

**INTERPOLATION AND MAPPING OF THE TOTAL ELECTRON
CONTENT OVER THE MALAYSIAN REGION**

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ABSTRACT

In the vast areas of satellite-related applications, the ionosphere is the main cause of error due to the Total Electron Content (TEC), which causes the ionospheric delay. As a result, estimating and mapping the TEC is vital for the application of various satellite-related fields, which are gaining momentum in Malaysia. Thus, this study aims to develop an efficient approach in mapping the TEC over Malaysian region using interpolation method. The TEC was mapped using three interpolation methods, namely inverse distance weighting (IDW), multiquadric and sphere multiquadric for different size of study area, different distribution and quantity of reference points. All the results from these three methods were compared with the TEC derived from IRI-2001 model and calculated for the root mean square (RMS) values. This study found that the effects of the quantity and distribution of reference points were conspicuous in the results obtained via both multiquadric and sphere multiquadric methods, whereas IDW was not conspicuously affected by both factors. This can be seen from the RMS values obtained from the IDW for both the well-distributed and randomly distributed two reference points were 3.1894 and 6.1681, whereas for the 18 reference points, the RMS yielded were 2.2436 and 2.5748 respectively. Furthermore, the results, especially via the multiquadric and IDW methods were more accurate in smaller study area's size, where the RMS yielded were 0.4685 and 0.0649 respectively for the $1^\circ \times 1^\circ$ area size. However, IDW seemed to consistently generate the more accurate result for all the study area size, where the RMS generated for all the area sizes studied ranges from 0.0649 to 1.2178. As a conclusion, the IDW method is the most suitable interpolation for the Malaysian region.

ABSTRAK

Dalam bidang aplikasi satelit yang luas ini, lapisan ionosfera merupakan punca utama berlakunya fenomena kelewatan ionosfera yang disebabkan oleh kandungan elektron penuh dan menyebabkan ralat bacaan. Oleh itu, penganggaran dan pemetaan kandungan elektron penuh amat penting untuk kegunaan pelbagai bidang aplikasi satelit, yang semakin mendapat peranan penting di Malaysia. Oleh sebab itu, tujuan kajian ini adalah untuk mengkaji kaedah paling berkesan untuk memetakan kandungan elektron penuh di Malaysia dengan menggunakan kaedah interpolasi. Kandungan elektron penuh dipetakan dengan menggunakan tiga kaedah interpolasi, iaitu kaedah *inverse distance weighting (IDW)*, *multiquadric* dan *sphere multiquadric* untuk pelbagai saiz kawasan kajian, dan pelbagai taburan serta kuantiti titik rujukan. Kesemua keputusan yang diperolehi daripada ketiga-tiga kaedah ini akan dibandingkan dengan kandungan elektron penuh yang diperolehi daripada model IRI 2001 untuk pengiraan nilai punca kuasa dua min (*Root Mean Square, RMS*). Kajian ini mendapati bahawa kesan kuantiti dan taburan titik-titik rujukan terhadap keputusan yang diperolehi melalui kaedah *multiquadric* dan *sphere multiquadric* amat jelas, manakala *IDW* tidak banyak dipengaruhi oleh kedua-dua faktor ini. Ini boleh dilihat dari nilai RMS yang diperolehi melalui kaedah *IDW* untuk kedua-dua taburan serata dan berselerak bagi dua titik rujukan iaitu 3.1894 dan 6.1681, manakala bagi 18 titik rujukan, RMS yang diperolehi masing-masing adalah 2.2436 dan 2.5748. Tambahan pula, keputusan-keputusan yang diperolehi terutamanya melalui kaedah *multiquadric* dan *IDW* adalah lebih jitu bagi kawasan kajian yang lebih kecil, di mana RMS yang diperolehi untuk saiz kajian $1^\circ \times 1^\circ$ adalah 0.4685 and 0.0649. Tetapi, *IDW* lebih menunjukkan kejituan yang konsisten bagi kesemua saiz kawasan kajian, di mana RMS yang diperolehi untuk semua saiz kawasan kajian adalah dari 0.0649 ke 1.2178. Kesimpulannya, kaedah *IDW* merupakan kaedah interpolasi yang paling sesuai untuk wilayah Malaysia.

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CHAPTER 1

INTRODUCTION

1.1 Overview on the Ionosphere

1.1.1 Definition of Ionosphere

The ionosphere is the ionized part of the atmosphere that extends from an altitude of around 50 km to more than 1000 km above the earth surface. The ionosphere is electrically neutral although there are a significant number of free thermal electrons and positive ions inside it. The term ionosphere was first used in 1926 by Sir Robert Watson-Watt in a letter to the secretary of the British Radio Research Board. The expression came into wide use during the period 1932-34 in papers and books. Before the term ionosphere gained worldwide acceptance, it was called the Kennelly-Heaviside layer, the upper conducting layer or ionized upper atmosphere (Hunsucker, 1991).

The activity of ionosphere will vary with altitude, latitude, longitude, universal time, season, solar cycle and magnetic activity. This variation is reflected in all ionospheric properties, such as electron density, ion and electron temperatures and ionospheric composition and dynamics. Normally the level of ionospheric activity is generally described in terms of electron density. The interaction of solar

radiation (the ultra-violet radiation of the Sun) or charged particles (such as X-rays and cosmic rays) with the Earth's atmosphere drives the ionospheric behaviour.

1.1.2 Structure of Ionosphere

The structure of the ionosphere is very complex due the physical and chemical processes within it. The sun's extreme ultraviolet (EUV) light, cosmic radiation and X-ray emissions encountering gaseous atoms and molecules in the atmosphere can impart enough energy for photo ionization to occur producing positively charged ions and negatively charged free electrons. Process recombination occurs in the ionosphere when the ions and electrons join again producing neutral atoms and molecules. But in the lower regions of the ionosphere, a process called attachment occurs when the free electrons combine with neutral atoms to produce negatively charged ions. The absorption of EUV light increases as altitude decreases. Due to the absorption and the increasing density of neutral molecules, a layer of maximum electron density is formed.

