

USING MATHEMATICAL THINKING APPROACH TO ENHANCE
PROBLEM SOLVING SKILLS IN PRIMARY SCHOOL

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DEDICATION

This thesis is dedicated to my parents, all the scarifies they have made and all the support they given to me. They also taught me that even the largest task can be accomplished if it is done one step at a time.

To my dear siblings, who assist me during the journey of completing my thesis. I am thankful for having all of them in my life.

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ABSTRACT

In mathematics education, problem-solving is one of the important skills that students need to be possessed in 21st century so that they are able to encounter with the daily life challenges. However, previous studies revealed that many primary students are unable to answer the problem-solving question and they are unable to perform mathematical thinking in the problem-solving process. The aim of this study is to determine the effectiveness of using mathematical thinking approach to enhance the problem-solving skills among the primary school students and also to determine the type of mathematical thinking process that is dominant by the students after receiving the mathematical thinking lesson. An experimental design of the quantitative research was employed in this study. There were 65 participants of primary four students who were divided into control group and treatment group were selected by using the purposive sampling. The control group consists of 30 students while the treatment group consists of 35 students. The instruments used in this study was the problem-solving question of pre-test and post-test. Statistical analysis using SPSS software was used in this study to analyse the test results of pre-test and post-test. The statistical analysis results showed that there is a significant different between the pre-test and post-test of treatment group. This showed that the using of mathematical thinking approach in mathematics teaching process enhances the students' problem-solving skills. Furthermore, repeated measure ANOVA was used to determine the type of mathematical thinking process dominant by the students. Based on the analysis results, it was found that the conjecturing process is dominant by the students while many students are still weak in generalising process. This study provides information about students' thinking process in learning mathematics for teachers to improve students' problem-solving skills.

ABSTRAK

Penyelesaian masalah merupakan salah satu kemahiran yang penting dalam pendidikan matematik dan perlu dikuasai oleh pelajar pada abad ke-21 agar mereka dapat menghadapi cabaran harian. Walau bagaimanapun, kajian yang sebelumnya menunjukkan bahawa banyak pelajar sekolah rendah tidak dapat menjawab soalan penyelesaian dan mereka tidak dapat melakukan pemikiran matematik dalam proses penyelesaian masalah. Tujuan kajian ini adalah untuk mengetahui keberkesanan penggunaan pendekatan pemikiran matematik untuk meningkatkan kemahiran menyelesaikan masalah di kalangan pelajar sekolah rendah dan juga untuk menentukan jenis proses pemikiran matematik yang didominasi oleh pelajar setelah menerima pelajaran pemikiran matematik. Reka bentuk eksperimen penyelidikan kuantitatif telah digunakan dalam kajian ini. Terdapat 65 orang pelajar darjah empat yang telah dibahagikan kepada kumpulan kawalan dan kumpulan rawatan. Kumpulan kawalan terdiri daripada 30 orang pelajar manakala kumpulan rawatan terdiri daripada 35 orang pelajar. Instrumen yang digunakan dalam kajian ini adalah soalan penyelesaian masalah ujian pra dan ujian pasca. Analisis statistik menggunakan perisian SPSS telah digunakan dalam kajian ini untuk menganalisis keputusan ujian pra dan ujian pasca. Hasil analisis statistik menunjukkan bahawa terdapat perbezaan yang signifikan antara ujian pra dan ujian pasca di kumpulan rawatan. Ini menunjukkan bahawa penggunaan pendekatan pemikiran matematik dalam proses pengajaran matematik telah meningkatkan kemahiran penyelesaian masalah pelajar. Selanjutnya, ujian ANOVA digunakan untuk menentukan jenis proses pemikiran matematik yang didominasi oleh para pelajar. Berdasarkan hasil analisis daripada ujian ANOVA, didapati bahawa proses membuat sangkaan didominasi oleh pelajar sementara banyak pelajar masih lemah dalam proses generalisasi. Kajian ini memberi maklumat tentang proses pemikiran matematik pelajar dalam pembelajaran matematik kepada guru untuk meningkatkan kemahiran menyelesaikan masalah pelajar.

