

This subsection presents two methodologies of AEWMA_{C} ($\text{AEWMA}_{\text{C}}^{(1)}$ and $\text{AEWMA}_{\text{C}}^{(2)}$) control charts proposed by Zaman et al. [8]. The AEWMA_{C} control charts diagnose different sizes of shift in the process location. This first form $\text{AEWMA}_{\text{C}}^{(1)}$ control chart depends on classical CUSUM statistic and Huber function. Similarly, the second form $\text{AEWMA}_{\text{C}}^{(2)}$ control chart utilized classical CUSUM statistic and Bi-square function. More details of $\text{AEWMA}_{\text{C}}^{(1)}$ and $\text{AEWMA}_{\text{C}}^{(2)}$ control charts are outlined in Sects. 2.5.1 and 2.5.2, respectively.

2.5.1 $\text{AEWMA}_C^{(1)}$ Control Chart

The $\text{AEWMA}_{\text{C}}^{(1)}$ control chart is also used to identify different sizes of shift in the process location. The plotting statistic of $\text{AEWMA}_{\text{C}}^{(1)}$ control chart is given as:

$$\text{AEWMA}_{Ci}^{(1)} = \left(1 - w_C^{(1)}(e_{1i})\right)\text{AEWMA}_{C(i-1)}^{(1)} + w_C^{(1)}(e_{1i})x_i, \tag{12}$$

where $w_C^{(1)}(e_{1i}) = \varnothing_1(e_{1i})/e_{1i}$ is the time varying parameter, $e_{1i} = x_i - C_{i-1}^+$ or $e_{1i} = x_i - C_{i-1}^-$ is a prediction error, and $\varnothing_1(e_{1i})$ known as Huber function for e_{1i} like $\varnothing_1(e_i)$. The control limits constants or quantities of classical EWMA control chart are similar to $\text{AEWMA}_{\text{C}}^{(1)}$ control chart except different control limits coefficient ($\text{L}_{\text{AEWMA}_{\text{C}}^{(1)}}$). In short, replace $\text{L}_{\text{AEWMA}_{\text{C}}^{(1)}}$ with L in Eqs. (5) and (7) to obtain control limits of $\text{AEWMA}_{\text{C}}^{(1)}$ control chart. The $\text{L}_{\text{AEWMA}_{\text{C}}^{(1)}}$ depends on λ, k, γ, μ_0, σ_0^2, and n for pre-fixed value of ARL_0. The {\text{LCL}}, CL, and {\text{UCL}} of $\text{AEWMA}_{\text{C}}^{(1)}$ control chart are denoted as $\text{LCL}_{\text{AEWMA}_{ci}^{(1)}}$, $CL_{\text{AEWMA}_{ci}^{(1)}}$, and $\text{UCL}_{\text{AEWMA}_{ci}^{(1)}}$, respectively. A process is said to be out-of-control if the $\text{AEWMA}_{ci}^{(1)} > \text{UCL}_{\text{AEWMA}_{ci}^{(1)}}$ or $\text{AEWMA}_{ci}^{(1)} < \text{LCL}_{\text{AEWMA}_{ci}^{(1)}}$, otherwise, in-control.

2.5.2 $\text{AEWMA}_C^{(2)}$ Control Chart

The $\text{AEWMA}_{\text{C}}^{(2)}$ control chart is also effective to identify different sizes of shift in the process location. The procedure of $\text{AEWMA}_{\text{E}}^{(2)}$ control chart depends on classical EWMA statistic and Bi-square function. The $\text{AEWMA}_{\text{E}}^{(2)}$ control chart plotting statistic is designed as follows:

$$\text{AEWMA}_{Ei}^{(2)} = \left(1 - w_C^{(2)}(e_{1i})\right)\text{AEWMA}_{E(i-1)}^{(2)} + w_C^{(2)}(e_{1i})x_i, \tag{13}$$

where $w_C^{(2)}(e_{1i}) = \varnothing_2(e_{1i})/e_{1i}$ is the time varying parameters and statistic $\varnothing_2(e_{1i})$ is Bi-square function for e_{1i} like$\varnothing_2(e_i)$. Like Subsection 2.5.1, replace control limits coefficient $\text{L}_{\text{AEWMA}_{\text{C}}^{(2)}}$ with L in Eqs. (5) and (7) to obtain control limits of $\text{AEWMA}_{\text{C}}^{(2)}$ control chart. The $\text{L}_{\text{AEWMA}_{\text{C}}^{(2)}}$ depends onλ,k,γ, μ_0, σ_0^2, and n for pre-fixed value ofARL_0. The {\text{LCL}}, CL, and {\text{UCL}} of the $\text{AEWMA}_{\text{C}}^{(2)}$ control chart are denoted as$\text{LCL}_{\text{AEWMA}_{ci}^{(2)}}$,$CL_{\text{AEWMA}_{ci}^{(2)}}$, and$\text{UCL}_{\text{AEWMA}_{ci}^{(2)}}$, respectively. A process said to be out-of-control if $\text{AEWMA}_{ci}^{(2)} > \text{UCL}_{\text{AEWMA}_{ci}^{(2)}}$ or$\text{AEWMA}_{ci}^{(2)} < \text{LCL}_{\text{AEWMA}_{ci}^{(2)}}$; else, in-control.

3 Proposed ACUSUM_C Control Charts

This section presents design structures of the proposed ACUSUM_{C} control charts to monitor shifts in the process location. The design structures of the proposed ACUSUM_{C} control charts depend on the classical CUSUM statistics and score functions. If the proposed ACUSUM_{C} control chart is designed with the Huber function, it denoted by $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ for the Bi-square function.

3.1 Proposed $\mathbf{ACUSUM_C^{(1)}}$ Control Chart

The plotting statistics of the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart can be presented as follows:

$$ACUSUM_{Ci}^{(1)+} = \max\left[0, x_i - \left(\mu_0 + w_C^{(1)}(e_{1i})\right) + \mathrm{ACUSUM}_{C(i-1)}^{(1)+}\right], \quad (14)$$

$$\mathrm{ACUSUM}_{Ci}^{(1)-} = \max\left[0, \left(\mu_0 - w_C^{(1)}(e_{1i})\right) - x_i + \mathrm{ACUSUM}_{C(i-1)}^{(1)-}\right], \quad (15)$$

where $\mathrm{ACUSUM}_{C(0)}^{(1)\pm} = 0$ are the initial values. The $\mathrm{ACUSUM}_{Ci}^{(1)+}$ and $\mathrm{ACUSUM}_{Ci}^{(1)-}$ are called one-sided upper and lower statistics of the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart, respectively. It is worthy to mention that control limit constants or quantities of the classical CUSUM control chart are similar to the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart except different control limit coefficient $\mathrm{h_{ACUSUM_C^{(1)}}}$ of the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart. Briefly, replace $\mathrm{h_{ACUSUM_C^{(1)}}}$ with h in Eq. (3) to obtain control limit of the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart. The $\mathrm{h_{ACUSUM_C^{(1)}}}$ depends on λ, k, γ, μ_0, σ_0^2, and n for pre-fixed value of $\mathrm{ARL_0}$ [28]. The {\ text{UCL}} control limit of the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart is denoted as $\mathrm{H_{ACUSUM_C^{(1)}}}$. If $\mathrm{ACUSUM}_{ci}^{(1)+} > \mathrm{H_{ACUSUM_C^{(1)}}}$ or $\mathrm{ACUSUM}_{ci}^{(1)-} > \mathrm{H_{ACUSUM_C^{(1)}}}$, the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart issues an alarm of out-of-control signal, otherwise in-control.

3.2 Proposed $\mathbf{ACUSUM_C^{(2)}}$ Control Chart

The plotting statistics of the proposed $\mathrm{ACUSUM_C^{(2)}}$ control chart are designed as below.

$$\mathrm{ACUSUM}_{Ci}^{(2)+} = \max\left[0, x_i - \left(\mu_0 + w_C^{(2)}(e_{1i})\right) + \mathrm{ACUSUM}_{C(i-1)}^{(2)+}\right], \quad (17)$$

$$\mathrm{ACUSUM}_{Ci}^{(2)-} = \max\left[0, \left(\mu_0 - w_C^{(2)}(e_{1i})\right) - x_i + \mathrm{ACUSUM}_{C(i-1)}^{(2)-}\right], \quad (18)$$

where $\mathrm{ACUSUM}_{C0}^{(2)\pm} = 0$ are the initial values. The $\mathrm{ACUSUM}_{Ci}^{(2)+}$ and $\mathrm{ACUSUM}_{Ci}^{(2)-}$ are one-sided upper and lower statistics of the proposed $\mathrm{ACUSUM_C^{(2)}}$ control chart, respectively. Like Sect. 3.1, replace control limit coefficient $\mathrm{h_{ACUSUM_C^{(2)}}}$ of the proposed $\mathrm{ACUSUM_C^{(2)}}$ with h in Eq. (3) to obtain control limit of proposed $\mathrm{ACUSUM_C^{(2)}}$ control chart. The $\mathrm{h_{ACUSUM_C^{(2)}}}$ depends on $\lambda, k, \gamma, \mu_0, \sigma_0^2$, and n for pre-fixed value of $\mathrm{ARL_0}$. The {\text{UCL}} of the proposed $\mathrm{ACUSUM_C^{(2)}}$ control chart, denoted as a $\mathrm{H_{ACUSUM_C^{(2)}}}$. If $\mathrm{ACUSUM}_{ci}^{(2)+} > \mathrm{H_{ACUSUM_C^{(2)}}}$ or $\mathrm{ACUSUM}_{ci}^{(2)-} > \mathrm{H_{ACUSUM_C^{(2)}}}$, the proposed $\mathrm{ACUSUM_C^{(2)}}$ control chart issues an alarm of out-of-control signal, otherwise in-control [28].

4 Performance Evaluation

This section contains methodologies of performances measures, explanation of parameters (k, λ, and γ) and their effect on proposed control harts performance, Monte Carlo simulation technique, special case of the proposed $\mathrm{ACUSUM_C^{(1)}}$ control chart, and how to design the proposed $\mathrm{ACUSUM_C^{(1)}}$ and $\mathrm{ACUSUM_C^{(2)}}$ control charts for users, practitioners, and engineers. More details are given in the following Sect. 4.1–4.5.

4.1 Performance Measures

Performance measures such as ARL which judge's performance of a control chart at a specific shift while EQL, RARL, and PCI measure overall effectiveness of control charts are considered. Their more details are given in subsequent Subsections 4.1.1–4.1.4.

4.1.1 Average Run Length

The most famous performance measure in statistical process control (SPC) is a ARL. It is categorized into two forms in-control ($\mathrm{ARL_0}$) and out-of-control (i.e., $\mathrm{ARL_1}$). The $\mathrm{ARL_0}$ represents an average of plotted points (sequence numbers) against control limit(s) under $H_0 : \mu = \mu_0$ (in-control) and $\mathrm{ARL_1}$ is an average of plotted points (sequence numbers) against control limit(s) under $H_1 : \mu = \mu_1$ (out-of-control). For a specific shift, smaller $\mathrm{ARL_1}$ value of a control chart against other control charts is preferred while $\mathrm{ARL_0}$ is at least same or greater [7, 29, 30].

4.1.2 Extra Quadratic Loss

The EQL measure provides overall performance of a control chart at a certain range of shifts [31]. Mathematically, it can be defined as follows:

$$EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta,$$

where δ represents shift in the process location, δ_{min} and δ_{max} are minimum and maximum values of shift, respectively, and $ARL(\delta)$ is an average run length of a control chart at a specific shift. The EQL measure uses integral technique over domain $\delta_{min} < \delta < \delta_{max}$ and square of a shift (i.e.,δ^2) as a weight to obtained overall performance value [32].

4.1.3 Relative Average Run Length

Like EQL, the RARL measure also presents overall effectiveness of a control chart. The RARL measures how close a particular control chart performs relative to the benchmark control chart. It can be determined by using below mathematical expression.

$$\mathrm{RARL} = \frac{1}{\delta_{max} - \delta_{min}} \int_{\delta_{min}}^{\delta_{max}} \frac{\mathrm{ARL}(\delta)}{\mathrm{ARL}_{\mathrm{bmk}}(\delta)} d\delta,$$

where $\mathrm{ARL}_{\mathrm{bmk}}(\delta)$ is ARL value of a benchmark control chart at a specific shift. A control chart with minimum ARL_1 value is considered as a benchmark control chart.

4.1.4 Performance Comparison Index

Ou et al. [33] designed PCI measure as a proportion of EQL of a control chart and EQL of a best control chart. Mathematically, it is defined below.

$$\mathrm{PCI} = \frac{\mathrm{EQL}}{\mathrm{EQL}_{\text{best chart}}},$$

where $\mathrm{EQL}_{\text{best chart}}$ presents the measure of best performing control chart. If $\mathrm{PCI} = 1$ for a control chart, this means it has the lowest EQL against other control charts.

4.2 Parameters (k, λ, *and* γ) and Their Effect

The optimal choices of parameters (k, λ, and γ) with coefficients $\mathrm{h}_{\mathrm{ACUSUM}_\mathrm{C}^{(1)}}$ and $\mathrm{h}_{\mathrm{ACUSUM}_\mathrm{C}^{(2)}}$ have significant effect on the performance of the proposed control charts. Many researchers observed specific effect of k and λ parameters on control charts performance. For example, k and λ of the classical CUSUM and EWMA control charts, respectively, at a specific value helps to detect some targeted shifts. In contrary, presence of more than one constant/parameter in a single control chart may leave a positive or negative effect on performance. Therefore, optimal combinations of parameters play significant role to a control chart performance. Find out the joint optimal combinations of parameters for sole control chart performance is a quite difficult task. However, by following the guidelines given by Capizzi and Masarotto [5], Abbas et al. [26], and Zaman et al. [34], the ranges of $1 \le \gamma \le 4$ [1, 1.5, 2, 2.5, 3, 3.5, 4], $0 < \lambda \le 1$ [0.05, 0.1, 0.2, 0.3, 0.4, 0.5], and $0.1 \le k \le 1.5$ [0.25, 0.5, 0.75, 1, 1.5] are considered for efficient performance of the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ and $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control charts. The main objective is balancing the sensitivity of the shifts in the process location. So, at a particular shift, if a parameter combination along $\mathrm{h}_{\mathrm{ACUSUM}_\mathrm{C}^{(1)}}$ /$\mathrm{h}_{\mathrm{ACUSUM}_\mathrm{C}^{(2)}}$ provides smaller ARL_1 against other parameters combinations that declared as optimal one while ARL_0 is at least same or greater.

4.3 Monte Carlo Simulation Technique

Algorithms are designed in MATLAB to obtain the desired ARLs of the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ and $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control charts. Monte Carlo simulations with 10^5 iterations are carried out for each displacement of shift [35]. Proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ and $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control chart numerical results are obtained using Zero and steady states. If a run length is initialized the targeted state known as zero state, while run length considered when control chart statistic has reached a steady state. The range of shift is set between 0 and 4. For the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ and $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control charts, only upward shift (i.e., $\delta > 0$) is considered because downward shift (i.e., $\delta < 0$) provides same behaviour as an upward shift by same absolute amount. The coefficients $\mathrm{L}_{\mathrm{ACUSUM}_\mathrm{C}^{(1)}}$ and $\mathrm{L}_{\mathrm{ACUSUM}_\mathrm{C}^{(2)}}$ values at different choices of k, λ, and γ when $\mathrm{ARL}_0 = 500$ are given in Table 1. Besides, only the ARLs of the $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ and $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control charts with their optimal parameters' combinations are presented in Tables 2 and 3, respectively, for comparison purposes. Similarly, overall performance measures to evaluate the performance of the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ against $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control charts are given in Table 4. Likewise, the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ and $\mathrm{ACUSUM}_\mathrm{C}^{(2)}$ control charts overall performance measures along other control charts are presented in Tables 5 and 6, respectively. Similarly, visually comparison is also presented in the form of Figures (see Figs. 1, 2, 3, 4, 5, 6, 7) for better understanding.