However, since there are many different atoms and molecules in the ionosphere and each of it have different rates of absorption, a series of distinct layers or regions of electron density exist. These are denoted by the letters D, E, F1 and F2, which are usually are collectively referred to as the bottom side of the ionosphere. The part of the ionosphere between the F2 layer and the upper boundary of the ionosphere is termed the topside of the ionosphere. Figure 1.1 below depicts the structure of the ionosphere.

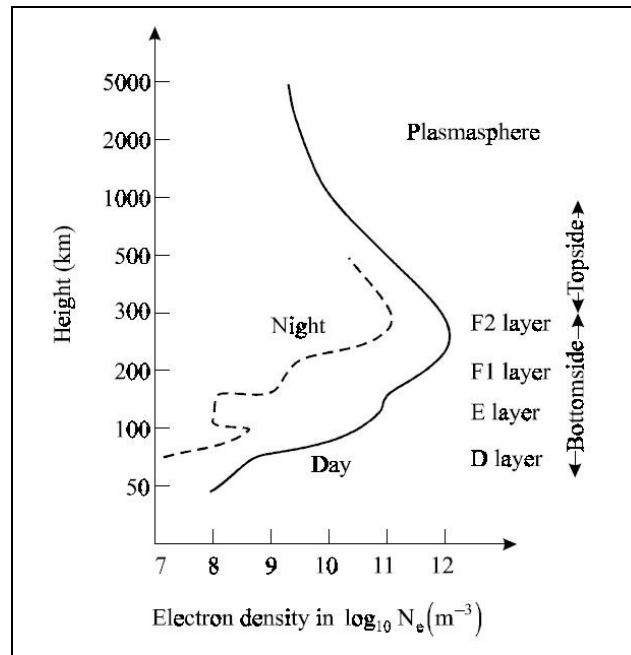


Figure 1.1: Structure of Ionosphere (Komjathy, 1997).

1.1.3 Characteristics of Ionosphere Structures

According to Komjathy (1997) the D layer extends from about 75 to 90 km above the earth. D-layer ionization is produced by solar UV light, X-rays and cosmic radiation at any time of day or night. Due to this, the electrons may become attached to molecules and atoms forming negative ions that cause the D layer to disappear, at night time. While during the day time, the electrons tend to detach themselves from the ions causing the D layer to reappear, as the consequence of sun's radiation. Since the electrons in D layer at the altitude of about 60 to 70 km are present by day but not by night, it causes a distinct diurnal variation in the electron density.

In Davies (1990), the lower part of D layer was referred to as the C layer where the cosmic radiation is the only source of ionization compared to the middle and

upper part of the D layer where both the cosmic radiation and X-ray emissions are present (Komjathy, 1997).

According to the National Oceanic and Atmospheric Administration (NOAA), the E layer extends from about 95 to 150 km above the earth. Since the ionization mostly depends on the level of solar activity and the zenith angle of the sun, ionization drops to low values at night. Although the E layer does not completely vanish at night, however, for practical purposes it is often assumed that its electron density drops to zero at night. Due to that, the E layer is said to be only present by day time. In that respect, the primary source of ionization is the sun's X-ray emissions, causing the electron densities in E layer showing distinct solar-cycle, seasonal and diurnal variations. According to NOAA, the E-layer effects are noted to be irregular. Other subdivisions of the E-layer, after isolating the irregular occurrence within this region into separate layers, are also labelled with an E prefix. These layers are the thick layer, E2, and a highly variable thin layer, Sporadic E. Ions in these regions consist of mainly O_2^+ .

The F1 layer is the lower part of the F layer, which extends from about 170 to 250 km above the earth. The main source of ionization in the F1 layer is the EUV light while the electron densities are primarily controlled by the zenith angle of the sun. Because of this, the F1 layer exists only during daylight hours and will disappear at night, so it's only observed during the day time. When it is present, it changes rapidly in a matter of minutes. During typical noon time, the mid-latitude electron densities range between 2.3×10^{11} and 3.3×10^{11} electrons/m³ in according to the solar activity (Komjathy, 1997).

From about 250 to 500 km above the earth is the region of the F2 layer. This layer is present 24 hours a day but varies in altitude with geographical location, solar activity, and local time. The critical frequency for this layer peaks after local noon time and decreases gradually, thus showing a linear dependency of the F2 layer on the number of solar sunspot. Here, the typical mid-latitude noon time electron

densities range between 2.8×10^{11} and 5.2×10^{11} electrons/m³ in according to the solar activity. The global spatial distribution of this layer also reveals a strong geomagnetic dependence rather than the solar zenith angle dependence (Komjathy, 1997).

The top side of the ionosphere starts at the height of maximum density of the F2 layer of the ionosphere and extends upward with decreasing density to a transitional height where O⁺ ions become less numerous than H⁺ and He⁺. The transition height varies but seldom drops below 500km at night or 800km during daytime, although it may lie as high as 1100km (<http://www.ngdc.noaa.gov/stp/IONO/ionostru.html>).

From above, the existence of the D, E, and F1 layers are noted to be primarily controlled by the solar zenith angle and showing a strong diurnal, seasonal and latitudinal variation. The diurnal variation of the D, E, and F1 layers also implies that they tend to reduce greatly in size or even will vanish at night time. In contrast, the F2 layer is present for 24 hours and is where the maximum electron density usually occurs. This happens as a consequence of the combination of the absorption of the EUV light and increase of neutral atmospheric density as the altitude decreases. Thus, this layer is commonly taken into consideration to represent the whole layer of the ionosphere, during the calculations of the ionospheric delay.