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LIST OF ABBREVIATIONS

DSKP	-	Dokumen Standard Kurikulum dan Pentaksiran
HOTS	-	Higher Order Thinking Skills
KSSR	-	Kurikulum Standard Sekolah Rendah
NCTM	-	National Council of Teachers of Mathematics
OECD	-	Organisation for Economic Co-operation and Development
NCTM	-	National Council of Teachers of Mathematics
PISA	-	Pakistan International Screen Awards
PT3	-	Pentaksiran Tingkatan 3
SPM	-	Sijil Pelajaran Malaysia
UPSR	-	Ujian Penilaian Sekolah Rendah

LIST OF SYMBOLS

ℓ	-	Litre
cm	-	Centimetre
m	-	Metre
RM	-	Ringgit Malaysia

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CHAPTER 1

INTRODUCTION

1.1 Introduction

In Malaysia mathematics curriculum, problem-solving is one of the major aspects that required students to integrate their mathematical knowledge, concepts, skills as well as making decision. Eventhough problem-solving is not a topic stated in the mathematics curriculum but it is a process which underlined the whole mathematics education to contextually assist students in learning the mathematical concept and skills (Tarzimah & Meerah, 2010). However, many students in Malaysia were seemed not to acquire the basic skills needed in mathematics. As a result, many students facing difficulties in learning mathematics especially in the mathematics problem-solving (Tarzimah & Meerah, 2010). Moreover, a recent study also revealed that Malaysian students were unable to make accurate perceptions and interactions, memorizing and retrieving facts, providing concentrations and using their logical thinking (Tarzimah & Meerah, 2010).

Tanius (2018) has stated that many primary students facing difficulty in problem-solving questions. They do not know how to find the key words from the questions, do not understand the questions and also could not figure out which mathematical knowledge or skills should be applied on the questions. As a result, they give up the question or they choose to skip long questions (Tanius., 2018). This also can be seen from the poor performance of Malaysian students in both international assessments, TIMSS and PISA. Based on their analysis, Malaysian students lack of chances to explore HOTS questions which required them to apply their mathematical knowledge and skill in problem-solving questions (Ministry of Education, 2013). Thus, Malaysia Ministry of Education has inserted the new element critical thinking into the new KSSR curriculum to ensure that the Malaysian curriculum is parallel with the international level (Tanius, 2018).

In the year 2013 - 2017, Malaysian Mathematics curriculum has been revised to enhance students' critical thinking skills, cognitive skills, reasoning skills and communication skills. The curriculum stated that problem-solving is an important part of mathematics learning and the ability of students to solve problems become a must objective that needed to be achieved by teachers at the end of the lesson (Irenika & Susanti, 2019). In order to assist teachers in their lesson planning, the Ministry of Education (2013) of Malaysia has also provided instructional materials and various teaching resources such as textbook and activity book which are well-planned and organized to help and guide teachers in planning their problem-solving lessons involving thinking process and activities in the mathematics classroom.

The new KSSR curriculum aims to help students to develop cognitively and emotionally while giving students a platform to develop their Higher Order Thinking Skills (HOTS). Therefore, more HOTS questions are included in UPSR (Primary 6 examination) to allow students to practically apply their mathematical knowledge and skills in problem-solving questions. However, the implementation of new KSSR did not bring up the results of UPSR in the year 2016. The UPSR 2016 shown that students' performance declined sharply with only 4890 (1.1%) scored straight compared to 38,344 (17.7%) in UPSR 2015 (The Star Online, 2016).

According to the Malaysia Mathematics Standard-Based Curriculum (DSKP) year 2020 for primary 4 mathematics, it aims to develop students who are mathematically inclined with the understanding of the concept of numbers, basic calculation skills, simple mathematical ideas and competency in applying mathematical knowledge and skills effectively to solve problems and make decision in overcoming the daily life challenges (*Dokumen Standard Kurikulum dan Pentaksiran Tahun 4*, 2020). In addition, it also stated in the objective to develop students' mathematical thoughtful learning, higher order thinking (HOTS), critical thinking, creative thinking, innovative thinking, reasoning and explore daily life mathematically.