4.4 Special Case of Proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ Control Chart

The classical CUSUM control chart to monitor the process location is a special case of the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ control chart at special value of parameter. When $|e_{1i}| \le \gamma$, the proposed $\mathrm{ACUSUM}_\mathrm{C}^{(1)}$ tends to the classical CUSUM control charts to monitor the process location.

Proof When $|e_{1i}| \le \gamma$, then $\emptyset_1(e_{1i})$ becomes,

$$\emptyset_1(e_{1i}) = \lambda e_{1i}. \tag{19}$$

Now substitute the resulted $\emptyset_1(e_{1i})$ in $w_C^{(1)}(e_{1i}) = \emptyset_1(e_{1i}) / e_{1i}$, that is,

$$w_C^{(1)}(e_{1i}) = \lambda e_{1i}/e_{1i} = \lambda. \tag{20}$$

Based on Eq. (20), replace $w_C^{(1)}(e_{1i})$ of Eqs. (14), and (15) with λ. Thus, new forms of Eqs. (14) and (15) are given as:

$$\mathrm{ACUSUM}_{Ci}^{(1)+} = \max\left[0, \bar{x}_i - (\mu_0 + \lambda) + \mathrm{ACUSUM}_{C(i-1)}^{(1)+}\right], \tag{21}$$

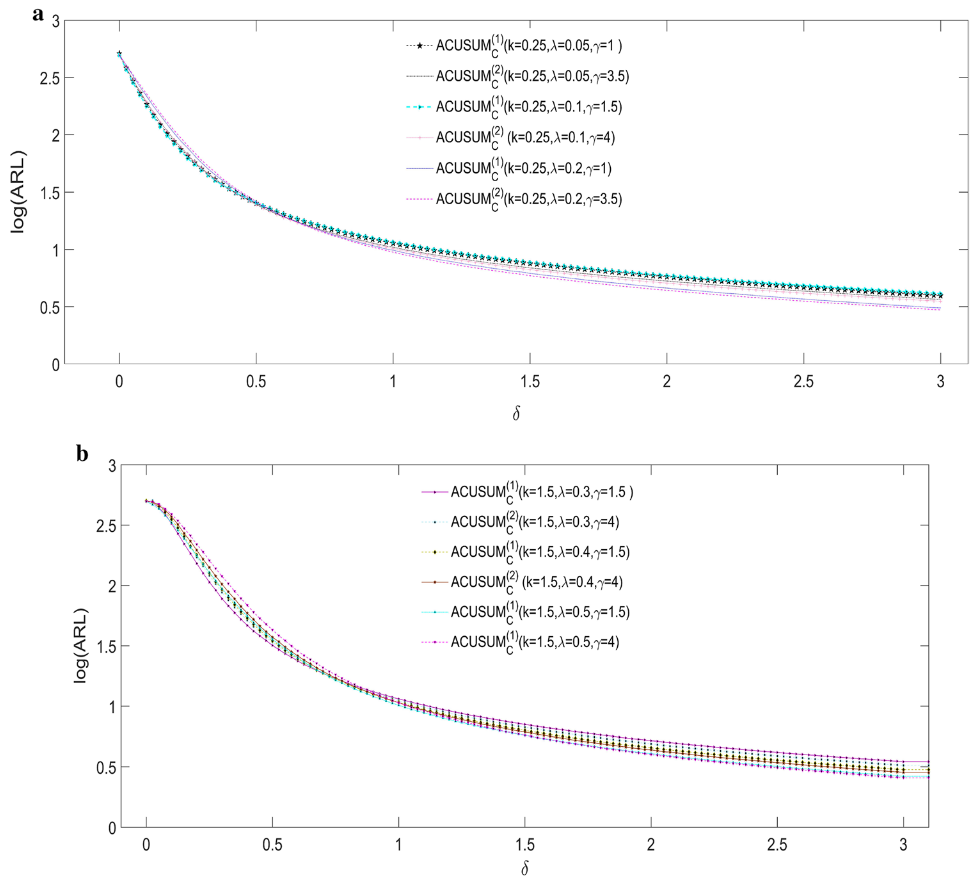


Fig. 1 **a** ARLs comparison between $\mathrm{ACUSUM}_C^{(1)}$ and $\mathrm{ACUSUM}_C^{(2)}$ control charts when $k = 0.25$, **b** ARLs comparison between $\mathrm{ACUSUM}_C^{(1)}$ and $\mathrm{ACUSUM}_C^{(2)}$ control charts when $k = 1.5$

$$\mathrm{ACUSUM}_{Ci}^{(1)-} = \max\left[0, \left(\mu_0 - \lambda\right) - \bar{x}_i + \mathrm{ACUSUM}_{C(i-1)}^{(1)-}\right]. \tag{22}$$

The $\mathrm{ACUSUM}_{Ci}^{(1)+}$ and $\mathrm{ACUSUM}_{Ci}^{(1)-}$ statistics in Eqs. (21)-(22) are identical to C_i^+ and C_i^- statistics in Eqs. (1)-(2), respectively, when $\lambda = K$. This shows that statistics of the proposed $\mathrm{ACUSUM}_C^{(1)}$ control chart becomes the statistics of the classical CUSUM control chart when $\left|e_{1i}\right| \le \gamma$ and $\lambda = K$. Because the statistics are identical, therefore their control limits formulation are also identical.

4.5 How to Construct Proposed $\mathrm{ACUSUM}_C^{(1)}$ and $\mathrm{ACUSUM}_C^{(1)}$ control charts

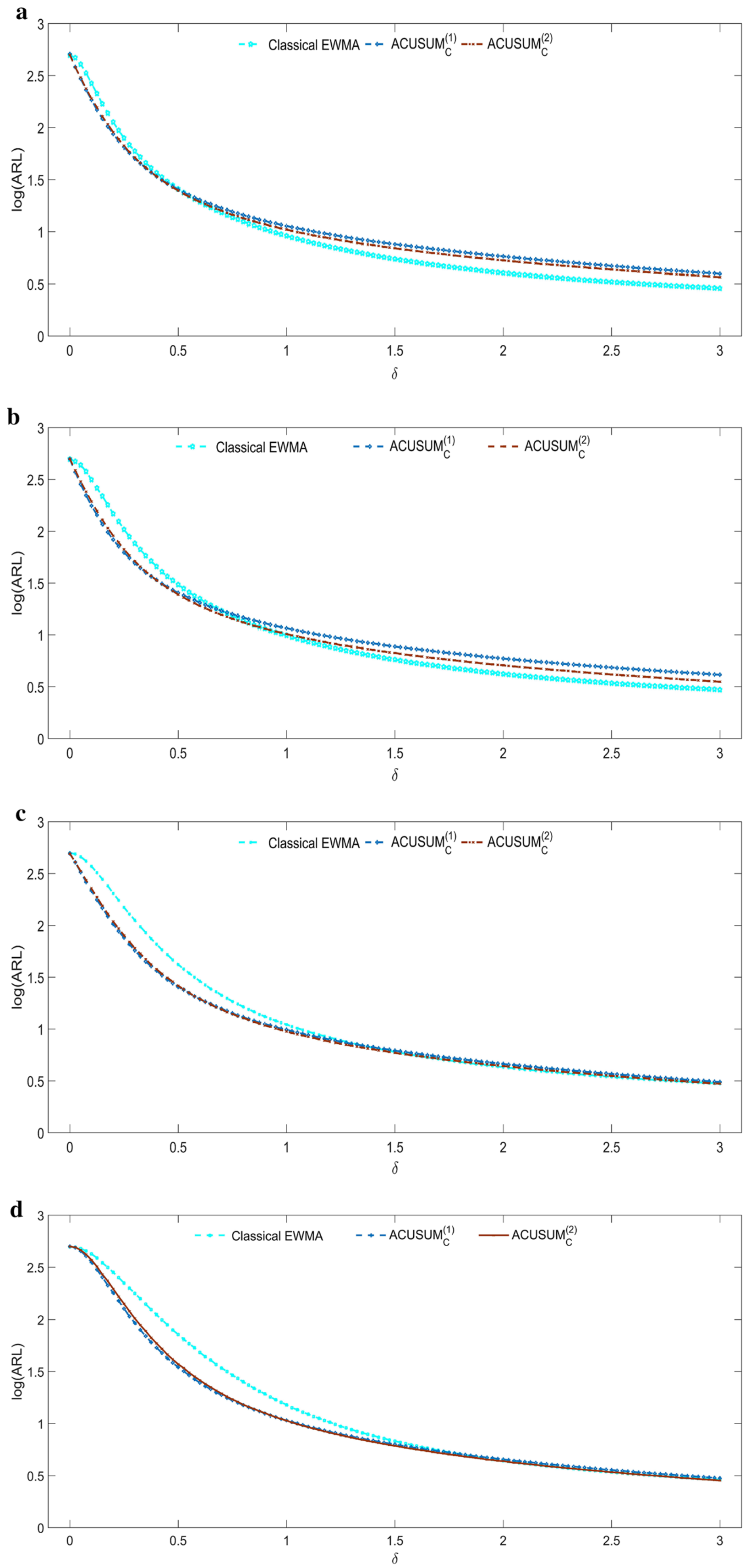
Guidelines how to construct the proposed $\mathrm{ACUSUM}_C^{(1)}$ and $\mathrm{ACUSUM}_C^{(1)}$ control charts play vital role to understand their methodologies which helps to implement them easily in real life as well. So, this section contains step-by-step general construct procedures of the proposed control charts for users, practitioners, and engineers. Sect. 4.5.1 provides step-by-step general construct procedure of the proposed $\mathrm{ACUSUM}_C^{(1)}$ control chart. Likewise, general construct procedure of the proposed $\mathrm{ACUSUM}_C^{(2)}$ control chart is presented in Sect. 4.5.2.

4.5.1 Construct Proposed $ACUSUM_C^{(1)}$ control chart

Let us assume desired ARL_0 is 500 and $\mathrm{h}_{\mathrm{ACUSUM}_C^{(1)}} = 3$. The general procedure to construct the proposed $\mathrm{ACUSUM}_C^{(1)}$ control chart is explained in following steps:

(i) Draw an observation from $x_i \sim N(\mu_0, \sigma_0^2)$ with $i = 1,2 \ldots$
(ii) Calculate C_i^+ and C_i^- statistics ($C_{i-1}^{\pm} = 0$) from Eqs. (1) and (2), respectively, based on x_i and $k = 0.5$.
(iii) Compute $e_{1i} = x_i - C_{i-1}^+$ or $e_{1i} = x_i - C_{i-1}^-$ statistic.
(iv) Select $\varnothing_1\left(e_{1i}\right)$ from Eq. (9) based on e_{1i} statistic and $\gamma = 1\ (\gamma \in (0, \infty))$ constant relation.
(v) Calculate $w_C^{(1)}\left(e_{1i}\right) = \varnothing_1\left(e_{1i}\right)/e_{1i}$ time-varying parameter.
(vi) Calculate the $\mathrm{ACUSUM}_{ci}^{(1)+}$ and $\mathrm{ACUSUM}_{ci}^{(1)-}$ statistics ($\mathrm{ACUSUM}_{C0}^{(1)\pm} = 0$) from Eqs. (14) and (15), respectively, based on x_i and $w_C^{(1)}\left(e_{1i}\right)$.
(vii) Calculate {\text{UCL}} control limit $\mathrm{H}_{\mathrm{ACUSUM}_C^{(1)}}$ based on $\mathrm{h}_{\mathrm{ACUSUM}_C^{(1)}}$ and σ_0.
(viii) Plot the $\mathrm{ACUSUM}_{ci}^{(1)\pm}$ statistics against $\mathrm{H}_{\mathrm{ACUSUM}_C^{(1)}}$. If the $\mathrm{ACUSUM}_{ci}^{(1)+} > \mathrm{H}_{\mathrm{ACUSUM}_C^{(1)}}$ or $\mathrm{ACUSUM}_{ci}^{(1)-} > \mathrm{H}_{\mathrm{ACUSUM}_C^{(1)}}$, note sample number of

Fig. 2 **a** ARLs comparison among $\text{ACUSUM}_C^{(1)}$, $\text{ACUSUM}_C^{(2)}$, and classical EWMA control charts when $\lambda = 0.05$. **b** ARLs comparison among $\text{ACUSUM}_C^{(1)}$, $\text{ACUSUM}_C^{(2)}$, and classical EWMA control charts when $\lambda = 0.1$. **c** ARLs comparison among $\text{ACUSUM}_C^{(1)}$, $\text{ACUSUM}_C^{(2)}$, and classical EWMA control charts when $\lambda = 0.2$. **d** ARLs comparison among $\text{ACUSUM}_C^{(1)}$, $\text{ACUSUM}_C^{(2)}$, and classical EWMA control charts when $\lambda = 0.4$



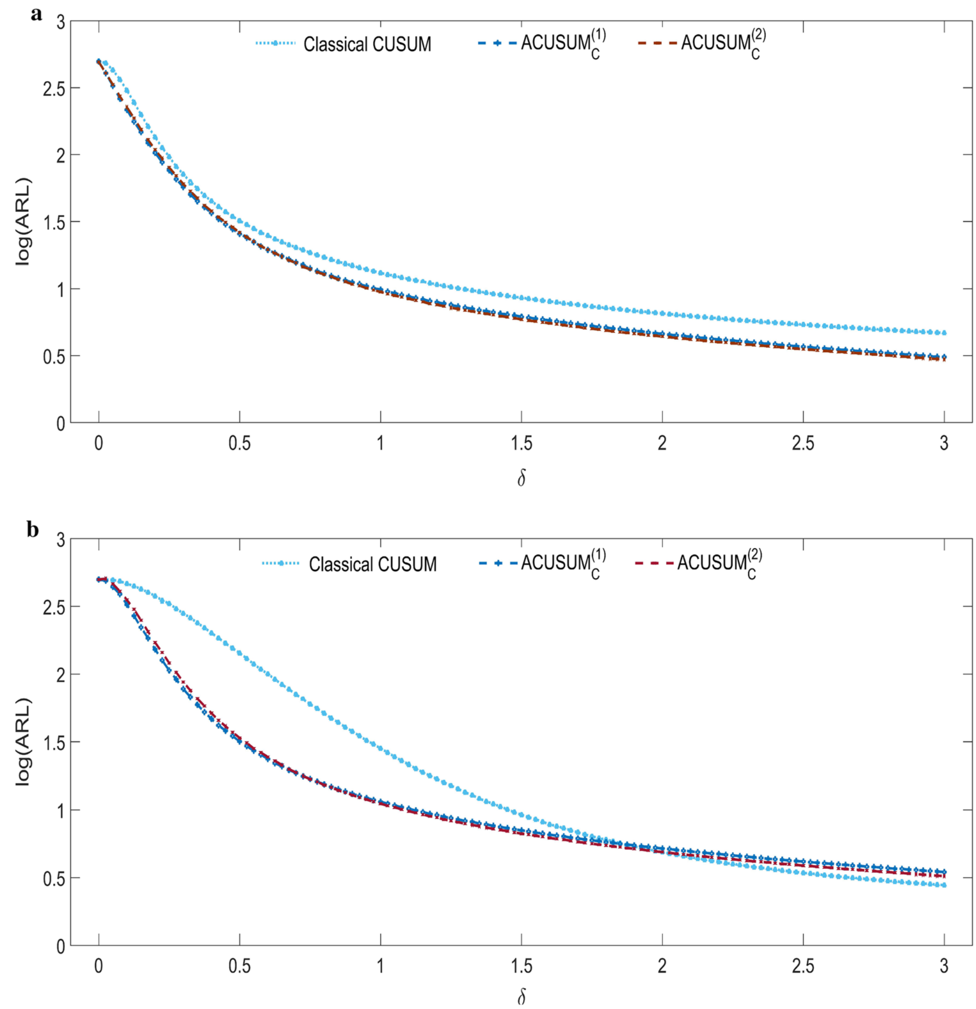


Fig. 3 **a:** ARLs comparison among $ACUSUM_C^{(1)}$, $ACUSUM_C^{(2)}$, and classical CUSUM control charts when $k = 0.25$. **b:** ARLs comparison among $ACUSUM_C^{(1)}$, $ACUSUM_C^{(2)}$, and classical CUSUM control charts when $k = 1.5$ 0

the $ACUSUM_{ci}^{(1)}$ or $ACUSUM_{ci}^{(1)-}$ statistic as a run length (RL).