1.1.4 Effects of Ionosphere Delay on Satellite Applications

The ionosphere is one of the main sources of error in the vast areas of satellite-related applications. Those satellite applications that will be affected by

the ionospheric effect are those which are directly related to satellite signal transmission system, such as navigational satellite operators example U.S. Global Positioning System (GPS), Russian Navigational System (GLONASS) and European Navigational System (Galileo). Other than that, radio and television operations utilising satellite communication; space weather forecasts; space and aero industries and the military are also significantly affected. In the space and aero industries, the ionosphere may affect the spacecraft designs, its internal and surface charging, sensor interference, satellite anomalies, loss of navigational signal phase and amplitude lock, besides affecting the planning of the electromagnetic environment in manned spacecraft's travels.

In the military, the ionospheric effect made its presence felt in terms of space communication and navigation as it causes loss of high frequency (HF) communications and direction finding, causes clutter in the horizon radar; disrupts targeting and extra or very low frequency (ELF/VLF) communications with submarines, besides contributed to reduced detection of missile launch. Furthermore, scientists using remote sensing measurement techniques -in astronomy, biology, geology, geophysics, seismology and many more fields were very much affected where discrepancies in their readings arises and thus compensation of the effects of the ionosphere on their observations are needed. Besides, those applications which does not utilise the satellite system but only involve signal and wave transmissions will be affected as well. For example, phone communication where it causes possible interference; radio communication agencies and amateur radio operators where efficiency of communication is compromised.

Irregularities of the effect of the ionospheric delay can range from a few meters to a few kilometres. It will scatter satellite radio signals when it is propagated through the ionosphere. This will lead to rapid phase and amplitude fluctuations and also variations in the angle of arrival and polarization of the radio signals, which are collectively known as ionospheric scintillation. Among the ionospheric scintillation that is important in satellite application are the amplitude scintillation and phase scintillation.

Amplitude scintillations, can reach 20 dB at 1.5 GHz during high solar activity times (Bishop *et al.*, 1996). It induces signal fading and cycle slips. When the signal fading exceeds the fade margin or the threshold limit of a receiving system, message errors in satellite communications are encountered and loss of lock occurs in navigational systems. The amplitude scintillation typically can last for several hours in the evening time, which is broken up with intervals of no fading in between (Klobuchar, 1991).

Phase scintillations will cause Doppler shifts and may degrade the performance of phase-lock loops. For example, in GPS navigation systems, the Doppler shifts caused by TEC variations can be up to 1-Hz/second, which may cause some narrow-band receivers to lose lock on the signals. This is due to the rapid frequency changes in the received signals, which are greater than the receiver bandwidth. The phase scintillation also may affect the resolution of space-based synthetic aperture radars. The duration of strong phase scintillation effects are limited from approximately one hour after local sunset to local midnight (Klobuchar, 1991).

Although the ionospheric scintillation is unlikely to affect all of the satellite in a receiver's field of view, they will have impact on the accuracy of the result of the navigation solution by degrading the geometry of the available constellation. This is the most severe test on a GPS receiver in the natural environment. Consequently, the coverage of both the satellites and the irregularities as well as the intensity of scintillation activity will all contribute to the accuracy of the result of the final solution (Klobuchar, 1991).

1.2 Problem Statement

Estimating and mapping the ionospheric delay for the whole country are being performed on a real time basis by many developed countries in the world. The time series of this type of map can be used to derive average monthly maps describing major ionospheric trends as a function of local time, season and spatial location (Weilgosz *et al*, 2003a). By analyzing these maps, ionospheric forecasting and broadcasting can be done and applied to many related fields or researches.

Unfortunately, this capability is currently absent in Malaysia where being in the tropical region, such information are vital for many applications. This research work will explore and develop the basic infrastructure such as ionospheric content computation, suitable interpolation method and finally mapping of the Total Electron Content (TEC) of the Malaysian region.

1.3 Research Objectives

The objectives of this research are:

1. To develop an efficient approach in mapping TEC over a regional area using interpolation methods.
2. To define the most suitable interpolation method for mapping the TEC over the Malaysian region.

1.4 Research Scope

The scope of this research is confined to the following areas:

1. Mapping the TEC over a regional area using an efficient interpolation method. The interpolation methods used here focus only on the inverse distance weighting, multiquadric and sphere multiquadric, as both the inverse distance weighting and multiquadric methods are among the commonly used method. Whereas sphere multiquadric method were included to study a method which interpolates from a spherical plane as opposed to the commonly used flat plane in the two former methods.
2. Producing a regional TEC map and numerical result that can be utilised for many satellite based observation applications.
3. TEC data from the model International Reference Ionosphere 2001 (IRI-2001) -an empirical model of the ionosphere based on all available data sources; please refer to section 4.2 for more details- rather than real observations were used in the interpolation processing step.
4. The research results were analysed using the Root Mean Square (RMS) method.
5. The grid size of the output TEC map is $0.5^{\circ} \times 0.5^{\circ}$.

1.5 Significance of Research

This research has its significance in terms of establishing the superiority of different interpolation methods in the mapping of the ionospheric effect on the

Malaysian region. This research also has its significance in terms of the development of a program to estimate the ionospheric effect for the whole of Malaysia, using the interpolation method which was found to be more suitable in the Malaysian context. The program is an alternative way for the amateur satellite users, especially Global Positioning System (GPS) single frequency users to compute the ionosphere errors in the satellite signals. Besides that, this program helps in terms of cost saving, as it provides an alternative to replace the expensive commercial software, such as Trimble Geomatics Office, Leica SKI-Pro, Waypoint GrafNav, EZSurv and RADAN.