As mentioned previously, the main goal of mathematics education is to develop an individual's mathematical thinking skills so that they are able to encounter real-life

challenges and can apply their mathematic knowledge to effectively solve a problem and making decision (Ministry of Education, 2003). According to Lutfiyya (1998), mathematical thinking refers to the use of thinking skills to understand ideas, discover the relationship between the ideas, draw a condition about the ideas and solve the problem involving the ideas. The ideas were generally referred to the mathematical concept or knowledge. On the other hand, Schoenfeld (1992) also stated that there are five aspects of cognition that involve in the mathematical thinking and problem-solving: (1) knowledge base; (2) problems-solving strategies; (3) Monitoring and control; (4) beliefs; and (5) practices.

Mathematical thinking is an act of sense-making and rest on the processes of generalising, specialising, convincing and conjecturing (Noraini & Liew, 2017). According to Irenika (2019), mathematical thinking is defined as the thinking ability that related to the use of reasoning and rational to develop mathematics arguments and procedures, the ability to develop strategies or methods, to understand mathematics contents and the ability to communicate the ideas. When a person is facing a problem, brain will start to work systematically to interpret and analyse the problem. This process is known as the mathematical thinking. Therefore, mathematical thinking is important and needed to be applied in the problem-solving process. Besides mathematical thinking can also be referred to a way on the ability of students to look at the big picture of a thing, stripping them down to their essentials and then analyse their underlying patterns either it is a numerical, structural or logical thing. In the process of mathematic learning, students need to see the underlying structures and recognise the mathematics which is normally related to real life and nature.

According to Halil (2017), mathematical thinking can be also a way an individual prefers to present, to understand and to think through the mathematics facts and connections by certain internal imaginations or external representations. Stacey (2014) stated that a successful and good problem solver need to experience by solving non-routine problems in a supportive environment, in addition, they need to reflect so that they are able to learn from the past experiences and also, they need to have effective heuristic strategies and good problem-solving habits as well as mathematical thinking processes. Mathematical thinking induces contradiction and help us to

understand ourselves and the world. Therefore, it can be said that mathematical thinking is a form of thinking that is realized not only in cases with numbers and abstract but also in our daily life (Halil, 2017).

Besides, mathematical thinking is also related to the ability of students in doing reasoning. According to Toni (2008), reasoning is one of the mathematical thinking that need to be acquired by students while solving problems. This is due to the reasoning ability enable students to think, to try out new solution and also to be able to prove their solutions through mathematical knowledge. Therefore, mathematical reasoning is also important when mathematical thinking approach is utilizing in a mathematics classroom. Moreover, Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. It enables students to recognise that mathematics make sense and can be understood. With mathematical reasoning, students able to learn how to evaluate situations, choose problem-solving strategies, draw a logical conclusion and describe the solution. Moreover, problem solving is one of the four mathematics proficiencies where students need to engage with the real-world problem and solve an open task, while mathematical reasoning is also one of the components that inquire students to develop their mathematical thinking.

As mentioned previously, mathematical thinking is the ability to think, analyse and connect mathematical facts or skills to solve a problem. Therefore, the development of mathematical thinking among student is important so that students' ability in problem-solving can be enhanced. Schoenfeld (1985) stated that problem-solving is a tool to advance the ability of students' thinking and mathematics problem-solving is complex cognitive that requires students to acquire knowledge through a process and then they store the knowledge so that it can be retrieved for later use (Endah & Surya, 2017). As stated, problem-solving is one of the important parts of mathematics education. The ability of students in mathematics problem-solving has become a goal of mathematics teachers in their mathematics classroom. The term "problem-solving" refers to the mathematics task that provides intellectual challenges to enhance students' mathematics understanding and development (Rita & Zuilardi, 2012). NCTM (2012) has also stated that problem-solving skills and ability among the

students should be developed when they are in primary school. Therefore, it is important for teachers to plan mathematic task that involves the use of mathematical thinking in problem-solving in their lesson plan.