(ix) Repeat from (i)-(viii) steps for 10^5 times and record RLs.

(x) Compute an average of 10^5 noted RLs that is called ARL_0.

(xi) If $ARL_0 = 500$ stop here, otherwise adjust $h_{ACUSUM_C^{(1)}}$ and repeat from (i)-(x) steps to obtain $ARL_0 = 500$.

(xii) To compute ARL_1 values, generate $x_i \sim N(\mu_1, \sigma_0^2)$ $(\delta = \mu_1 > \mu_0)$ and repeat from (ii)-(x) steps.

ARLs of the proposed $ACUSUM_C^{(1)}$ control chart for other values of k and γ can be calculated by following aforementioned steps (see Table 1).

4.5.2 Construct Proposed $ACUSUM_C^{(2)}$ control chart

If ARL_0 is 500 and $h_{ACUSUM_C^{(2)}} = 3$, ARLs of the proposed $ACUSUM_C^{(2)}$ control chart can be calculated by following given below steps:

(i) Draw an observation from $x_i \sim N(\mu_0, \sigma_0^2)$ with $i = 1,2\ldots$

(ii) Calculate C_i^+ and C_i^- statistics ($C_{i-1}^{\pm} = 0$) from Eqs. (1) and (2), respectively, based on x_i and $k = 0.5$.

(iii) Compute $e_{1i} = x_i - C_{i-1}^+$ or $e_{1i} = x_i - C_{i-1}^-$ statistic.

(iv) Select $\varnothing_2(e_{1i})$ from Eq. (11) based e_{1i} statistic and $\gamma = 1$ $(\gamma \in (0, \infty))$ constant relation.

(v) Calculate $w_C^{(2)}(e_{1i}) = \varnothing_2(e_{1i})/e_{1i}$ time-varying parameter.

(vi) Calculate the $ACUSUM_{ci}^{(2)+}$ and $ACUSUM_{ci}^{(2)-}$ statistics ($ACUSUM_{C0}^{(2)\pm} = 0$) from Eqs. (17) and (18), respectively, based on x_i and $w_C^{(2)}(e_{1i})$.

(vii) Calculate {\text{UCL}} control limit $H_{ACUSUM_C^{(2)}}$ based on $h_{ACUSUM_C^{(2)}}$ and σ_0.

(viii) Plot the $ACUSUM_{ci}^{(2)\pm}$ statistics against $H_{ACUSUM_C^{(2)}}$. If the $ACUSUM_{ci}^{(2)+} > H_{ACUSUM_C^{(2)}}$ or $ACUSUM_{ci}^{(2)-} > H_{ACUSUM_C^{(2)}}$, note sample number of the $ACUSUM_{ci}^{(2)}$ or $ACUSUM_{ci}^{(2)-}$ statistic as a run length (RL).

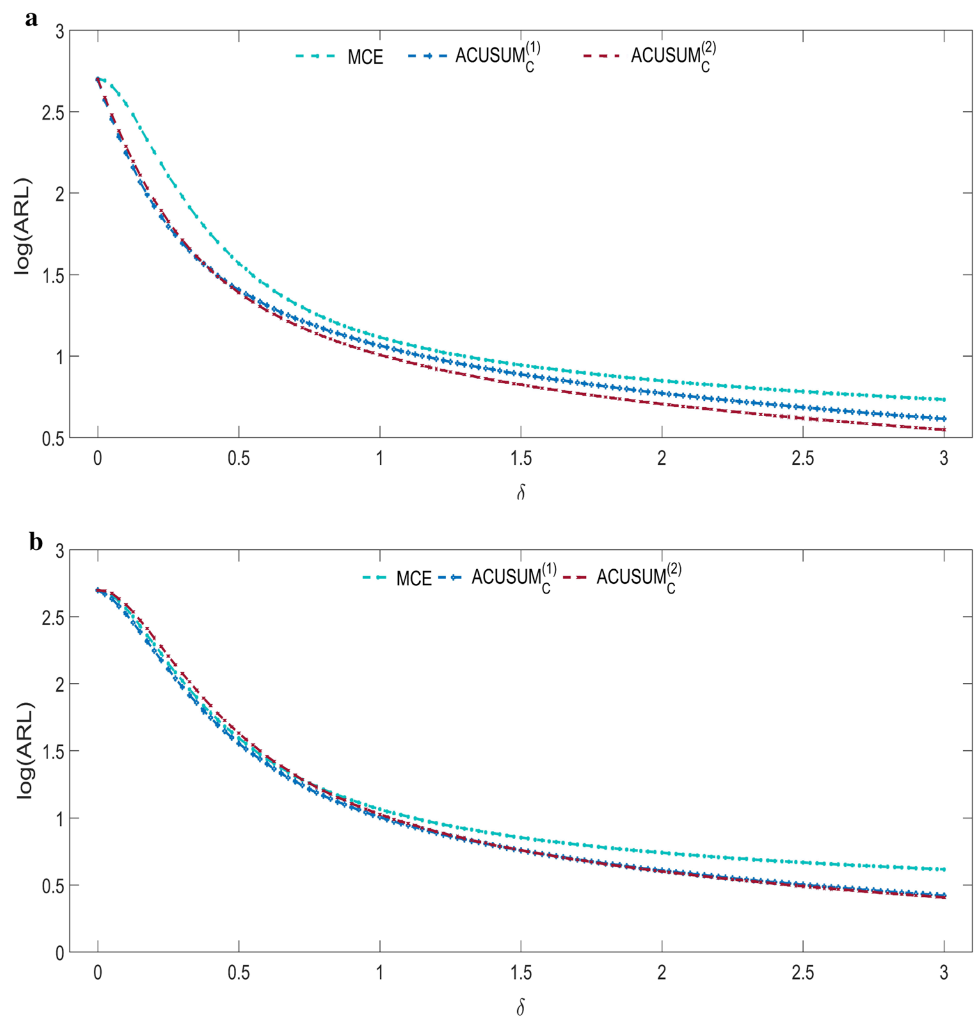


Fig. 4 **a:** ARLs comparison among $\text{ACUSUM}_C^{(1)}$, $\text{ACUSUM}_C^{(2)}$, and MCE control charts when $\lambda = 0.1$. **b:** ARLs comparison among $\text{ACUSUM}_C^{(1)}$, $\text{ACUSUM}_C^{(2)}$, and MCE control charts when $\lambda = 0.5$

(ix) Repeat from (i–viii) steps for 10^5 times and record RLs.
(x) Compute an average of 10^5 noted RLs that is called ARL_0.
(xi) If $\text{ARL}_0 = 500$, otherwise adjust $h_{\text{ACUSUM}_C^{(2)}}$ and repeat from (i–x) steps to obtain $\text{ARL}_0 = 500$.
(xii) To compute ARL_1 values of proposed $\text{ACUSUM}_C^{(2)}$ control chart, generate $x_i \sim N(\mu_1, \sigma_0^2)\,(\delta = \mu_1 > \mu_0)$ and repeat from (ii–x) steps.

The ARLs of proposed $\text{ACUSUM}_C^{(2)}$ control chart for other values of k and γ can be calculated by following aforementioned steps (see Table 1).

5 Comparative Analysis of Control Charts

This section contains analysis among control charts. For instance, comparison between proposed $\text{ACUSUM}_C^{(1)}$ and $\text{ACUSUM}_C^{(2)}$ control charts is given in Sect. 5.1. Besides, comparative analysis of the proposed $\text{ACUSUM}_C^{(1)}$ and $\text{ACUSUM}_C^{(2)}$ control charts against counterparts is provided in Sect. 5.2.

5.1 Comparison Between $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ Control Charts

Following some imperative points are noted while comparing proposed control charts.

(i) The $\text{ACUSUM}_C^{(1)}$ control chart performs better as compared to $\text{ACUSUM}_C^{(2)}$ control chart for small shifts with small values of k and γ (see Tables 2–3). For example, at $\lambda = 0.05$, the $\text{ACUSUM}_C^{(1)}$ control chart outperform against $\text{ACUSUM}_C^{(2)}$ control chart for small value of $k = 0.25$ and $\gamma = 1.00$ when $\delta \leq 0.50$ (see Fig. 1a).
(ii) Similarly, at $\lambda = 0.10$, the $\text{ACUSUM}_C^{(1)}$ control chart retains superiority against $\text{ACUSUM}_C^{(2)}$ control chart for small shifts $\delta \leq 0.50$. For example, at $k = 0.25$ and $\gamma = 1.5\ 0$, the $\text{ACUSUM}_C^{(1)}$ control chart has smaller

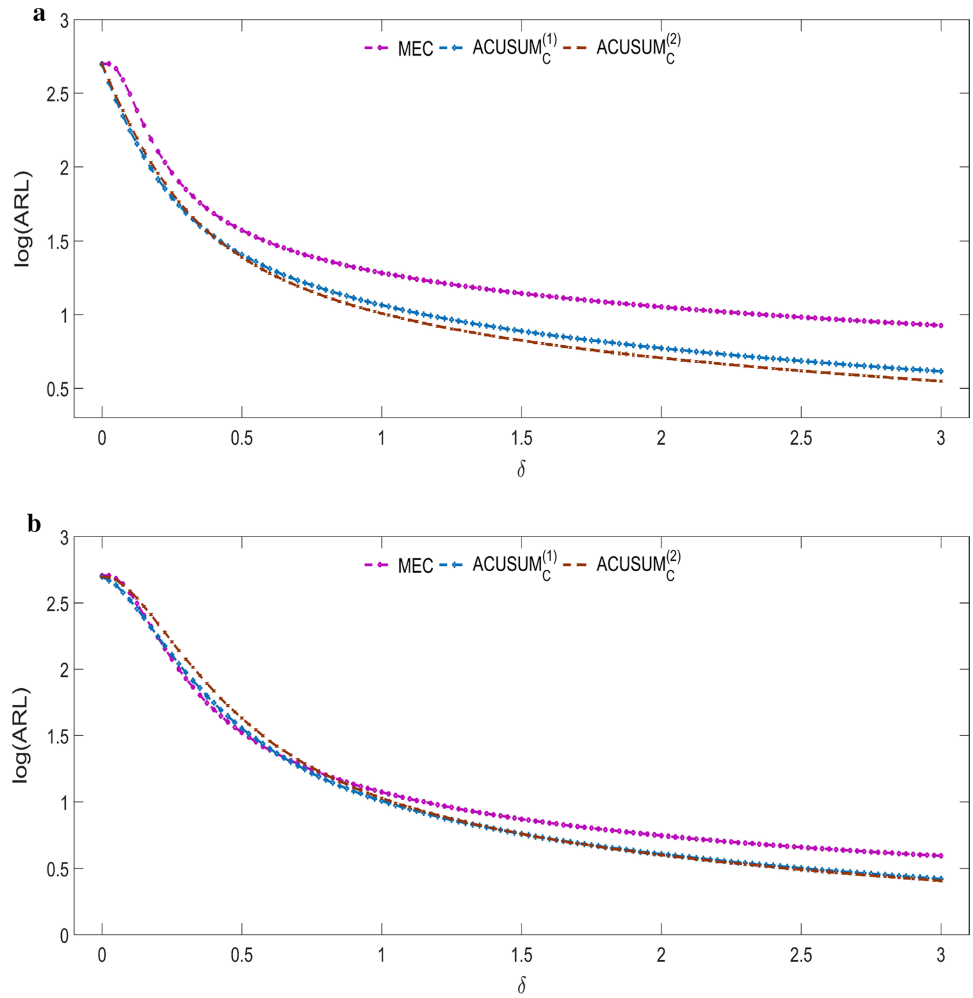


Fig. 5 **a:** ARLs comparison among $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$, $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$, and MEC control charts when $\lambda = 0.1$. **b:** ARLs comparison among $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$, $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$, and MEC control charts when $\lambda = 0.5$

ARL_1 against $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control chart (see Tables 2–3 and Fig. 1a).

(iii) Likewise, at $\lambda > 0.10$, the $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart keeps dominance over $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control chart for small shifts $\delta \le 0.75$. For example, at $k = 1.50$, $\lambda = 0.30$ and $\gamma = 1.5\,0$, the $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart has smaller ARL_1 against $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control chart (see Tables 2–3 and Fig. 1b).

(iv) By following $k = 1$, $\gamma = 1$, and $\lambda = 0.1$, $k = 0.50$, $\gamma = 4$, and $\lambda = 0.50$, and $k = 0.25$, $\gamma = 4$, and $\lambda = 0.50$ parameters combinations, the $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart has smaller EQL, PCI, and RARL values against $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart. Other than these parameters combinations, the $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control chart outperform (see Table 4).

(v) Concisely, the $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart performs better against $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control chart at different values of parameters.

5.2 Proposed Versus Other Control Charts

This subsection contains comparative analysis of the proposed $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ and $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control charts against counterparts such as classical EWMA [3] and CUSUM [2], MCE [21], MEC [5, 8, 26]$\mathrm{AEWMA}_{\mathrm{C}}\mathrm{AEWMA}_{\mathrm{E}}$, and Hybrid EWMA [27] control charts. More details on comparative analysis are provided in Sects. 5.2.1–5.2.7.