1.6 Study Area

The study area for this research was the whole region of Malaysia. The coordinate of the study area ranges from 0°N to 8°N latitude and 99°E to 120°E longitude, which can be seen in Figure 1.2 below:

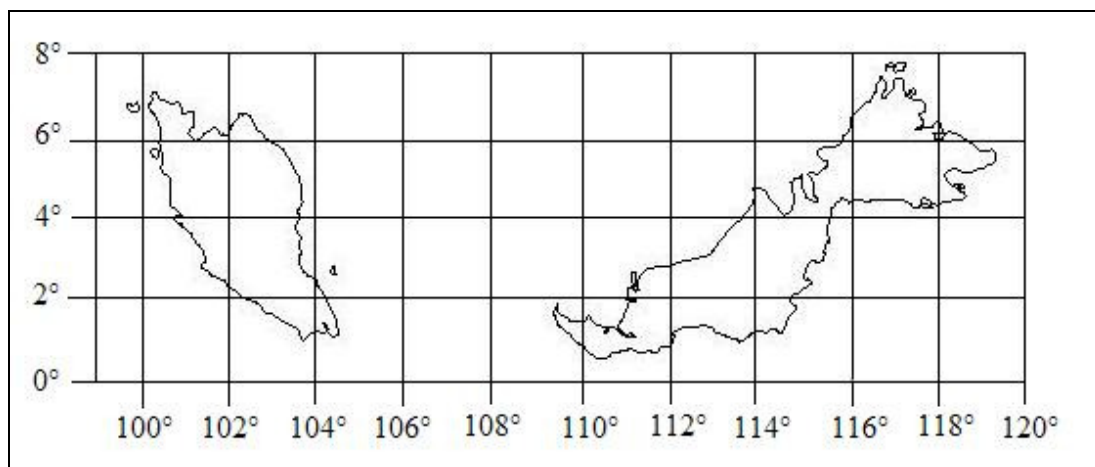


Figure 1.2: Study Area.

REFERENCES

- Adam, O. and Léonard, O. (2002). Explicit Thermodynamic Properties using Radial Basis Functions Neural Networks, *Proceedings of the 2nd SIAM International Conference on Data Mining*. 11-13 April. Arlington, VA, USA.
- Ahrens, B. (2005). Interactive Comment on “Distance in Spatial Interpolation of Daily Rain Gauge Data”. *Hydrology and Earth System Sciences Discussions*, 2, 967-970.
- ArcGIS: The Complete Enterprise GIS. (2006). <http://www.esri.com/software/arcgis/>
- Barnes, R.I., Gardiner-Garden, R.S. and Harris, T.J. (2000). Real Time Ionospheric Models for the Australian Defence Force. *Proceedings of WARS'00 (Workshop on the Applications of Radio Science)*. 27-29 April. La Trobe University, Beechworth.
- Bartier, P.M. and Keller, C.P. (1996). Multivariate Interpolation to Incorporate Thematic Surface Data using Inverse Distance Weighting (IDW). *Computers & Geosciences*, 22(7), 195-199.
- Baxter, B.J.C. (1992). *The Interpolation Theory of Radial Basis Functions*. Ph.D. Thesis. Cambridge University, U.K.

- Bertoni, F., Sahai, Y., Limal, W.L.C., Fagundes, P.R., Pillat, V.G; Becker-Guedes, F., Abalde, J.R. (2006). IRI-2001 Model Predictions Compared with Ionospheric Dates Observed at Brazilian Low Latitude Station. *Annales Geophysicae*, 24, 2191-2200.
- Bilitza, D. (ed.) (1990). *International Reference Ionosphere 1990*. Greenbelt, Maryland: NSSDC 90-22.
- Bilitza, D. (2001). International Reference Ionosphere 2000. *Radio Science*, 36(2), 162-175.
- Buhmann, M.D. (2003). *Radial Basis Functions: Theory and Implementations*. Cambridge, U.K.: Cambridge University Press.
- Chaplot, V., Darboux, F., Bourennane, H., Leguédois, S., Silvera, N. and Phachomphon, K. (2006). Accuracy of Interpolation Techniques for the Derivation of Digital Elevation Models in Relation to Landform Types and Data Density. *Geomorphology*, 77, 126-141.
- Chen, R.H. and Wu, Z.M. (2006). Applying Multiquadric Quasi-Interpolation to Solve Burgers' Equation. *Applied Mathematics and Computation*, 172(1), 472-484.
- Cherrie, J.B., Beatson, R.K. and Newsam, G.N. (2002a). Fast Evaluation of Radial Basis Functions: Methods Multiquadrics in R_n . *SIAM Journal of Scientific Computing*, 23(5), 1549-1571.
- Cherrie, J.B., Beatson, R.K. and Newsam, G.N. (2002b). Fast Evaluation of Radial Basis Functions: Methods for Generalized Multiquadrics in R_n . *SIAM Journal on Scientific Computing*, 23(5), 1549-1571. SICS
- Cilliers, P.J., Coetzee, P.J., Olckers, J.M. (2004). Ionospheric Mapping for HF Communications and HF Direction Finding. *Proceedings of AFRICON 2004*. 15-17September. Gaborone, Botswana, 145-154.

- Cilliers, P.J., Opperman, B.D.L. (2005). Towards Near Real-time TEC Mapping over South Africa, *Proceedings of the 50th Annual Conference of the South African Institute of Physics*, 5-7 July. Pretoria, South Africa.
- Collins F.C. Jr., and Balstad, P.V. (1996). A Comparison of Spatial Interpolation Techniques in Temperature Estimation. *Proceedings of Third International. Conf. /Workshop on Integrating GIS and Environmental Modelling*. 21-25 January. Santa Fe, New Mexico.
- da Costa, A.M., Boas, J.W.V. and da Fonseca E.S. Jr. (2004). GPS Total Electron Content Measurements at Low Latitudes in Brazil for Low Solar Activity. *Geofísica Internacional*, 43(1), 129-137.
- Das, A.K., Mohanty, U.C., Das, S., Mandal, M. and Kalsi, S.R. (2003). Circulation Characteristics of a Monsoon Depression during BOBMEX-99 using High-Resolution Analysis. *Proceedings of the Indian Academy of Sciences: Earth and Planetary Sciences 112*. June. India, 165–184
- Davis, J.C. (1986). *Statistics and Data Analysis in Geology*. New York: Wiley.
- Definition of the Ionospheric Regions (Structures). (2006).
<http://www.ngdc.noaa.gov/stp/IONO/ionostru.html>
- Ding, H., Shu, C. and Tang, D.B. (2005). Error Estimates of Local Multiquadric-Based Differential Quadrature (LMQDQ) Method Through Numerical Experiments. *International Journal for Numerical Methods Engineering*, 63, 1513-1529.
- Ducic, V., Artru, J., Murakami, M. and Lognonn, P. (2002). Imagery of the Ionosphere using Dense GPS Networks. *Proceedings of International Union of Geophysics and Geodesy general assembly*. 30 June-22 July. Sapporo, Japan.

- Erol, B. and Çelik R.N. (2004). Modelling Local GPS/Levelling Geoid with the Assessment of Inverse Distance Weighting and Geostatistical Kriging Methods. *Proceedings of Geo-Imagery Bridging Continents XXth ISPRS Congress*. 12-23 July. Istanbul, Turkey.
- Escudero, A., Schlesier, A.C., Rlus, A., Flares, A., Rubek, F., Larsen, G.B., Syndergaard, S. and Hseg, p. (2001). Ionospheric Tomography using Orsted GPS Measurements - Preliminary Results. *Physics and Chemistry of the Earth (A)*, 26(3), 173-176.
- Essex, E.A. (2002). GPS For Ionospheric Sensing: Space and Ground Based. *Journal of Global Positioning Systems*, 1(1), 66-67.
- Fedoseyev, A.I., Friedman, M.J. and Kansa, E.J. (2000). Continuation for Nonlinear Elliptic Partial Differential Equations Discretized by the Multiquadric Method. *International Journal of Bifurcation and Chaos*, 10(2), 481-492.
- Fedrizzi, M., Langley, R.B., Komjathy, A., Santos, M.C., de Paula, E.R. and Kantor, I.J. (2001). The Low-Latitude Ionosphere: Monitoring its Behaviour with GPS. Institute of Navigation. *Proceedings of ION GPS -2001*. 11-14 September, Salt Lake City, 2468-2475.
- Feltens, J. and Jakowski, N. (2001). The International GPS Service (IGS) Ionosphere Working Group Activities. *Proceedings of Antarctic Geodesy Symposium 2001*. 17-21 July. Saint-Petersburg, Russia.
- Ferreira, A.J.M. (2003). A Formulation of the Multiquadric Radial Basis Function Method for the Analysis of Laminated Composite Plates. *Composite Structures*, 59(3), 385-392.
- Ferreira, A.J.M., Martins, P.A.L.S. and Roque, C.M.C. (2005a). Solving Time-Dependent Engineering Problems with Multiquadrics. *Journal of Sound and Vibration*, 280, 595-610.

- Ferreira, A.J.M., Roque, C.M.C. and Jorge, R.M.N. (2005b). Analysis of Composite Plates by Trigonometric Shear Deformation Theory and Multiquadrics. *Computers and Structures*, 87(27), 2225-2237.
- Fogel, D.N. (1996). Image Rectification with Radial Basis Functions: Application to RS/GIS Data Integration. *Proceedings of the Third International Conference on Integrating GIS and Environmental Modelling*. 21-26 January. Santa Fe, New Mexico, 1-19.
- Fogel, D. N. and Tinney, L. R. (1996) Image. Registration using Multiquadric Functions, the finite Element Method, Bivariate Mapping Polynomials and Thin Plate Spline. Technical Report, *National Center for Geographic Information and Analysis*. March. University of California, Santa Barbara.
- Fornberg, B., Flyer, N., Hovde, S. and Piret, C. (2007). Locality Properties of Radial Basis Function Expansion Coefficients for Equispaced Interpolation. *IMA Journal of Numerical Analysis*, Advance Access published on July 16, 1-25. doi:10.1093/imanum/drm014.
- Franke, R. (1982). Scattered Data Interpolation: Tests of Some Methods. *Journal of Computational Mathematics*, 48, 181-200.
- Gao, Y., Heroux, P. and Kouba, J. (1994). Estimation of GPS Receiver and Satellite L1/L2 Signal Delay Biases using Data from CACS. *Proceedings of KIS94, the International Symposium on Kinematic Systems in Geodesy, Geomatics, and Navigation*. 30 August-2 September. Banff, Alberta, Canada, 109-117.
- Gao, Y. and Liu, Z.Z. (2002). Precise Ionosphere Modeling using Regional GPS Network Data. *Journal of Global Positioning Systems*, 1(1), 18-24.
- Garcia, R., Crespon, F., Godet, P.E., Jeansou, E., Moreaux, G., Helbert, J., Lognonn, P. (2006). <http://ganymede.ipgp.jussieu.fr/spectre> :Continuous 2D Monitoring of the Ionosphere by GPS Data, 3D Tomographic Developments, Database Evaluation and Applications. *Geophysical Research Abstracts*, 8.