5.2.1 Proposed Versus Classical EWMA Control Chart

Proposed $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart recognizes out-of-control signals in advance against the classical EWMA control for all shifts at certain values of λ. For example, at small shifts (i.e.,$\delta \in (0,\ 0.75)$) and small values of $\lambda \in (0.05{,}0.1)$ (see Figs. 2a-2b), the proposed $\mathrm{ACUSUM}_{\mathrm{C}}^{(1)}$ control chart performs efficient against the classical EWMA control chart, but it becomes more effective for all shifts for large value of $\lambda \in (0.2{,}0.5)$ (see Tables 2 and 7 and Fig. 2c–d). Similarly, the proposed $\mathrm{ACUSUM}_{\mathrm{C}}^{(2)}$ control chart also shows superior

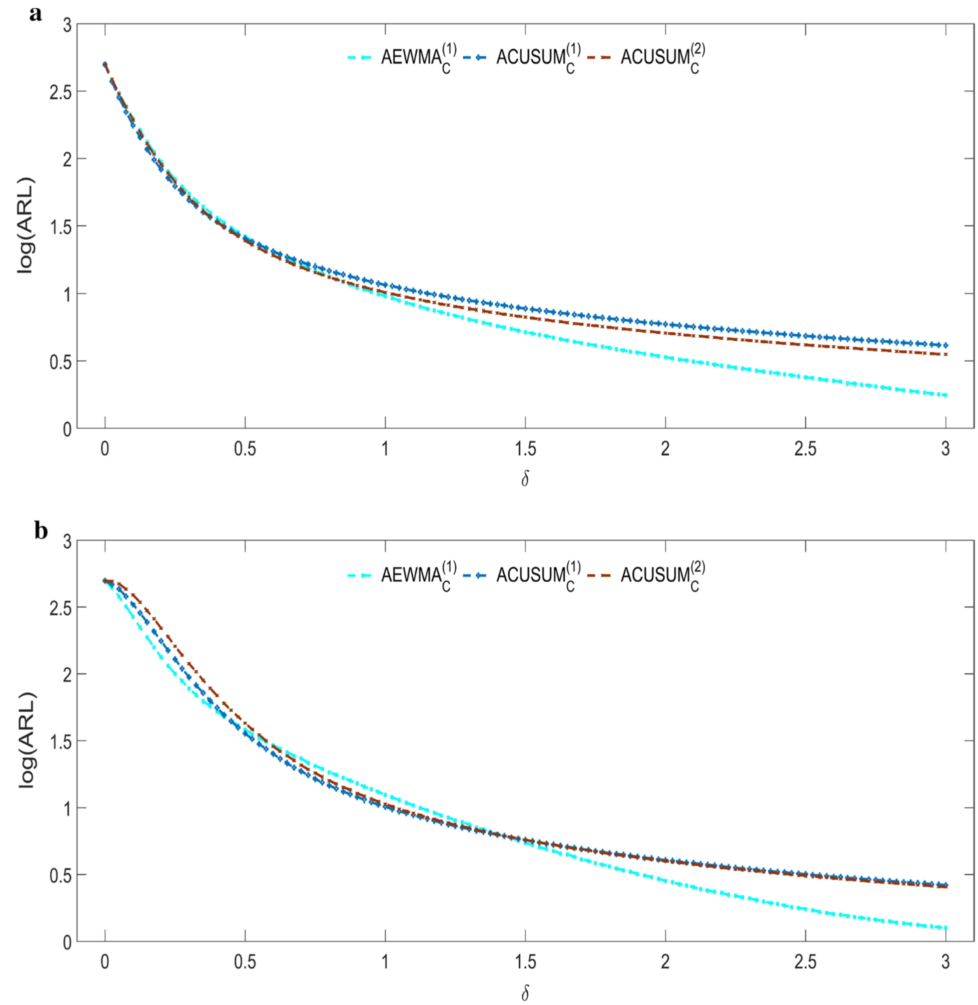


Fig. 6 **a:** ARLs comparison among ACUSUM$_C^{(1)}$, ACUSUM$_C^{(2)}$, and AEWMA$_C^{(1)}$ control charts when $\lambda = 0.1$. Figure 6b: ARLs comparison among ACUSUM$_C^{(1)}$, ACUSUM$_C^{(2)}$, and AEWMA$_C^{(2)}$ control charts when $\lambda = 0.5$

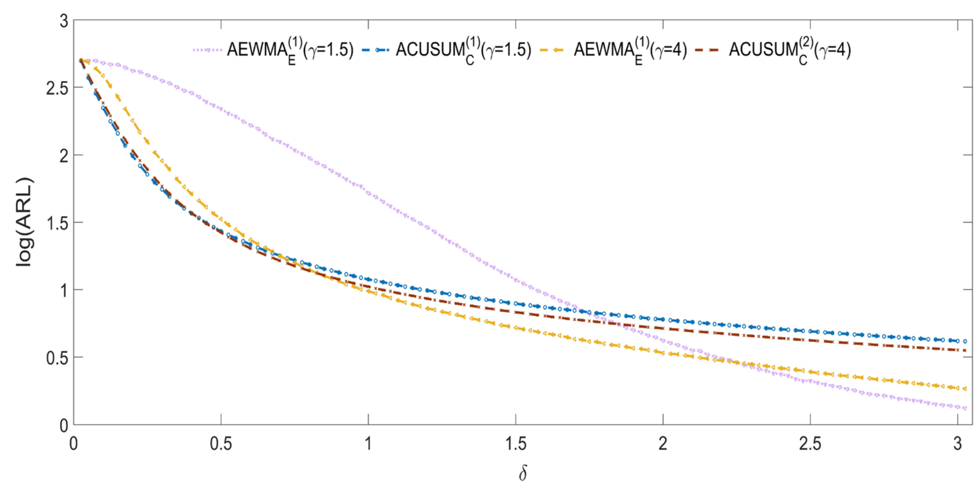


Fig. 7 ARLs comparison among ACUSUM$_C^{(1)}$ and ACUSUM$_C^{(2)}$, and AEWMA$_E^{(1)}$ control charts when $\lambda = 0.1$

performance against the classical EWMA control chart (see Tables 3 and 7 and Fig. 2a–d). Besides, with respect to overall performance, at $\lambda = 0.05$, the EQL values of the classical EWMA and proposed ACUSUM$_C^{(1)}$ control charts are 17.61 and 18.99, respectively. This comparison shows the proposed ACUSUM$_C^{(1)}$ control chart has inferior performance as compared to the classical EWMA control chart (see Table 5). Similarly, the classical EWMA control chart persists superiority against the proposed ACUSUM$_C^{(2)}$ control

Table 1 Coefficients $L_{ACUSUM_C^{(1)}}$ and $L_{ACUSUM_C^{(2)}}$ values at different choices of k, λ, and γ when $ARL_0 = 500$

k	γ=1	1.5	2	2.5	3	3.5	4	k	γ=1	1.5	2	2.5	3	3.5	4
$L_{ACUSUM_C^{(1)}}$															
	λ=0.05							λ=0.3							
0.25	8.00	10.55	12.20	13.35	14.35	15.25	16.15	0.25	4.95	5.70	6.15	6.43	6.63	6.78	6.92
0.50	9.40	13.03	15.71	18.90	18.38	18.96	19.26	0.50	5.38	6.24	6.77	7.07	7.28	7.40	7.48
0.75	10.87	15.23	17.76	18.90	19.35	19.58	19.76	0.75	5.76	6.72	7.21	7.41	7.49	7.52	7.55
1.00	11.57	16.17	18.42	19.34	19.59	19.77	19.81	1.00	7.40	6.91	7.31	7.48	7.52	7.54	7.56
1.50	12.02	16.62	18.92	19.62	19.65	19.78	19.83	1.50	5.98	6.98	7.38	7.48	7.55	7.55	7.58
	λ=0.1							λ=0.4							
0.25	7.10	8.92	10.1	10.9	11.42	11.9	12.44	0.25	4.37	4.89	5.19	5.42	5.56	5.66	5.75
0.50	8.12	10.62	12.35	13.3	13.9	14.26	14.46	0.50	4.64	5.24	5.59	5.80	5.90	5.98	6.02
0.75	9.21	12.11	13.65	14.25	14.6	14.7	14.73	0.75	4.89	5.52	5.85	5.99	6.04	6.07	6.08
1.00	9.74	12.65	14.065	14.55	14.7	14.8	14.75	1.00	4.99	5.63	5.93	6.03	6.07	6.08	6.09
1.50	9.99	12.95	14.25	14.68	14.72	14.8	14.75	1.50	4.98	5.65	5.95	6.03	6.07	6.08	6.09
	λ=0.2							λ=0.5							
0.25	5.80	6.92	7.58	7.98	8.28	8.52	8.73	0.25	3.905	4.28	4.52	4.65	4.76	4.84	4.89
0.50	6.46	7.81	8.66	9.15	9.49	9.69	9.79	0.50	4.10	4.51	4.75	4.9	4.98	5.03	5.05
0.75	7.10	8.63	9.39	9.71	9.88	9.93	9.96	0.75	4.25	4.68	4.9	5.01	5.04	5.06	5.07
1.00	7.40	8.95	9.60	9.87	9.93	9.95	9.97	1.00	4.30	4.74	4.94	5.04	5.06	5.08	5.07
1.50	7.50	9.05	9.71	9.92	9.95	9.98	9.98	1.50	4.28	4.76	4.97	5.04	5.06	5.08	5.07
$L_{ACUSUM_C^{(2)}}$															
	λ=0.05							λ=0.3							
0.25	3.265	3.78	4.45	5.24	6.01	6.75	7.42	0.25	2.99	3.23	3.55	3.89	4.22	4.52	4.78
0.50	3.38	4.02	4.91	5.88	6.86	7.8	8.7	0.50	3.04	3.35	3.74	4.16	4.56	4.91	6.15
0.75	3.45	4.24	5.3	6.55	7.78	8.98	10	0.75	3.09	3.43	3.9	4.41	4.87	5.27	6.81
1.00	3.46	4.31	5.51	6.91	8.27	9.57	10.77	1.00	3.09	3.45	3.95	4.5	5	5.43	5.8
1.50	3.33	4.23	5.53	7.03	8.54	9.89	11.1	1.50	3.01	3.35	3.86	4.46	5	5.45	5.85
	λ=0.1							λ=0.4							
0.25	3.2	3.65	4.22	4.86	5.52	6.75	6.66	0.25	2.92	3.1	3.32	3.58	3.83	4.03	4.23
0.50	3.28	3.85	4.59	5.41	6.21	6.93	7.64	0.50	2.96	3.17	3.46	3.76	4.06	4.32	4.53
0.75	3.35	4.02	4.93	5.95	6.93	7.86	8.66	0.75	2.99	3.23	3.57	4.41	4.27	4.56	4.79
1.00	3.46	4.07	5.1	6.23	7.32	8.3	9.17	1.00	3.00	3.26	3.6	3.98	4.34	4.65	4.89
1.50	3.24	3.98	5.07	6.3	7.5	8.56	9.38	1.50	3.01	3.15	3.51	3.92	4.3	4.63	4.9
	λ=0.2							λ=0.5							
0.25	3.09	3.41	3.84	4.32	4.75	5.16	5.55	0.25	2.85	2.98	3.15	3.35	3.52	3.68	3.81
0.50	3.15	3.57	4.1	4.68	5.24	5.74	6.15	0.50	2.89	3.04	3.24	3.46	3.68	3.86	4.03
0.75	3.21	3.68	4.33	5.03	5.7	7.86	6.81	0.75	2.91	3.08	3.32	3.57	3.82	4.02	4.18
1.00	3.22	3.72	4.42	5.21	5.94	6.58	7.1	1.00	2.91	3.09	3.34	3.6	3.86	4.08	4.26
1.50	3.11	3.61	4.35	5.2	6	6.67	7.22	1.50	2.86	3.02	3.26	3.54	3.82	4.05	4.23

chart, too (see Table 6). In contrary as $\lambda > 0.05$, for example at $\lambda = 0.10$, the classical EWMA control chart shows inferior performance as compared to the proposed $ACUSUM_C^{(2)}$ control chart. For example, EQL, PCI, and RARL values of the classical EWMA control chart are 21.47, 2.79, and 2.71 while $ACUSUM_C^{(2)}$ control chart keeps smaller (see Table 6). Likewise, at $\lambda = 0.5$, the $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts also show superiority against the classical EWMA control chart (see Tables 5 and 6). Shortly, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts performance increases as $\lambda > 0.05$ increases against the classical EWMA control chart.

5.2.2 Proposed Versus Classical CUSUM Control Chart

The $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have efficient diagnostic ability against the classical CUSUM control chart. For example, at $k = 0.25$, Fig. 3a shows that proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have smaller ARL_1 values for all shifts against the classical CUSUM

Table 2 ARLs properties of proposed $ACUSUM_c^{(1)}$ control chart at different choices of γ, k, and λ when $ARL_0 = 500$

State	γ, $h_{ACUSUM_C^{(1)}}$, k	δ												γ, $h_{ACUSUM_C^{(1)}}$, k	δ											
		0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00		0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00
	$\lambda = 0.05$													$\lambda = 0.2$												
Zero	1,8, 0.25	64.8	25.4	15.4	11.3	9.02	7.59	6.60	5.79	5.19	4.71	3.95	3.01	1,5.8,0.25	75.7	25.6	14.2	9.82	7.57	6.20	5.28	4.60	4.10	3.68	3.09	2.33
Steady	1,8, 0.25	55.4	20.7	12.0	8.30	6.39	5.03	4.10	3.54	2.96	2.56	1.94	1.18	1,5.8,0.25	65.5	20.9	10.6	6.73	4.65	3.56	2.72	2.12	1.74	1.37	1.09	1.00
Zero	1.5,10.55, 0.25	63.0	27.2	17.4	12.9	10.4	8.75	7.60	6.75	6.09	5.55	4.74	3.65	1.5,6.92,0.25	68.2	24.9	14.4	10.2	7.94	6.55	5.64	4.96	4.42	4.01	3.39	2.61
Steady	1.5,10.55, 0.25	52.8	22.0	13.5	9.70	7.71	6.30	5.30	4.64	4.09	3.64	2.98	2.08	1.5,6.92,0.25	58.7	20.1	10.8	7.1	5.21	3.97	3.10	2.55	2.08	1.78	1.22	1.02
Zero	1,9.4, 0.5	71.3	28.5	17.8	13.2	10.5	8.79	7.59	6.72	6.00	5.41	4.55	3.43	1,6.46,0.5	83.6	27.8	15.6	10.7	8.31	6.81	5.82	5.06	4.49	4.04	3.38	2.54
Steady	1,9.4, 0.5	59.6	22.8	13.6	9.70	7.55	6.16	5.23	4.53	3.94	3.48	2.80	1.88	1,6.46,0.5	71.8	22.3	11.5	7.53	5.38	4.20	3.36	2.75	2.24	1.92	1.28	1.10
Zero	1.5,13.03, 0.5	72.9	32.5	21.0	15.8	12.7	10.6	9.23	8.19	7.37	6.72	5.71	4.38	1.5,7.81,0.5	77.5	27.6	16.0	11.3	8.87	7.31	6.25	5.49	4.91	4.45	3.76	2.90
Steady	1.5,13.03, 0.5	60.5	25.9	16.6	12.2	9.70	8.13	6.98	6.19	5.54	5.02	4.23	3.15	1.5,7.81,0.5	65.4	21.8	12.0	8.02	5.92	4.73	3.82	3.20	2.76	2.36	1.78	1.08
Zero	1,10.87, 0.75	98.5	38.7	26.3	19.7	14.9	11.7	9.53	8.01	6.92	6.11	4.97	2.50	1,7.1,0.75	117	34.9	18.8	12.7	9.57	7.61	6.28	5.38	4.68	4.18	3.45	2.61
Steady	1,10.87, 0.75	83.2	30.8	19.6	14.3	10.7	8.40	6.83	5.71	4.90	4.28	3.40	2.45	1,7.1,0.75	102	28.2	14.2	9.12	6.41	4.85	3.82	3.03	2.57	2.11	1.58	1.10
Zero	1.5,15.23, 0.75	92.1	42.9	33.1	26.3	20.3	16.0	13.0	10.9	9.37	8.26	6.68	3.12	1.5,8.63,0.75	99.4	33.1	19.3	13.6	10.5	8.44	7.08	6.05	5.31	4.74	3.92	3.01
Steady	1.5,15.23, 0.75	76.6	34.5	24.5	19.1	15.0	11.9	9.70	8.30	7.19	6.31	5.11	3.76	1.5,8.63,0.75	85.2	26.6	14.6	9.83	7.31	5.67	4.55	3.82	3.22	2.77	2.12	1.41
Zero	1,11.57, 1	88.5	35.4	22.2	16.6	13.4	11.1	9.45	8.15	7.14	6.35	5.21	3.21	1,7.4,1	102	32.2	17.8	12.2	9.30	7.52	6.32	5.45	4.79	4.28	3.55	2.71
Steady	1,11.57,1	74.2	28.3	17.3	12.6	9.93	8.11	6.89	5.89	5.13	4.56	3.63	2.67	1,7.4,1	88.8	26.0	13.5	8.66	6.27	4.79	3.91	3.16	2.65	2.29	1.77	1.04
Zero	1.5,16.17, 1	88.0	40.5	26.9	20.7	17.3	14.8	12.7	11.0	9.64	8.56	6.99	3.33	1.5,8.95,1	94.0	31.9	18.4	13.0	10.1	8.29	7.03	6.11	5.38	4.83	4.01	3.09
Steady	1.5,16.17, 1	74.7	32.8	21.3	16.1	13.3	11.3	9.66	8.37	7.36	6.59	5.41	4.00	1.5,8.95,1	79.6	25.4	14.1	9.50	7.25	5.70	4.70	3.93	3.34	2.91	2.30	1.56
Zero	1,12.02, 1.5	85.5	35.1	22.0	16.1	12.8	10.6	9.00	7.88	6.99	6.31	5.26	4.50	1,7.5,1.5	98.1	31.5	17.5	12.0	9.24	7.50	6.30	5.45	4.80	4.31	3.60	2.75
Steady	1,12.02, 1.5	72.4	28.0	17.3	12.5	9.75	7.98	6.80	5.86	5.15	4.58	3.78	2.82	1,7.5,1.5	85.4	25.4	13.2	8.69	6.29	4.94	3.97	3.22	2.77	2.31	1.80	1.09
Zero	1.5,16.62, 1.5	87.5	40.1	26.3	19.6	15.7	13.2	11.4	10.1	9.07	8.23	6.91	4.60	1.5,9.05,1.5	91.0	31.4	18.2	12.9	10.0	8.17	6.94	6.04	5.35	4.81	4.04	3.11
Steady	1.5,16.62, 1.5	73.9	32.8	21.0	15.6	12.4	10.4	8.95	7.87	7.04	6.40	5.38	4.05	1.5,9.05,1.5	77.4	24.9	13.9	9.54	7.10	5.58	4.61	3.82	3.35	2.93	2.28	1.59
	$\lambda = 0.1$													$\lambda = 0.3$												
Zero	1,7.1, 0.25	68.3	25.1	14.9	10.7	8.43	7.01	6.03	5.30	4.76	4.28	3.60	2.73	1,4.95,0.25	90.9	28.1	14.4	9.50	7.10	5.74	4.85	4.17	3.69	3.31	2.75	2.11
Steady	1,7.1, 0.25	58.0	20.4	11.4	7.57	5.66	4.41	3.57	2.92	2.44	2.08	1.47	1.23	1,4.95,0.25	78.6	23.1	10.7	6.37	4.22	2.95	2.12	1.58	1.19	1.04	1.00	1.00
Zero	1.5,8.92, 0.25	62.7	25.5	15.8	11.6	9.23	7.74	6.70	5.93	5.33	4.86	4.13	3.19	1.5,5.7,0.25	80.5	26.4	14.2	9.63	7.29	5.91	5.01	4.36	3.88	3.51	2.96	2.25
Steady	1.5,8.92, 0.25	53.5	20.4	12.0	8.51	6.46	5.27	4.34	3.65	3.21	2.79	2.13	1.37	1.5,5.7,0.25	70.7	21.3	10.4	6.44	4.40	3.11	2.42	1.87	1.48	1.09	1.01	1.00
Zero	1,8.12, 0.5	72.4	27.8	16.7	12.0	9.48	7.91	6.80	5.97	5.33	4.82	4.03	3.05	1,5.38,0.5	99.9	30.0	15.4	10.2	7.64	6.17	5.21	4.49	3.96	3.56	2.95	2.22
Steady	1,8.12, 0.5	61.8	22.0	12.6	8.8	6.61	5.23	4.42	3.80	3.20	2.75	2.13	1.37	1,5.38,0.5	86.4	24.3	11.4	6.9	4.74	3.40	2.60	2.05	1.60	1.24	1.10	1.00
Zero	1.5,10.62, 0.5	71.2	29.2	18.5	13.6	10.8	9.09	7.86	6.95	6.23	5.67	4.81	3.69	1.5,6.24,0.5	90.1	28.7	15.3	10.4	7.88	6.40	5.43	4.73	4.21	3.78	3.19	2.41
Steady	1.5,10.62, 0.5	59.4	23.3	14.2	10.2	8.05	6.59	5.61	4.83	4.32	3.90	3.24	2.25	1.5,6.24,0.5	77.6	23.0	11.3	7.07	5.00	3.65	2.89	2.32	1.85	1.59	1.10	1
Zero	1,9.21, 0.75	103	36.1	22.1	16.0	12.2	9.69	7.95	6.77	5.87	5.20	4.26	3.19	1,5.76,0.75	130	35.8	17.6	11.3	8.32	6.51	5.38	4.59	4.01	3.57	2.96	2.24
Steady	1,9.21, 0.75	87.4	28.7	16.6	11.7	8.64	6.72	5.40	4.50	3.88	3.33	2.61	1.75	1,5.760.75	115	29.4	13.1	7.77	5.19	3.70	2.80	2.09	1.67	1.30	1.10	1.03
Zero	1.5,12.11, 0.75	90.9	37.3	24.8	19.0	15.0	12.0	9.95	8.44	7.36	6.51	5.32	3.96	1.5,6.720.75	115	33.5	17.4	11.7	8.72	6.95	5.79	4.96	4.35	3.89	3.24	2.46
Steady	1.5,12.11, 0.75	75.8	29.9	18.8	14.0	10.9	8.74	7.22	6.13	5.31	4.69	3.81	2.76	1.5,6.720.75	99.9	27.0	13.0	8.01	5.61	4.14	3.24	2.62	2.14	1.81	1.22	1.09
Zero	1,9.74, 1	90.5	33.2	20.1	14.5	11.4	9.40	7.96	6.88	6.06	5.41	4.45	3.33	1,7.4,1	118	33.4	16.8	11.0	8.18	6.50	5.43	4.65	4.09	3.64	3.03	2.30
Steady	1,9.74, 1	77.2	26.5	15.4	10.7	8.29	6.67	5.54	4.69	4.08	3.57	2.87	2.00	1,7.4,1	75.9	25.4	14.2	9.81	7.32	5.83	4.82	4.11	3.58	3.13	2.42	1.76