- Garcia, R., Ducic, V., Crespon, F., Helbert, J., Moreaux, G. and Lognonn, P. (2004). 3D Ionospheric Model and Postseismic Atmospheric Signals from GPS Data. *Geophysical Research Abstracts*, 6.
- Godefa, A. (2006). *Spatial Interpolation*.
http://www-users.cs.umn.edu/~godefa/CSci8701/E3_I.doc
- Gold, C.M. (1988). Point and Area Interpolation and the Digital Terrain Model. *Proceedings Trends and concerns of spatial sciences, Second Annual International Symposium*. University of New Brunswick, Canada, 133-147.
- Gotway, C.A., Ferguson, R.B., Hergert, G.W. and Peterson, T.A. (1996). Comparison of Kriging and Inverse-Distance Methods for Mapping Soil Parameters. *Soil Science Society of America Journal*, 60, 1237-1247.
- Hajj, G.A., Lee, L.C., Pi, X. and Romans, L.J. (2000). COSMIC GPS Ionospheric Sensing and Space Weather. *Special issue of TAO (Terrestrial, Atmospheric and Oceanic Science)*, 11(1), 235-272.
- Hardy, R.L. (1971). Multiquadric Equations of Topography and Other Irregular Surfaces. *Journal of Geophysical Research*, 76, 1905-1915.
- Hardy, R.L. (1990). Theory and Applications of Multiquadric-Biharmonic Method. *Computers & Mathematics with Applications*, 19, 163-208.
- Hardy, R.L. and Goepfert, W.M. (1975). Least Squares Prediction of Gravity Anomalies, Geoidal Undulations, and Deflections of the Vertical Multiquadric Harmonic Functions, *Geophysical Research Letters*, 2, 423-426.
- Heise, S., Jakowski, N., Wehrenpfennig, A., Reigber, Ch. and König, R. (2001): Preliminary Results on Ionosphere/Plasmasphere Imaging Based on GPS Data Obtained Onboard CHAMP. *Proceedings of The International Beacon Satellite Symposium*. 3-6 June. Boston.

- Heise, S., Jakowski, N., Wehrenpfennig, A., Reigber, C. and Lühr, H. (2003). Initial Results on Ionosphere/Plasmasphere Sounding Based on GPS Data Obtained Onboard CHAMP. *First CHAMP Science Mission Results for Gravity, Magnetic and Atmospheric Studies*. Springer Verlag, Berlin, Heidelberg, New York, 408-414.
- Hernández-Pajares, M. (2003). *Performance of IGS Ionosphere TEC Maps*.
www.igs2004.unibe.ch/abstracts/Atmosphere_Iono.pdf
- Hernández-Pajares, M. (2004). IGS Ionosphere WG: An Overview. *Proceedings of Final Meeting COST271 Action*. 26-27 August. Abingdon, UK.
- Herpin U., Lieth H. and Markert B. (1995). *Monitoring der Schwermetallbelastung in der Bundesrepublik Deutschland mit Hilfe von Moosanalysen*. Berlin: UBA-Texte, 31/95.
- Hon, Y.C., Cheung, K.F., Mao, X.Z. and Kansa E.J. (1999). A Multiquadric Solution for Shallow Water Equation. *ASCE Journal of Hydraulic Engineering*, 125(5), 524-533. American Society of Civil Engineers
- Hunsucker, R.D. (1991). *Radio Techniques for Probing the Ionosphere*. New-York: Springer-Verlag.
- Ionosondes and Digisondes. (2006). <http://www.amfmdx.net/fmdx/ionosonde.html>
- Jakowski, N., Heise, S., Wehrenpfennig, A. and Schliiter, S. (2001). TEC Monitoring by GPS - A Possible Contribution to Space Weather Monitoring. *Physics and Chemistry of the Earth (C)*, 26(8), 609-613.
- Jiao, P.N. and Wu, J. (1996). Ionospheric Models and Data for the Chinese Subcontinent. *Advances in Space Research*, 18(6), 191-200.

- Jin, R., Chen, W. and Simpson, T. (2000). Comparative Studies of Metamodeling Techniques under Multiple Modeling Criteria. *Structural and Multidisciplinary Optimization*, 22(5), 1-13.
- Kansa, E.J. (1990a). Multiquadrics- A Scattered Data Approximation Scheme with Applications to Computational Fluid Dynamics: I. Surface Approximations and Partial Derivative Estimates. *Computers & Mathematics with Applications*, 19, 127-145.
- Kansa, E.J. (1990b). Multiquadrics- A Scattered Data Approximation Scheme with Applications to Computational Fluid Dynamics: II. Solutions to Parabolic, Hyperbolic, and Elliptic Partial Differential Equations. *Computers & Mathematics with Applications*, 19, 147-161.
- Kitsiou, D. and Karydis, M., (2000). Categorical Mapping of Marine Eutrophication Based on Ecological Indices. *The Science of the Total Environment*, 255, 113-127.
- Klobuchar, J.A. (1991). Ionospheric effects on GPS. *GPS World*, 2(4), 48-51.
- Komjathy, A. (1997). *Global Ionospheric Total Electron Content Mapping using the Global Positioning System*. Ph.D. Thesis. University of New Brunswick, Canada.
- Kravchenko, A. and Bullock, D.G. (1999). A Comparative Study of Interpolation Methods for Mapping Soil Properties. *Journal of Agronomy*, 91,393-400.
- Lam, N. (1983). Spatial Interpolation Methods: A Review, *The American Cartographer*, 10 (2), 129-149.
- Lammeren, R.V. (2002). *Chapter 15: Data Integration 3: Digital elevation models*. http://www.geo-informatie.nl/courses/grs10306/Lectures/2002/Ch15_IGI_GIS_DEM.doc