Table 2 (continued)

State	γ, $h_{ACUSUM_C^{(1)}}$, k	δ 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00	γ, $h_{ACUSUM_C^{(1)}}$, k	δ 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00
Zero	1.5,12.64, 1	84.7	35.3	22.4	16.7	13.5	11.3	9.68	8.46	7.45	6.65	5.48	4.09	1.5,6.91,1	109	32.4	17.0	11.4	8.61	6.92	5.81	5.00	4.43	3.96	3.32	2.52
Steady	1.5,12.64, 1	71.4	28.4	17.6	12.8	10.1	8.42	7.12	6.15	5.38	4.81	3.95	2.93	1.5,6.91,1	95.0	26.0	12.8	7.95	5.58	4.32	3.33	2.74	2.23	1.84	1.36	1.10
Zero	1,9.99, 1.5	86.7	32.7	19.8	14.2	11.1	9.14	7.74	6.75	5.98	5.37	4.49	3.39	1,5.98,1.5	113	33.0	16.6	11.0	8.21	6.57	5.46	4.69	4.11	3.68	3.07	2.32
Steady	1,9.99, 1.5	73.3	26.1	15.1	10.7	8.21	6.61	5.46	4.71	4.09	3.65	2.91	2.06	1,5.98,1.5	99.6	26.8	12.3	7.63	5.15	3.82	2.88	2.44	1.90	1.46	1.03	1.01
Zero	1.5,12.95, 1.5	83.6	35.0	22.2	16.3	12.9	10.7	9.17	8.04	7.18	6.49	5.45	4.14	1.5,6.98,1.5	107	31.9	16.9	11.4	8.65	6.97	5.83	5.03	4.45	4.01	3.35	2.56
Steady	1.5,12.95, 1.5	70.6	28.2	17.4	12.6	9.91	8.13	6.94	5.99	5.32	4.82	3.99	2.99	1.5,6.98,1.5	93.7	25.7	12.6	8.04	5.60	4.31	3.45	2.78	2.27	1.94	1.39	1.10

control chart (see Tables 2–3 and 7). Similarly, at $k = 1.50$, Fig. 3b also presents superiority of proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts over the classical CUSUM control chart for $\delta \le 1.75$ shifts (see Tables 2–3 and 7). With regard to the overall performance, at $k = 0.25$ and $\lambda = 0.05$, PCI and RARL values of the classical CUSUM control chart are 1.25 and 1.22, respectively, which are larger as compared to the $ACUSUM_C^{(1)}$ control chart (see Table 5). Furthermore, as $\lambda > 0.05$ and $k > 0.25$, proposed $ACUSUM_C^{(1)}$ control chart retains overall performance superiority against the classical CUSUM control chart (see Tables 5). Correspondingly, proposed $ACUSUM_C^{(2)}$ control chart also has overall good detection ability against the classical CUSUM control chart, too (see Tables 6). In brief, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts outperform against the classical CUSUM control chart for a single shift and for overall performance as well as.

5.2.3 Proposed Versus MCE Control Chart

The proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts outperform against the MCE control chart. For example, at $\lambda = 0.1$, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(1)}$ control charts have smaller ARL_1 values against the MCE control chart for all shifts (see Tables 2 and 7, Fig. 4a). Similarly, as $\lambda > 0.1$, for instance, at $\lambda = 0.5$, the proposed $ACUSUM_C^{(1)}$ retains superior performance against MCE control chart for all shifts, but the proposed $ACUSUM_C^{(2)}$ control chart performs better only for $\delta > 0.75$ shifts (see Tables 3 and 7, Fig. 4b). In term of overall performance comparison, the proposed $ACUSUM_C^{(1)}$ control chart has lower EQL, PCI, and RARL as compared to MCE control chart. For example, at $\lambda = 0.1$, 18.90, 2.46, and 2.34 are EQL, PCI, and RARL values, respectively, of the MCE control chart while the proposed $ACUSUM_C^{(1)}$ control chart has smaller values (see Table 5). Besides, at $\lambda = 0.5$, proposed $ACUSUM_C^{(1)}$ control chart performs better as well. Similarly, at $\lambda = 0.1$ and 0.5, proposed $ACUSUM_C^{(2)}$ control chart also shows superior performance as compared to MCE control chart (see Table 6). Concisely, the proposed $ACUSUM_C^{(1)}$ outperforms against MCE control chart for all shifts while proposed $ACUSUM_C^{(2)}$ control chart shows superiority at certain values of parameters for specific shifts.

5.2.4 Proposed Versus MEC Control Chart

The proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ show superior performance against the MEC control chart. For example, at $\lambda = 0.1$, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have smaller ARL_1 values against the MEC control chart for all shifts (see Tables 2–3 and 7, Fig. 5a). Similarly, as $\lambda > 0.1$, for instance, at $\lambda = 0.5$, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ preserve superiority against MCE control

Table 3 ARLs properties of proposed $ACUSUM_c^{(2)}$ control chart at different choices of γ, k, and λ when $ARL_0 = 500$

State	γ, $h_{ACUSUM_C^{(2)}}$, k	δ 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00	γ, $h_{ACUSUM_C^{(2)}}$, k	δ 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00
		$\lambda = 0.05$													$\lambda = 0.2$											
Zero	3.5,6.75,0.25	66.8	24.7	14.5	10.5	8.3	6.9	5.99	5.31	4.77	4.34	3.66	2.77	3.5,5.16,0.25	80.1	26.1	14.1	9.4	7.25	5.94	5.04	4.41	3.92	3.54	2.97	2.24
Steady	3.5,6.75,0.25	58.2	20.2	11.2	7.6	5.6	4.4	3.63	2.94	2.42	2.08	1.46	1.21	3.5,5.16,0.25	70.1	21.5	10.4	6.5	4.44	3.26	2.43	1.84	1.37	1.21	1.11	1.04
Zero	4,7.42,0.25	63.9	24.7	14.8	10.8	8.5	7.2	6.21	5.51	4.95	4.52	3.85	2.96	4,5.55,0.25	77.7	25.8	14.0	9.59	7.36	6.00	5.13	4.50	4.01	3.65	3.08	2.34
Steady	4,7.42,0.25	55.6	20.0	11.3	7.8	5.9	4.6	3.82	3.16	2.66	2.28	1.66	1.34	4,5.55,0.25	66.1	21.0	10.5	6.56	4.62	3.50	2.61	1.99	1.57	1.34	1.21	1.10
Zero	3.5,7.8,0.5	71.7	27.2	16.4	11.9	9.43	7.87	6.82	6.03	5.42	4.92	4.14	3.12	3.5,5.74,0.5	88.4	28.3	15.2	10.4	7.97	6.49	5.53	4.82	4.30	3.89	3.24	2.42
Steady	3.5,7.8,0.5	61.5	21.6	12.4	8.5	6.49	5.22	4.42	3.63	3.19	2.86	2.15	1.31	3.5,5.74,0.5	75.4	22.7	11.5	7.0	4.95	3.72	3.10	2.47	1.91	1.51	1.31	1.14
Zero	4,8.7,0.5	70.4	27.6	16.9	12.4	9.9	8.27	7.17	6.34	5.73	5.22	4.43	3.37	4,6.15,0.5	83.8	27.5	15.2	10.4	8.0	6.58	5.60	4.92	4.39	3.96	3.35	2.54
Steady	4,8.7,0.5	59.7	21.6	12.9	9.0	6.87	5.73	4.81	4.12	3.69	3.20	2.64	1.71	4,6.115,0.5	71.7	21.9	11.1	7.2	5.2	3.97	3.11	2.58	2.05	1.64	1.42	1.21
Zero	3.5,8.98,0.75	110	39.5	34.2	23.4	14.5	10.8	8.54	7.09	6.12	5.35	4.35	3.22	3.5,6.21,0.75	128	37.2	19.6	13.0	9.35	7.30	5.97	5.05	4.40	3.92	3.23	2.45
Steady	3.5,8.98,0.75	70	24.6	14.2	10.1	7.8	6.4	5.39	4.66	4.10	3.63	2.91	2.04	3.5,6.21,0.75	89.0	25.2	12.5	7.87	5.69	4.36	3.56	2.86	2.41	2.01	1.47	1.15
Zero	4,10,0.75	101	39.3	39.3	26.9	16.1	11.8	9.39	7.81	6.70	5.88	4.75	3.50	4,6.81,0.75	121	35.5	19.5	13.2	9.64	7.55	6.20	5.27	4.62	4.10	3.39	2.58
Steady	4,10,0.75	67	24.8	14.7	10.5	8.2	6.7	5.84	5.08	4.46	4.00	3.32	2.31	4,6.81,0.75	83	24.2	12.6	8.1	5.91	4.62	3.69	3.10	2.51	2.20	1.76	1.23
Zero	3.5,9.57,1	92.4	33.6	20.4	15.0	11.9	9.86	8.34	7.15	6.25	5.57	4.55	3.38	3.5,6.58,1	110	32.8	17.4	11.7	8.83	7.11	5.94	5.10	4.49	4.02	3.35	2.53
Steady	3.5,9.57,1	74.3	26.0	15.0	10.7	8.3	6.86	5.86	5.02	4.45	4.00	3.27	2.27	3.5,6.58,1	93	26.2	13.0	8.3	6.05	4.59	3.78	3.09	2.61	2.16	1.63	1.23
Zero	4,10.77,1	89.3	34.5	21.4	16.0	12.9	10.8	9.19	7.91	6.91	6.12	5.00	3.71	4,7.1,1	105	32.4	17.5	11.9	9.07	7.35	6.17	5.32	4.68	4.20	3.49	2.67
Steady	4,10.77,1	71.5	26.7	16.2	11.5	9.0	7.5	6.39	5.56	5.03	4.52	3.69	2.65	4,7.1,1	88	25.4	13.0	8.5	6.27	4.86	3.93	3.21	2.75	2.38	1.81	1.34
Zero	3.5,8.56,1.5	87.7	32.9	19.9	14.4	11.3	9.26	7.87	6.89	6.08	5.48	4.58	3.46	3.5,6.67,1.25	104	31.6	17.1	11.6	8.75	7.09	5.94	5.13	4.52	4.06	3.38	2.58
Steady	3.5,8.56,1.5	74.3	26.5	15.5	11.1	8.7	7.07	6.10	5.27	4.67	4.13	3.46	2.36	3.5,6.67,1.25	93	26.1	13.1	8.3	5.99	4.72	3.68	3.09	2.57	2.12	1.69	1.34
Zero	4,9.83,1.5	85.3	33.7	20.9	15.2	12.0	9.91	8.47	7.43	6.61	5.96	4.98	3.78	4,7.22,1.5	101	31.4	17.2	11.9	9.00	7.32	6.16	5.33	4.71	4.23	3.55	2.73
Steady	4,9.83,1.5	72.6	27.2	16.3	11.7	9.30	7.65	6.58	5.74	5.16	4.61	3.89	2.80	4,7.22,1.5	87	25.6	13.0	8.5	6.31	5.03	4.05	3.37	2.83	2.44	1.91	1.51
		$\lambda = 0.1$													$\lambda = 0.3$											
Zero	3.5,6.75,0.25	71.1	24.9	14.2	10.0	7.84	6.52	5.61	4.95	4.43	4.02	3.40	2.56	3.5,4.52,0.25	95.9	29.1	14.5	9.35	6.99	5.60	4.70	4.08	3.60	3.24	2.70	2.07
Steady	3.5,6.75,0.25	74.4	24.2	12.9	8.6	6.56	5.14	4.34	3.51	3.05	2.62	2.01	1.22	3.5,4.52,0.25	84.0	23.9	10.8	6.27	4.12	2.95	2.09	1.70	1.47	1.30	1.22	1.08
Zero	4,6.66,0.25	67.4	24.5	14.3	10.2	8.03	6.66	5.76	5.10	4.59	4.17	3.54	2.71	4,4.78,0.25	90.6	27.9	14.2	9.33	6.99	5.59	4.74	4.10	3.64	3.29	2.76	2.12
Steady	4,6.66,0.25	57.8	19.9	11.0	7.3	5.36	4.07	3.26	2.78	2.22	1.85	1.35	1.27	4,4.78,0.25	80.6	22.9	10.7	6.26	4.24	2.83	2.19	1.71	1.48	1.32	1.20	1.10
Zero	3.5,6.93,0.5	75.1	27.1	15.7	11.2	8.77	7.27	6.26	5.52	4.94	4.49	3.76	2.83	3.5,4.91,0.5	107	31.3	15.5	10.0	7.48	6.01	5.05	4.36	3.86	3.45	2.89	2.17
Steady	3.5,6.93,0.5	63.9	21.6	11.7	7.7	5.84	4.56	3.80	3.13	2.68	2.27	1.69	1.25	3.5,4.91,0.5	93	25.5	11.5	6.8	4.54	3.31	2.36	1.84	1.46	1.27	1.17	1.09
Zero	4,7.64,0.5	72.9	27.1	16.0	11.52	9.07	7.55	6.52	5.75	5.16	4.69	3.97	3.03	4,5.21,0.5	100	30.1	15.3	10.1	7.51	6.04	5.08	4.42	3.91	3.53	2.96	2.24
Steady	4,7.64,0.5	63.3	21.6	11.9	8.14	6.26	4.91	4.09	3.48	3.01	2.59	2.04	1.26	4,5.21,0.5	88	24.5	11.3	6.8	4.56	3.35	2.56	1.96	1.49	1.29	1.19	1.10
Zero	3.5,7.86,0.75	116	37.9	25.2	17.5	12.1	9.13	7.39	6.18	5.33	4.72	3.85	2.90	3.5,5.27	139	38.5	18.3	11.4	8.21	6.34	5.22	4.43	3.85	3.45	2.85	2.17
Steady	3.5,7.86,0.75	75	24.2	13.4	9.2	6.9	5.45	4.58	3.88	3.40	3.02	2.32	1.40	3.5,5.27	106	27.4	12.3	7.2	4.98	3.66	2.69	2.11	1.63	1.31	1.15	1.03
Zero	4,8.66,0.75	106	36.9	26.2	18.7	12.8	9.80	7.89	6.64	5.74	5.07	4.14	3.11	4,6.81	135	37.4	18.0	11.5	8.34	6.48	5.36	4.55	3.98	3.55	2.96	2.24
Steady	4,8.66,0.75	72	24	13.7	9.5	7.3	5.9	4.99	4.30	3.79	3.25	2.63	1.76	4,6.81	100	26.8	12.2	7.2	5.08	3.79	2.87	2.28	1.81	1.51	1.19	1.07
Zero	3.5,8.3,1	97.3	32.7	18.9	13.5	10.5	8.58	7.22	6.21	5.46	4.87	4.01	3.03	3.5,5.43,1	123	34.8	16.9	10.8	7.93	6.30	5.19	4.47	3.93	3.51	2.92	2.22

Table 3 (continued)

State	$\gamma, h_{ACUSUM_C^{(2)}}, k$	δ 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00	$\gamma, h_{ACUSUM_C^{(2)}}, k$	δ 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	3.00	4.00
Steady	3.5,8.3,1	78.9	25.1	14.1	9.6	7.26	5.88	4.90	4.23	3.69	3.25	2.58	1.67	3.5,5.43,1	109	28.4	12.8	7.6	5.19	3.81	2.93	2.36	1.86	1.38	1.13	1.00
Zero	4,9.17,1	92.9	32.7	19.5	14.0	11.1	9.07	7.73	6.67	5.86	5.24	4.32	3.25	4,5.8,1	121	33.8	16.9	10.9	8.12	6.43	5.35	4.58	4.05	3.62	3.03	2.29
Steady	4,9.17,1	75.1	25.2	14.5	10.2	7.8	6.28	5.37	4.62	4.03	3.57	2.86	1.93	4,5.8,1	105	27.6	12.7	7.6	5.32	3.93	2.97	2.45	1.97	1.65	1.15	1.00
Zero	3.5,8.56,1.25	92.7	31.8	18.7	13.2	10.2	8.37	7.07	6.13	5.44	4.88	4.07	3.10	3.5,5.45,1.25	120	33.9	16.6	10.8	7.94	6.30	5.25	4.49	3.95	3.55	2.95	2.25
Steady	3.5,8.56,1.25	79.6	25.8	14.5	9.9	7.6	6.11	5.16	4.36	3.89	3.46	2.72	1.84	3.5,5.45,1.25	108	27.6	12.7	7.5	5.16	3.73	2.85	2.34	1.88	1.48	1.13	1.09
Zero	4,9.38,1.5	88.9	32.0	19.0	13.6	10.6	8.72	7.41	6.46	5.74	5.16	4.32	3.29	4,5.86,1.5	118	33.4	16.8	10.9	8.14	6.49	5.40	4.64	4.10	3.66	3.06	2.33
Steady	4,9.38,1.5	75.2	25.6	14.7	10.2	8.0	6.48	5.47	4.67	4.10	3.73	2.98	2.08	4,5.86,1.5	106	27.4	12.6	7.7	5.37	3.99	3.04	2.51	2.03	1.70	1.23	1.03

chart but only for $\delta > 0.75$ shifts (see Tables 2–3 and 7, Fig. 5b). From the perspective of overall performance, for example, at $\lambda = 0.1$, the proposed $ACUSUM_C^{(1)}$ control chart has lower EQL, PCI, and RARL values as compared to the MCE control chart. For instance, 36.79, 4.78 and 4.58 are EQL, PCI, and RARL values; respectively, of the MEC control chart those are larger as compared to the proposed $ACUSUM_C^{(1)}$ control chart (see Table 5). As $\lambda > 0.10$, like $\lambda = 0.5$, 18.78, 1.67 and 1.66 are values of EQL, PCI, and RARL, respectively, of the proposed $ACUSUM_C^{(1)}$ control chart, and these are smaller against the MCE control chart values (see Table 5). Similarly, at $\lambda = 0.1$ and 0.5, proposed $ACUSUM_C^{(2)}$ control chart also shows superior performance as compared to MEC control chart (see Table 6). Shortly, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts are more effective against MEC control chart for all shifts at small value of λ, and for small shift at large value of λ.

5.2.5 Proposed Versus $AEWMA_C$ Control Charts

At $\lambda = 0.1$, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have smaller ARL_1 ($\delta = 0.25, 0.5$ and 0.75) values against the $AEWMA_C^{(1)}$ control chart. But as $\delta >$ 0.75, more specifically at $\delta = 1$, 1.5 and 1.75, proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts show inferior detection ability against $AEWMA_C^{(1)}$ control chart (see Tables 2–3 and 7, Fig. 6a). Similarly, at $\lambda = 0.5$, proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have inferior detection ability, but only for small shift ($\delta = 0.25$) against the $AEWMA_C^{(1)}$ control chart. In contrary, for $0.25 \leq \delta \leq 1.5$ shifts, proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have good detection ability against the $AEWMA_C^{(1)}$ control chart (see Fig. 6b). An $AEWMA_C^{(1)}$ control chart has superior performance against the $AEWMA_C^{(2)}$ control chart. So, with this relation, proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts also have better detection ability against an $AEWMA_C^{(2)}$ control chart for $0.05 < \delta < 1.00$ shifts see Tables 2–3 and 7 and Fig. 6b). According to overall performance, the EQL, PCI, and RARL of the $AEWMA_C$ control charts are smaller as compared to proposed $ACUSUM_C$ control charts (see Tables 5 and 6). Concisely, proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control chart perform better against $AEWMA_C^{(1)}$ and $AEWMA_C^{(2)}$ for different shifts at different values of parameters.

5.2.6 Proposed Versus $AEWMA_E^{(1)}$ Control Chart

The proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts perform better against the $AEWMA_E^{(1)}$ control chart, but only for small sizes of shift such as $\delta = 0.25, \ldots, 1.25$ at any choices of k and λ (see Tables 3–4 and 7, Fig. 7). As $\delta > 0.75$, the $AEWMA_E^{(1)}$ control chart performs superior by detecting an earlier signal as compared to proposed

Table 4 EQL, RARL, and PCI values of proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts at different choices of k, γ, and λ when $ARL_0 = 500$

	EQL λ= 0.05	PCI	RARL	γ	k	EQL λ= 0.1	PCI	RARL	γ	k	EQL λ= 0.5	PCI	RARL	γ	k
$ACUSUM_C^{(1)}$	18.99	1.08	1.07	1.0	0.25	17.52	1.06	1.06	1.0	0.25	13.56	1.00	1.00	1.0	0.25
$ACUSUM_C^{(2)}$	35.84	2.04	2.08	3.5	0.25	27.76	1.69	1.69	3.5	0.25	14.56	1.07	1.08	1.0	0.25
$ACUSUM_C^{(1)}$	17.54	1.00	1.00	1.5	0.25	16.47	1.00	1.00	1.5	0.25	15.19	1.12	1.21	1.5	0.25
$ACUSUM_C^{(2)}$	27.93	1.59	1.75	4.0	0.25	23.02	1.40	1.48	4.0	0.25	14.32	1.06	1.07	4.0	0.25
$ACUSUM_C^{(1)}$	22.12	1.26	1.23	1.0	0.5	19.53	1.19	1.16	1.0	0.5	13.57	1.00	1.00	1.0	0.5
$ACUSUM_C^{(2)}$	26.40	1.51	1.51	3.5	0.5	22.65	1.38	1.38	3.5	0.5	14.26	1.05	1.06	3.5	0.5
$ACUSUM_C^{(1)}$	18.21	1.04	1.03	1.5	0.5	16.94	1.03	1.02	1.5	0.5	14.70	1.08	1.15	1.5	0.5
$ACUSUM_C^{(2)}$	23.40	1.33	1.36	4.0	0.5	20.67	1.25	1.28	4.0	0.5	14.16	1.04	1.05	4.0	0.5
$ACUSUM_C^{(1)}$	21.91	1.25	1.24	1.0	0.75	19.68	1.19	1.18	1.0	0.75	14.15	1.04	1.05	1.0	0.75
$ACUSUM_C^{(2)}$	34.87	1.99	1.95	3.5	0.75	27.28	1.66	1.63	3.5	0.75	14.57	1.07	1.07	3.5	0.75
$ACUSUM_C^{(1)}$	19.87	1.13	1.13	1.5	0.75	18.33	1.11	1.11	1.5	0.75	14.10	1.04	1.05	1.5	0.75
$ACUSUM_C^{(2)}$	25.53	1.46	1.47	4.0	0.75	21.96	1.33	1.35	4.0	0.75	14.30	1.06	1.06	4.0	0.75
$ACUSUM_C^{(1)}$	26.80	1.53	1.49	1.0	1.0	22.82	1.39	1.35	1.0	1.0	14.22	1.05	1.04	1.0	1.0
$ACUSUM_C^{(2)}$	25.90	1.48	1.48	3.5	1.0	22.36	1.36	1.36	3.5	1.0	14.30	1.05	1.06	3.5	1.0
$ACUSUM_C^{(1)}$	20.93	1.19	1.18	1.5	1.0	19.06	1.16	1.15	1.5	1.0	14.09	1.04	1.04	1.5	1.0
$ACUSUM_C^{(2)}$	22.75	1.30	1.32	4.0	1.0	20.50	1.24	1.26	4.0	1.0	14.21	1.05	1.06	4.0	1.0
$ACUSUM_C^{(1)}$	26.81	1.53	1.59	1.0	1.5	22.71	1.38	1.43	1.0	1.5	14.35	1.06	1.07	1.0	1.5
$ACUSUM_C^{(2)}$	32.93	1.88	1.84	3.5	1.5	26.45	1.61	1.58	3.5	1.5	14.68	1.08	1.08	3.5	1.5
$ACUSUM_C^{(1)}$	25.36	1.45	1.60	1.5	1.5	21.61	1.31	1.41	1.5	1.5	14.31	1.06	1.07	1.5	1.5
$ACUSUM_C^{(2)}$	24.45	1.39	1.40	4.0	1.5	21.47	1.30	1.31	4.0	1.5	14.34	1.06	1.06	4.0	1.5

$ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts. But, as $\lambda > 0.5$, proposed $ACUSUM_C^{(1)}$ control chart performs better against to $AEWMA_E^{(1)}$ control chart for small-to-moderate shifts. In contrary, as $\delta > 1.5$, the $AEWMA_E^{(1)}$ control chart holds smaller ARL_1 against proposed $ACUSUM_C^{(1)}$ control chart. So, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts perform more effective for small shifts against the $AEWMA_E^{(1)}$ control chart. In term of overall performance comparison, the proposed $ACUSUM_C^{(1)}$ control chart also has lower EQL, PIC, and RARL values as compared to an $AEWMA_E$ control chart at $\lambda = 0.5$ while the $ACUSUM_C^{(2)}$ control chart has larger (see Tables 5–7). In short, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts outperform against $AEWMA_E^{(1)}$ control chart particularly for small shifts.

5.2.7 Proposed Versus Hybrid EWMA Control Chart

The proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts have superior performance against Hybrid EWMA control chart for small-to-moderate shifts. For instant, at $\lambda = 0.1$ and $\delta = 0.25, 0.5, 0.75, \ldots 2.5$, proposed $ACUSUM_C^{(1)}$ control chart holds smaller ARL_1. But as $\delta > 2.5$, the Hybrid EWMA control chart detects signal earlier against the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts. (See Tables 3–4 and 7). With regard to overall performance comparison, for instance, at $\lambda = 0.1$, 17.77, 2.31 and 2.24 are EQL, PCI, and RARL values, respectively, of the Hybrid control chart which are larger against proposed $ACUSUM_C^{(1)}$ control chart at $k = 0.25, \gamma = 1$, and $\lambda = 0.10$ (see Table 5). In contrary, at $\lambda = 0.5$, the Hybrid control chart has smaller EQL, PCI, and RARL values as compared to proposed $ACUSUM_C^{(1)}$ control chart (see Tables 5). Like proposed $ACUSUM_C^{(1)}$ control chart, the proposed $ACUSUM_C^{(2)}$ control chart also has same overall performance efficiency against the Hybrid control chart (see Tables 6). In brief, the proposed $ACUSUM_C^{(1)}$ and $ACUSUM_C^{(2)}$ control charts perform more efficiently against the Hybrid control chart for small-to-moderate shifts at the certain values of k, γ, and λ.