- Leandro, R.F.; Santos, M.C., Leandro, R.F., Santos, M.C. (2004). Regional Computation of TEC using A Neural Network Model. *Proceedings of American Geophysical Union, Spring Meeting 2004*. May. Montreal.
- Legates, D.R. and Willmont, C.J. (1990). Mean Seasonal and Spatial Variability in Global Surface Air Temperature. *Theoretical Application in Climatology*, 41, 11-21.
- Li, T.H. (1999). Multiscale Representation and Analysis of Spherical Data by Spherical Wavelets. *SIAM Journal on Scientific Computing*, 21(3), 924-953.
- Lin, L.S. (2002). Quality Control Issues on Real-time Estimation of Ionospheric Delay using GPS Measurements. *Proceedings of the 23rd Asian Conference on Remote Sensing*. 25-29 November. Kathmandu, Nepal, 146-152.
- Liu, C.T., Tsai, L.C. and Tsai, W.H. (2000). Computerized Ionospheric Tomography using the GPS/MET and NNSS Data. *Proceedings of the 23rd Asian Conference on Remote Sensing*. 25-29 November. Kathmandu, Nepal.
- Lloyd, C.D. (2005). Assessing the Effect of Integrating Elevation Data into the Estimation of Monthly Precipitation in Great Britain. *Journal of Hydrology*, 308, 128-150.
- Lloyda, C.D. and Atkinsonb, P.M. (2004). Increased Accuracy of Geostatistical Prediction of Nitrogen Dioxide in the United Kingdom with Secondary Data. *International Journal of Applied Earth Observation and Geoinformation*, 5, 293-305.
- Maguire, D.J., Goodchild, M.F. and Rhind, D.W. (Eds.) (1991). *Geographical Information Systems: Principles and Applications*. New York: Longman Scientific and Technical.

- Mannucci, A.J., Iijima, B., Sparks, L., Pi, X.Q., Wilson, B. and Lindqwister, U. (1999). Assessment of Global TEC Mapping using A Three-Dimensional Electron Density Model. *Journal of Atmospheric and Solar-Terrestrial Physics*, 61(16), 1227-1236.
- Mannucci, A.J., Wilson, B.D. and Edwards C.D. (1993). A New Method for Monitoring the Earth Ionosphere Total Electron Content using the GPS Global Network, *Proceedings of ION GPS'93*. 22-24 September. Salt Lake City, UT, 1323-1332.
- Martin, D. (1996). *Geographic Information Systems: Socioeconomic Applications*. (2nd ed.) London: Routledge.
- McDonald, D.B., Grantham, W.J. and Tabor, W.L. (2000). Response Surface Model Development for Global/Local Optimization using Radial Basis Functions. *Proceedings of the 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*. 6-8 September. Long Beach, CA.
- Meyer, C. (2006). Evaluating Water Quality using Spatial Interpolation Method. *Proceedings of the 2006 ESRI User Conference*. 7-11 August. Pinellas County, Florida, U.S.A.
- Meyer, F., Bamler, R., Jakowski, N. and Fritz, T. (2006). The Potential of Low-Frequency SAR Systems for Mapping Ionospheric TEC Distributions. *IEEE Geoscience and Remote Sensing Letters*, 3(4).
- Milillo, T. M. and Gardella, J.A. Jr. (2006). Spatial Statistics and Interpolation Methods for TOF SIMS Imaging. *Applied Surface Science*, 252, 6883-6890.
- Moeketsi, D.M., Combrinck, W.L., McKinnell, L.A., Fedrizzi, M. (2007). Mapping GPS-Derived Ionospheric Total Electron Content over Southern Africa during Different Epochs of Solar Cycle 23. *Advances in Space Research*, 39, 821-829.

- Mouat, C.T. and Beatson, R.K. (2002). RBF Collocation. *Research report UCDMS 2002/3*. Canterbury University, Christchurch, New Zealand.
- Nam, M. D. and Thanh, T. C. (2001). Numerical Solution of Differential Equations using Multiquadric Radial Basis Functions Networks, *Neural Networks*, 14, 185–199.
- Nuss, W.A. and Titley, D.W. (1994). Use of Multiquadric Interpolation for Meteorological Objective Analysis. *Monthly Weather Review*, 122, 1611-1631. American Meteorological Society.
- Olea, R.A. (1999). *Geostatistics for Engineers and Earth Scientists*. Dordrecht, The Netherlands: Kluwer Academic Publishers Group.
- Opperman, B.D.L, Cilliers, P.J., McKinnell, L.A. and Haggard, R. (2007). Development of a Regional GPS-Based Ionospheric TEC Model for South Africa. *Advances in Space Research*, 39, 808-815.
- Ping, J., Kono, Y., Matsumoto, K., Otsuka, Y., Saito, A., Shum, C., Heki, K. and Kawano, N. (2002). Regional Ionosphere Map Over Japanese Islands. *Earth, Planets and Space*, 54, 13-16.
- Pottmann, H. and Eck, M. (1990). Modified Multiquadric Methods for Scattered Data Interpolation over a Sphere. *Journal of Computer Aided Design*, 7, 313-321.
- Press, W. H., Flannery, B.P., Teukolsky, S.A. and Vetterling, W.T. (1992). *Numerical Recipes in C: The Art of Scientific Computing*. (2nd ed.) New York: Cambridge University Press.
- Reinstorf, F., Binder, M., Schirmer, M., Grimm-Strele, J., and Walther, W. (2005). Comparative Assessment of Regionalisation Methods of Monitored Atmospheric Deposition Loads, *Atmospheric Environment*, 39 (20), 3661-3674.