Table 5 EQL, RARL, and PCI values of proposed ACUSUM$_C^{(1)}$ and other control at different choices of k, γ, and λ when $ARL_0 = 500$

	λ= 0.05					λ= 0.1					λ= 0.5				
	EQL	PCI	RARL	γ	k	EQL	PCI	RARL	γ	k	EQL	PCI	RARL	γ	k
ACUSUM$_c^{(1)}$	18.99	1.08	1.04	1	0.25	17.52	2.28	2.20	1	0.25	13.56	1.20	1.22	1	0.25
ACUSUM$_c^{(1)}$	22.12	1.26	1.19	1.5	0.25	19.53	2.54	2.45	1.5	0.25	13.57	1.21	1.21	1.5	0.25
ACUSUM$_c^{(1)}$	21.91	1.24	1.20	1	0.5	19.68	2.56	2.48	1	0.5	14.15	1.26	1.27	1	0.5
ACUSUM$_c^{(1)}$	26.80	1.52	1.45	1.5	0.5	22.82	2.97	2.86	1.5	0.5	14.22	1.26	1.27	1.5	0.5
ACUSUM$_c^{(1)}$	26.81	1.52	1.53	1	0.75	22.71	2.95	2.88	1	0.75	14.35	1.27	1.29	1	0.75
ACUSUM$_c^{(1)}$	35.84	2.04	2.01	1.5	0.75	27.76	3.61	3.51	1.5	0.75	14.56	1.29	1.31	1.5	0.75
ACUSUM$_c^{(1)}$	26.40	1.50	1.46	1	1	22.65	2.95	2.86	1	1	14.26	1.27	1.28	1	1
ACUSUM$_c^{(1)}$	34.87	1.98	1.89	1.5	1	27.28	3.55	3.43	1.5	1	14.57	1.29	1.31	1.5	1
ACUSUM$_c^{(1)}$	25.90	1.47	1.43	1	1.5	22.36	2.91	2.82	1	1.5	14.30	1.27	1.29	1	1.5
ACUSUM$_c^{(1)}$	32.93	1.87	1.78	1.5	1.5	26.45	3.44	3.32	1.5	1.5	14.68	1.30	1.32	1.5	1.5
Classical EWMA	17.61	1.00	1.00	n/a	n/a	–	–	–	n/a	n/a				n/a	n/a
Classical CUSUM	22.02	1.25	1.22	n/a	0.25	–	–	–	n/a	n/a				n/a	n/a
Classical CUSUM	24.67	1.40	1.93	n/a	1.5	–	–	–	n/a	n/a				n/a	n/a
Classical EWMA				n/a	n/a	15.16	1.97	1.92	n/a	n/a	22.60	2.01	2.03	n/a	**n/a**
MEC				n/a	n/a	36.79	4.78	4.58	–	0.5	18.78	1.67	1.66		0.5
MCE				n/a	n/a	18.90	2.46	2.39	–	0.5	15.17	1.35	1.36		0.5
AEWMAc$^{(1)}$				n/a	n/a	11.60	1.51	1.49	4	0.2	11.26	1.00	1.00	4	0.2
AEWMAc$^{(2)}$				n/a	n/a	10.53	1.37	1.36	31.12	0.09	12.55	1.11	1.11	31.12	0.080
AEWMAE$^{(1)}$				n/a	n/a	7.69	1.00	1.00	4		13.89	1.23	1.28	4	
Hybrid EWMA				n/a	n/a	17.77	2.31	2.24			12.43	1.10	1.11		

Table 6 EQL, RARL, and PCI values of proposed ACUSUM$_c^{(2)}$ and other control charts at different choices of k, γ, and λ when $ARL_o = 500$

	$\lambda = 0.05$					$\lambda = 0.1$					$\lambda = 0.5$				
	EQL	PCI	RARL	γ	k	EQL	PCI	RARL	γ	k	EQL	PCI	RARL	γ	k
ACUSUM$_c^{(2)}$	26.4	1.5	1.5	3	0.25	16.47	2.14	2.07	3	0.25	15.19	1.35	1.39	3	0.25
ACUSUM$_c^{(2)}$	17.5	1.0	1.0	3.5	0.25	16.94	2.20	2.13	3.5	0.25	14.70	1.31	1.34	3.5	0.25
ACUSUM$_c^{(2)}$	33.3	1.9	1.9	1	0.5	18.33	2.38	2.31	1	0.5	14.10	1.25	1.27	1	0.5
ACUSUM$_c^{(2)}$	19.9	1.1	1.1	3	0.5	19.06	2.48	2.40	3	0.5	14.09	1.25	1.27	3	0.5
ACUSUM$_c^{(2)}$	40.0	2.3	2.2	3.5	0.75	21.61	2.81	2.75	3.5	0.75	14.31	1.27	1.29	3.5	0.75
ACUSUM$_c^{(2)}$	25.4	1.4	1.6	3	0.75	21.61	2.81	2.75	3	0.75	14.32	1.27	1.29	3	0.75
ACUSUM$_c^{(2)}$	38.1	2.2	2.1	3.5	1	23.02	2.99	2.92	3.5	1	14.16	1.26	1.27	3.5	1
ACUSUM$_c^{(2)}$	23.4	1.3	1.4	3	1	20.67	2.69	2.61	3	1	14.30	1.27	1.29	3	1
ACUSUM$_c^{(2)}$	35.9	2.0	2.0	3.5	1.5	21.96	2.86	2.77	3.5	1.75	14.21	1.26	1.28	3.5	1.5
ACUSUM$_c^{(2)}$	22.8	1.3	1.3	3	1.5	20.50	2.67	2.59	3	1.75	14.34	1.27	1.29	3	1.5
Classical EWMA	17.6	1.0	1.0	n/a	n/a	–	–	–	n/a	n/a				n/a	n/a
Classical CUSUM	22.0	1.3	1.3	n/a	0.25	–	–	–	n/a	n/a				n/a	n/a
Classical CUSUM	24.7	1.4	2.1	n/a	1.5	–	–	–	n/a	n/a				n/a	n/a
Classical EWMA				n/a	n/a	21.47	2.79	2.71	n/a	n/a	22.60	2.01	2.03	n/a	n/a
MEC				n/a	n/a	36.79	4.78	4.58	–	0.5	18.78	1.67	1.66		0.5
MCE				n/a	n/a	18.90	2.46	2.39	–	0.5	15.17	1.35	1.36		0.5
AEWMA$_c^{(1)}$				n/a	n/a	11.60	1.51	1.49	4	0.2	11.26	1.00	1.00	4	0.2
AEWMA$_c^{(2)}$				n/a	n/a	10.53	1.37	1.36	31.12	0.09	12.55	1.11	1.11	31.12	0.080
AEWMA$_E^{(1)}$				n/a	n/a	7.69	1.00	1.00	4		13.89	1.23	1.28	4	
Hybrid EWMA				n/a	n/a	17.77	2.31	2.24			12.43	1.10	1.11		

Table 7 ARLs properties of other control charts when $ARL_o = 500$

	Classical EWMA		Classical CUSUM		MEC		MCE	
	$\lambda = 0.05$	$\lambda = 0.1$	$k = 0.25$ $h=8.59$	$k = 1.5$ $h=1.71$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda_C = 0.1$ $L_C = 9.660$	$\lambda_C = 0.5$
	$L = 2.820$	$L = 2.820$			$b^* = 37.42$	$b^* = 11.20$		$L_C = 7.430$
					$a^* = 0.500$	$a^* = 0.500$	$k = 0.500$	$k = 0.500$
δ								
0.00	500	500	500	502	499	510	500	500
0.25	79.5	106	96.5	332	79.6	99.8	127	144
0.50	25.8	31.5	32.1	143	35.7	30.8	36.0	38.2
0.75	13.7	15.9	18.5	60.2	24.2	16.7	17.6	16.6
1.00	9.13	10.3	13.2	28.3	18.9	11.5	11.8	10.0
1.25	6.82	8.21	10.3	15.2	15.9	8.86	8.89	7.83
1.50	5.47	6.10	8.60	9.23	13.9	7.31	7.30	5.51
1.75	4.61	5.24	7.39	6.39	12.4	6.26	6.80	5.23
2.00	4.03	4.40	6.56	4.90	11.3	5.53	5.49	4.83
2.25	3.62	3.91	5.90	4.00	10.4	4.97	5.12	3.56
2.50	3.31	3.40	5.41	3.44	9.71	4.54	4.49	3.39
2.75	3.07	3.23	5.01	3.06	9.09	4.19	4.12	3.10
3.00	2.88	2.90	4.69	2.80	8.56	3.91	3.83	2.52

	$AEWMA_c^{(1)}$		$AEWMA_c^{(2)}$		$AEWMA_E$		Hybrid EWMA	
	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.5$			$\lambda_1 = 0.1$	$\lambda_2 = 0.5$
	$\gamma = 4.00$	$\gamma = 4.00$	$\gamma = 31.12$	$\gamma = 31.12$	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda_2 = 0.10$	$\lambda_1 = 0.100$
	$k = 0.20$	$k = 0.20$	$k = 0.09$	$k = 0.08$	$\gamma = 4.00$	$\gamma = 4.00$	$L = 7.230$	$L = 4.340$
δ	$L_{AEWMA_c^{(1)}} = 4.497$	$L_{AEWMA_c^{(1)}} = 3.217$	$L_{AEWMA_c^{(1)}} = 3.980$	$L_{AEWMA_c^{(1)}} = 3.280$	$L = 2.825$	$L = 3.080$		
0.00	501	500	501	500.09	499	504.73	502	501.30
0.25	70.2	100	69.7	99.12	102	258.46	86.0	100.46
0.50	26.4	38.1	28.9	45.08	27.9	88.54	27.9	28.61
0.75	14.6	20.6	15.9	25.21	12.7	35.49	15.9	14.28
1.00	9.57	12.5	10.0	15.16	7.32	16.67	11.2	8.99
1.25	6.85	8.09	6.84	9.52	4.75	9.02	8.79	6.50
1.50	5.19	5.50	4.97	6.20	3.32	5.56	7.24	5.11
1.75	4.13	3.89	3.73	4.23	2.45	3.71	6.10	4.14
2.00	3.38	2.85	2.93	3.06	1.90	2.67	5.33	3.51
2.25	2.82	2.19	2.37	2.26	1.55	2.02	4.74	3.07
2.50	2.39	1.75	1.93	1.82	1.31	1.63	4.25	2.70
2.75	2.04	1.46	1.61	1.50	1.17	1.38	3.84	2.46
3.00	1.77	1.27	1.39	1.29	1.09	1.21	3.52	2.23

6 Real Life Examples

This section explains the employment procedure of the proposed control charts with real-life data. Section 6.1 introduces potential fields where proposed control charts can be contributed to identify the possible special causes. Variable of interest is given in Sect. 6.2. Likewise, Sect. 6.3 introduces the properties of metal layer thickness (real-life data). Implementation of control charts with real-life data is provided in Sect. 6.4. Lastly, results and discussion are given in Sect. 6.5.

6.1 Potential Fields for Proposed Control Charts

The proposed control charts can be implemented to detect possible special causes from an ongoing process in many industries. Few of them are illustrated as: manufacturing industry, public health monitoring, chemical process monitoring, quality improvement in the banking sector, insurance industry, etc. The semiconductor wafer manufacturing industry is one of them where metal layer thickness is very important quality characteristic of semiconductor wafers.

6.2 Variable of Interest

Silicon is famous material in the world; it is mostly used as a semiconductor into many fields such as electronic and semiconductor makes most of the electronic circuits [28]. It is a very thin device, the thickness of a metal layer on silicon wafers is measured in µm (micrometre), and the tolerance of the one such product is specified ±0.0050 inches, which is a very small measure unit (see Fig. 8). The warpage, grinding, rigidity, coating, chemical vapour deposition methods, and other environmental condition play the main role to maintain targeted layer thickness of a metal layer on silicon wafers. A little trouble in these processes can lead to significant deviation in the metal layer thickness.

6.3 Properties of Metal Layer Thickness Å

The proposed ACUSUM$_c^{(1)}$ and ACUSUM$_c^{(1)}$, and other control charts deal only with standard normally distributed data. Therefore, the metal layer thickness data of semiconductor first is converted into standardized form (Z-score). Afterwards, Kolmogorov–Smirnov test is used to confirm data

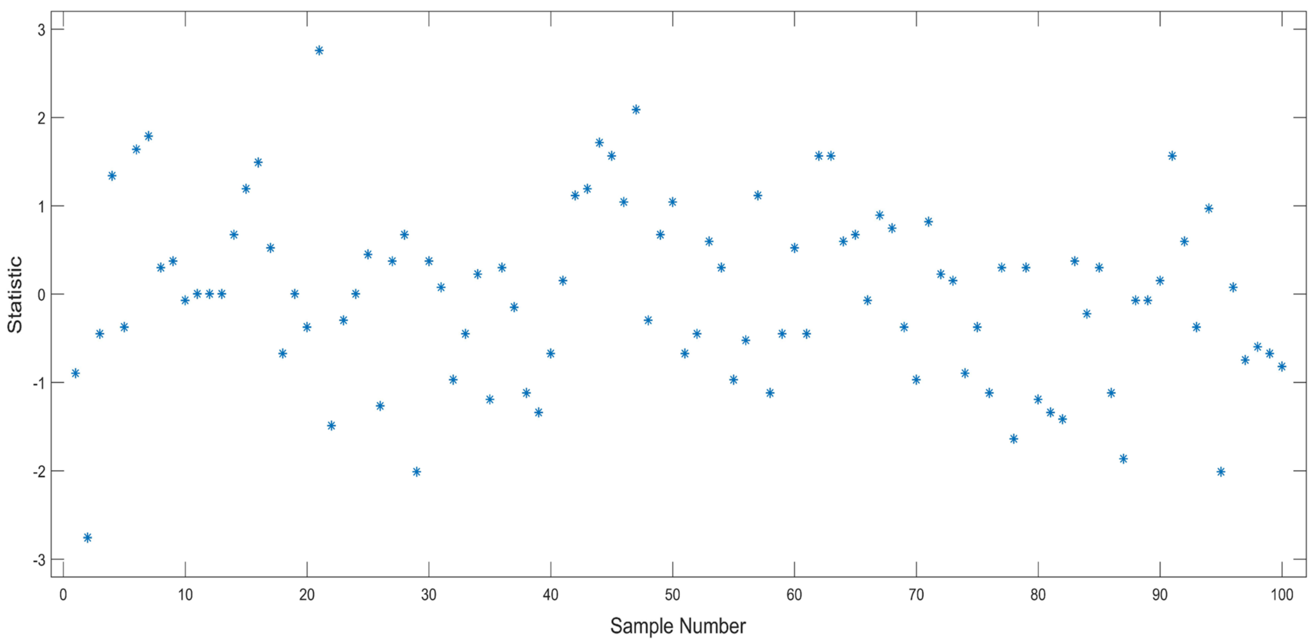
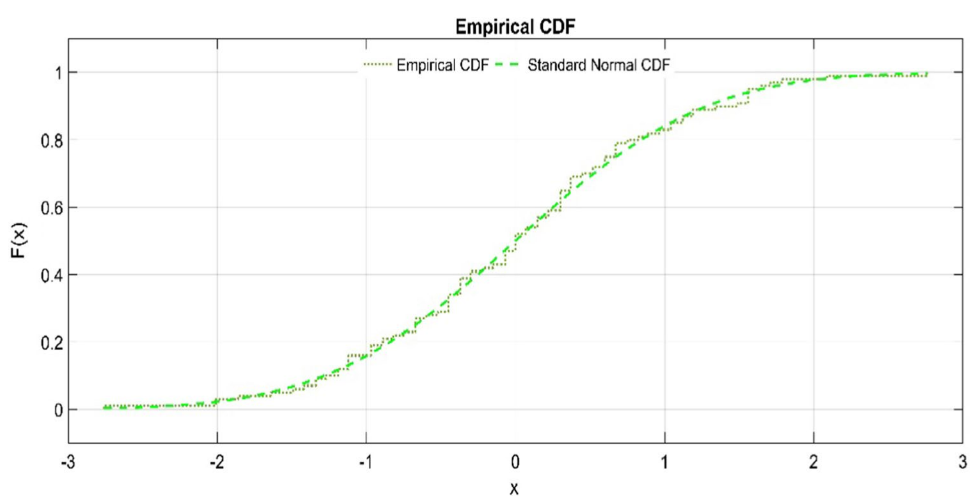


Fig. 8 Graphical presentation of thickness of a metal layer on silicon wafers

Fig. 9 Empirical CDF of layer thickness Å on semiconductor wafers



normality. The metal layer thickness data is approximately standard normal and insignificant with *p-value* = 0.64. Furthermore, Fig. 9 depicts the empirical cumulative distribution function (CDF) and standard normal CDF is almost similar.