- Richard M. Jones, (1999). *Introduction to MFC Programming with Visual C++*. New Jersey: Prentice Hall.
- Robinson, T.P., Metternicht, G. (2005). Testing the Performance of Spatial Interpolation Techniques for Mapping Soil Properties. *Computers and Electronics in Agriculture*, 50, 97-108.
- Ruffini, G., Flores, A., Cardellach, E., Cucurull, L. and Rius, A. (1998). 4D Imaging of the Ionosphere using GPS Data and an Application to Ionospheric Calibration of Radar Altimeters. *Proceedings of ESA WPP-155, ESA Workshop on Space Weather*. 11-13 November. ESTEC, Noordwijk, The Netherlands.
- Salleh, S., Zomaya, A.Y., Olariu, S. and Sanugi B. (2005). *Numerical Simulations and Case Studies using Visual C++.NET*. Canada: John Wiley & Sons, Inc.
- Schaer, S. (1999). *Mapping and Predicting the Earth's Ionosphere using the Global Positioning System*. Ph.D. Thesis. University of Berne, Switzerland.
- Schmidt, M. (2001). *Grundprinzipien der Wavelet-Analyse und Anwendungen in der Geodäsie*. Shaker Verlag, Aachen: Habilitationsschrift.
- Schmidt, M., Bilitza, D., Shum, C.K. and Zeilhofer, C. (2007). Regional 4-D Modeling of the Ionospheric Electron Density. *Advances in Space Research*, doi:10.1016/j.asr.2007.02.050.
- Schlüter, S., Stolle, C., Jakowski, N. and Jacobi, C. (2003). Monitoring the 3 Dimensional Ionospheric Electron Distribution Based on GPS Measurements. *First CHAMP Mission Results for Gravity, Magnetic and Atmospheric Studies*. Springer Verlag, Berlin, Heidelberg, New York, 521-527.
- Spokas, K., Graff, C., Morcet, M. and Aran, C. (2003). Implications of the Spatial Variability of Landfill Emissions Rates on Geospatial Analyses, *Waste Management*, 23(7), 599-607.

- Stanislawski, I., Juchnikowski, G., Cander, L.R., Ciruolo, L., Bradley, P.A., Zbyszynski, Z. and Swiatek, A. (2002). The Kriging Method of TEC Instantaneous Mapping. *Advances in Space Research*, 29(6), 945-948.
- Technical Committee ISO/TC 20, *Aircraft and Space Vehicles*, Subcommittee SC 14, *Space Systems and Operations*. (2005). *Space Environment (Natural and Artificial)-Earth's Ionosphere model: International Reference Ionosphere and Extensions to the Plasmasphere*. www.oma.be/ISO/standards/ts16457.pdf
- Thiessen, A.H. (1998): Estimating Daily Regional Rainfall Fields with Multiquadric Functions: Accuracy of Interpolation and Decision Making, *Journal of Geographic Information and Decision Analysis*, 2(2), 168-181.
- Tsai, L.C., Liu, C.H., Tsai, W.H. and Liu, C.T. (2002). Tomographic Imaging of the Ionosphere using the GPS/MET and NNSS Data. *Journal of Atmospheric and Solar-Terrestrial Physics*, 64, 2003-2011.
- Tusat, E. and Turgut, B. (2004). Investigation Ambiguity Resolution in GPS and the Effects of Ionospheric Modeling on Base Vector Components. *Proceedings of FIG Working Week 2004*, 22-27 May. Athens, Greece, 1-11.
- Wackernagel, H. (1995). *Multivariate Geostatistics*. New York: Springer-Verlag.
- Wang, B.P. (2004). Parameter Optimization in Multiquadric Response Surface Approximations. *Structural and Multidisciplinary Optimization*, 26, 219-223.
- Wang, S.W. and Armstrong, M.P. (2003). A Quadtree Approach to Domain Decomposition for Spatial Interpolation in Grid Computing Environments. *Parallel Computing*, 29, 1481-1504.
- Wang, S.Y. and Wang, M.Y. (2005). Radial Basis Functions and Level Set Method for Structural Topology Optimization. *International Journal for Numerical Methods in Engineering*, 65 (12), 2060-2090.

- Walker, R. (1993). *AGI Standards Committee GIS Dictionary*. Association for Geographic Information.
- Weber, D.D. and Englund, E.J. (1992). Evaluation and Comparison of Spatial Interpolators. *Mathematics Geology*, 24, 381-391.
- Weber, D.D., and Englund, E.J. (1994). Evaluation and Comparison of Spatial Interpolators II. *Mathematics Geology*, 26, 589-603.
- Wielgosz, P., Grejner-Brzezinska, D., Kashani, I. (2003a). Regional Ionosphere Mapping with Kriging and Multiquadric Methods. *Journal of Global Positioning Systems*, 2(1), 48-55.
- Wielgosz, P., Kashani, I., Grejner-Brzezinska, D., Zanimonskiy, Y. and Cisak, J. (2003b). Regional Ionosphere Modeling Using Smoothed Pseudoranges. *Proceedings of the 5th International Antarctic Geodesy Symposium*. 15-17 September. Lviv, Ukraine.
- WinGslib. (2006). <http://www.statios.com/Training/>
- Wollenhaupt, N.C., Wolkowski, R.P. and Clayton, M.K. (1994). Mapping Soil Test Phosphorus and Potassium for Variable-Rate Fertilizer Application. *Journal. Prodduction Agriculture*, 7, 441-448.
- Zain, A.F.M., Ho, Y.H., Abdullah, M., Rhazali, Z.A., Abdullah, S. and Marsimin, M.F. (2005). First Ionospheric Experimental Campaign and Observation at Fraser's Hill, Malaysia: Total Electron Content (TEC) and Scintillation Measurements. *Proceedings of 2005 Asia-Pacific Conference on Applied Electromagnetics*. 20-21 December. Johor Bahru, Johor, Malaysia.
- Zhao, C.Y., Nan Z.R. and Cheng G.D. (2005). Methods for Modelling of Temporal and Spatial Distribution of Air Temperature at Landscape Scale in the Southern Qilian Mountains, China. *Ecological Modelling*, 189, 209-220.