6.4 Implementation of Control Charts with Real-Life Data

The proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$, classical CUSUM, classical EWMA, and $\text{AEWMA}_{\text{C}}^{(1)}$ control charts are implemented with real-life data (see Table 8). Three different scenarios such as one with false alarm, and others two out-of-control situations by adding 1σ and 3σ to the last 30 observations are introduced. So, for the first 70 observations, the process is considered in-control. Moreover, all control charts have ARL_0 equal to 500.

6.5 Results and Discussion

After implementing the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ control chart with real-life data for three different scenarios original data and numerical results are given in Table 8. The classical CUSUM and EWMA and $\text{AEWMA}_{\text{C}}^{(1)}$ control charts are also implemented with real-life data and only their out-of-control statistics are given in Table 9. At 1σ, the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ control chart detects first out-of-control signal at ninety-first order and in total 10 out-of-control signals with zero false alarm (see Table 9). Similarly, the classical CUSUM control chart detects first signal at ninety-fourth order and in total 3 out-of-control signals with 3 false alarms (see Table 9). Likewise, the classical EWMA control chart detects first signal at ninety-first order and in total 4 out-of-control signals, but with the cost of 1 false alarm (see Table 9). Correspondingly, the $\text{AEWMA}_{\text{C}}^{(1)}$ control chart detects first signal at ninety-second order and in total 3 out-of-control signals with 1 false alarms (see Table 9). It shows, the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ ($\lambda = 0.2$, $k = 0.5$, and $\gamma = 1.5$)

Table. 8: Numerical example of proposed $\text{ACUSUM}_{\text{c}}^{(1)}$ control chart along other control charts when $\text{ARL}_\text{o} = 500$

					$\text{ACUSUM}_{\text{c}}^{(1)}$ ($\lambda = 0.2$, $k = 0.5$, and $\gamma = 1.5$,)															
					False Alarm								1σ							
	x_i				$ACUSUM_{Ci}^{(1)+}$				$ACUSUM_{Ci}^{(1)-}$				$ACUSUM_{Ci}^{(1)+}$				$ACUSUM_{Ci}^{(1)-}$			
1	438	433	441	435	0.00	0.00	4.68	0.00	0.69	1.07	0.00	1.78	0.00	0.00	4.68	4.41	0.69	1.07	0.00	0.00
2	413	455	444	454	0.00	0.17	3.48	0.10	2.88	0.50	0.00	1.28	0.00	0.17	3.48	5.51	2.88	0.50	0.00	0.00
3	444	459	458	428	0.00	0.64	3.49	0.00	3.13	0.00	0.00	2.65	0.00	0.64	3.49	4.23	3.13	0.00	0.00	0.00
4	468	423	454	454	1.14	0.00	3.16	0.10	1.59	1.56	0.00	2.15	1.14	0.00	3.16	5.33	1.59	1.56	0.00	0.00
5	445	455	437	434	0.57	0.17	1.46	0.00	1.76	0.99	0.24	3.14	0.57	0.17	1.46	4.60	1.76	0.99	0.24	0.00
6	472	451	443	432	1.94	0.04	0.43	0.00	0.00	0.72	0.25	4.28	1.94	0.04	0.43	3.85	0.00	0.72	0.25	0.00
7	474	437	465	431	3.53	0.00	1.35	0.00	0.00	1.49	0.00	5.50	3.53	0.00	1.35	3.23	0.00	1.49	0.00	0.22
8	454	444	435	455	3.40	0.00	0.00	0.17	0.00	1.74	0.57	4.93	3.40	0.00	0.00	4.40	0.00	1.74	0.57	0.00
9	455	453	444	447	3.41	0.02	0.00	0.00	0.00	1.32	0.82	4.95	3.41	0.02	0.00	4.98	0.00	1.32	0.82	0.00
10	449	434	457	454	2.89	0.00	0.32	0.10	0.00	2.31	0.10	4.45	2.89	0.00	0.32	6.08	0.00	2.31	0.10	0.00
11	450	454	444	435	2.68	0.10	0.00	0.00	0.00	1.81	0.35	5.37	2.68	0.10	0.00	5.54	0.00	1.81	0.35	0.00
12	450	448	471	425	2.48	0.00	1.33	0.00	0.00	1.76	0.00	6.88	2.48	0.00	1.33	4.23	0.00	1.76	0.00	0.41
13	450	435	471	449	2.28	0.00	2.69	0.00	0.00	2.68	0.00	6.75	2.28	0.00	2.69	4.96	0.00	2.68	0.00	0.00
14	459	432	458	449	2.75	0.00	3.08	0.00	0.00	3.82	0.00	6.62	2.75	0.00	3.08	5.69	0.00	3.82	0.00	0.00
15	466	441	459	452	3.74	0.00	3.52	0.00	0.00	4.29	0.00	6.27	3.74	0.00	3.52	6.64	0.00	4.29	0.00	0.00
16	470	452	449	471	5.03	0.00	2.94	1.33	0.00	3.94	0.00	4.48	5.03	0.00	2.94	9.00	0.00	3.94	0.00	0.00
17	457	465	462	458	5.35	0.92	3.63	1.73	0.00	2.62	0.00	3.68	5.35	0.92	3.63	10.21	0.00	2.62	0.00	0.00
18	441	466	460	445	4.14	1.91	4.18	1.14	0.14	1.23	0.00	3.83	4.14	1.91	4.18	10.14	0.14	1.23	0.00	0.00
19	450	473	445	463	3.94	3.42	3.23	1.91	0.00	0.00	0.00	2.66	3.94	3.42	3.23	11.53	0.00	0.00	0.00	0.00
20	445	471	437	423	3.37	4.78	1.73	0.00	0.17	0.00	0.44	4.10	3.37	4.78	1.73	9.69	0.17	0.00	0.44	0.17
21	487	464	461	451	5.57	5.29	2.35	0.00	0.00	0.00	0.00	3.83	5.57	5.29	3.26	10.08	0.00	0.00	0.00	0.00
22	430	478	453	440	3.40	6.97	2.37	0.00	0.81	0.00	0.00	4.38	3.40	6.97	4.28	9.57	0.81	0.00	0.00	0.00
23	446	446	452	442	2.90	5.87	2.32	0.00	0.91	0.00	0.00	4.78	2.90	5.87	5.23	9.22	0.91	0.00	0.00	0.00
24	450	459	438	441	0.00	5.83	1.23	0.00	0.00	0.00	0.69	5.25	0.00	5.83	4.78	8.81	0.00	0.00	0.00	0.00
25	456	464	445	439	0.25	6.16	0.66	0.00	0.00	0.00	0.86	5.87	0.25	6.16	5.08	8.25	0.00	0.00	0.00	0.00

Table. 9: Detection ability of different control charts

Control charts	λ	k	γ/L	out-of-control	False alarms	First Signal point order	δ control points
$\text{ACUSUM}_{\text{C}}^{(1)}$	0.2	0.5	1	3	1	92nd	1σ
	0.2	0.5	1.5	10	0	91st	1σ
$\text{ACUSUM}_{\text{C}}^{(2)}$	0.2	0.5	3.5	1	1	94th	1σ
	0.2	0.5	4	2	1	92nd	1σ
Classical EWMA	0.2	n/a	2.962	4	1	92nd	1σ
	0.3	n/a	3.02	4	1	91st	1σ
Classical CUSUM	n/a	0.5	5.08	3	3	94th	1σ
$\text{AEWMA}_{\text{C}}^{(1)}$ CUSUM error	0.2	0.2	3.6565	3	1	92nd	1σ
$\text{ACUSUM}_{\text{C}}^{(2)}$	0.2	0.5	1	27	1	73rd	3σ
	0.2	0.5	1.5	30	0	71st	3σ
$\text{ACUSUM}_{\text{C}}^{(2)}$	0.2	0.5	3.5	29	1	72nd	3σ
	0.2	0.5	4	29	1	72nd	3σ
Classical EWMA	0.2	n/a	2.962	29	1	72nd	3σ
	0.3	n/a	3.02	29	1	72nd	3σ
Classical CUSUM	n/a	0.5	5.08	29	1	72nd	3σ
$\text{AEWMA}_{\text{C}}^{(1)}$	0.2	0.2	3.6565	30	1	71st	3σ

control chart performs better (see Figs. 10a-10c). Besides, other control charts have either higher detection order, less total detection points, or high false alarm (see Table 9). In addition, at 3σ, the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ ($\lambda = 0.2$, $k = 0.5$, and $\gamma = 1.5$) control chart keeps the superiority (see Table 9). In Fig. 10c the jump of statistic $\text{ACUSUM}_{C71}^{(1)+}$ at 71 order is due to adaptive nature of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ control chart which leads to superiority. The objective of the study is to detect different sizes of shift with an adaptive idea in the process location. So, it shows the edge over other control charts with adaptive idea in detecting out-of-control signals for real-life data as well. The changes in warpage, grinding, rigidity, coating and chemical vapour deposition methods, and other environmental condition factors might be caused to disturb the metal layer thickness and give out-of-control signals. These kinds of abnormalities may be identified by using appropriate techniques from SPC such as newly proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts.

7 Summary, Conclusion, and Recommendation

From last few years, an adaptive CUSUM (ACUSUM) control chart became famous to detect different sizes of shift in the process location. The ACUSUM control chart using classical EWMA statistic and score function, symbolized as ACUSUM_{E} control chart, is advanced form of the classical CUSUM control chart. The ACUSUM_{E} control chart is efficient to identify different sizes of shift, but the classical EWMA statistic does not help to differentiate a precise shift [20] as the classical CUSUM statistic can do. So, this study proposed two ACUSUM control charts, symbolized as $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ to further improve the detection ability of a specific as well as for different sizes of shift in the process location. Methodologies of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts are based on score (Huber and Bi-square) functions [5] and the classical CUSUM statistic [2]. In more details, some vital points are given as below.

1. The proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts initially adaptively renew the reference parameters values using classical CUSUM statistic and then to assign a weight on it using score functions.
2. Performance of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts depends on the optimal choices of k, λ, and γ parameters.
3. It is importance to mentioned here, the classical CUSUM control chart is a special case of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ control chart when λ=K. Algorithms are developed in MATLAB to obtain numerical results using Monte Carlo simulation technique.
4. Performance measures such as average run length for a specific shift, extra quadratic loss, relative average run length, and performance comparison index for overall performance are used to evaluate the performance of the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts against other control charts.
5. The analysis reveals, the proposed $\text{ACUSUM}_{\text{C}}^{(1)}$ and $\text{ACUSUM}_{\text{C}}^{(2)}$ control charts outper-

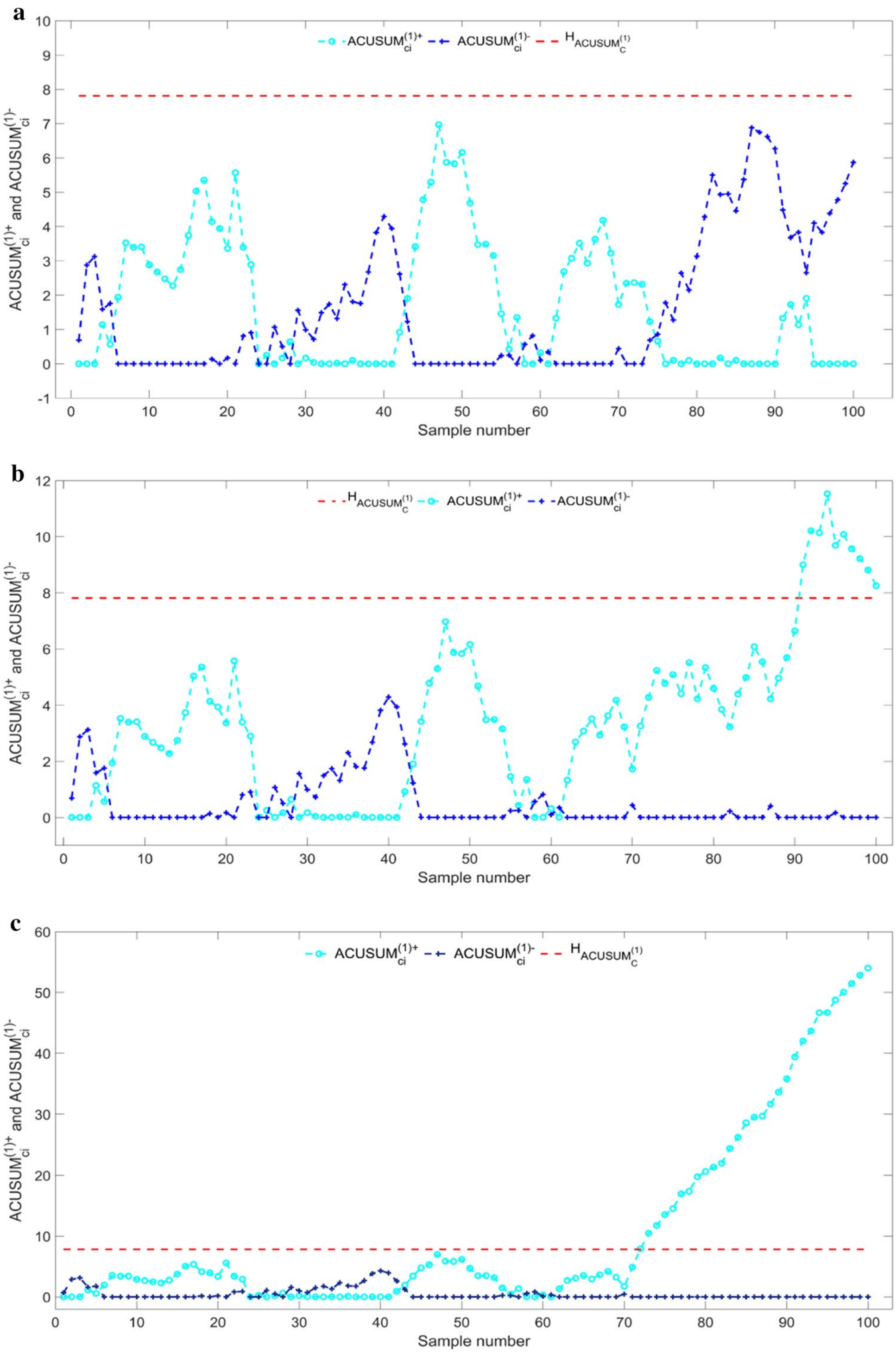


Fig. 10 **a** Graphical presentation of proposed ACUSUM$_c^{(1)}$ control chart with real-data at false alarm. **b** Graphical presentation of proposed ACUSUM$_c^{(1)}$ control chart with real-data by adding 1σ to last 30 observations. **c:** Graphical presentation of proposed ACUSUM$_c^{(1)}$ control chart with real-data by adding 3σ to last 30 observations

form as compared to existing (classical EWMA and CUSUM, MCE, MEC, AEWMA$_C$, AEWMA$_E$, and Hybrid EWMA) control charts.

6. Besides, the superiority of the proposed ACUSUM$_C^{(1)}$ and ACUSUM$_C^{(2)}$ control charts is also endorsed by real-life data of semiconductor wafer manufacturing industry.

The scope of the proposed study may be extended in other directions such as multivariate, non-parametric, and rankest sampling techniques.

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