According to Polya (1945), there are four basic steps in doing problem-solving, which are understand the problem; planning to solve the problem; carry out the plan and interpreting the answers. As mentioned previously, the ability of students in problem-solving is one of the main goals that needed to be achieved in mathematics education. Therefore, students' ability in mathematical problem solving can be defined as the ability of the students to understand problems, plan strategies to the problems, carry out the strategies and re-examine the solution (Rustam, 2019).

There was a studied stated that exposing students with daily problem-solving activities enhanced students thinking skills and reasoning skills. In this research, the researcher exposed her students with mathematics word problems daily and encouraged her students to solve the problem through a series of thinking process. At the end of the research, she found that through the daily problem-solving activities, the students' ability in problem-solving was enhanced (Julie, 2008). When students are engaged in problem-solving, they explore the problem, from the exploration, they develop the strategy or methods and solution through thinking about the problem. From these methods and solutions, students develop the reasoning skills and prove their thinking to be reasonable and valid. Throughout the problem-solving stages, we noticed that mathematical thinking process took place as students need to think of about the problems, analyse the information and think of about the solution to prove their thinking.

1.2 Background of The Study

In Malaysia, the Ministry of education (2013) stated that there is a need to develop primary school students' mathematical thinking skill so that they can fulfil the

vision of the National Education Philosophy (Noraini and Liew, 2017). As mentioned previously, the purpose of mathematics education is to develop students' ability to solve daily life problems and making decision. In addition, mathematics is also a subject that can be used to train students in developing their thinking skills that lead to further ability to solve real-life problems. However, many students were lack of mathematical problem-solving skills (Phonapichat, 2014). This was proven by the poor performance of mathematics achievement in the national test results.

A studied to analyse the difficulties faced by elementary students in mathematics problem-solving was done by Phonapichat (2014) revealed that students have difficulties in understanding the keywords in the mathematics problems and thus they are unable to interpret them in mathematics sentences. Besides, Phonapichat (2014) also found that students were unable to figure out what to assume and what information from the problem is needed to be solved. As a result, students tend to guess the answer when they do not understand the problems without going through any thinking process. In addition, Phonapichat (2014) also found that students do not like to read long mathematics questions and also, they do not have the patient to read mathematical problems.

In Malaysia, there are certain primary school were using English language as their media of conducting mathematics subject. This leads to many students cannot understand the questions as English is not their mother tongue. As a result, many students unable to understand the mathematics lessons and also could not able to read and understand the problem-solving question. There was a studied about the relationship between the difficulties of mathematical problem-solving and the students' reading skills was revealed that the students' ability in problem-solving and reading skills are interrelated. It can be said that the better the reading skills, the better the ability of the students to understand the problem-solving question (Walker, 2008). Besides, Pearce and Bruun (2016) also showed that students' abilities to read and understand the mathematics problem solving questions was the most frequently cited difficulty. In addition, the research also suggested that both reading and mathematics skills are needed overall for students' reasoning abilities.

On the other hand, many students also encountered difficulties in understanding the mathematical language and lead to misconception (Phonapichat, 2014). In Malaysia, teachers will receive various kind of excuses from students about why they did not complete their mathematics homework. The excuses such as “Mathematics problems are difficult, I do not know how to solve the problem, I do not understand the questions, I did not finish my homework because I do not know how to do or I hate mathematics.” This situation was commonly occurred in most of the Malaysia mathematics classroom (Tarzimah & Meerah, 2010). As a result, students tend to wait for answers from teachers or simply guess the answers without going through any thinking and analysing process. In Indonesia, a studied has stated that low mathematics problem-solving ability among students caused students do not like mathematics subject (Rustam, 2019). In addition, this studied also stated that the mathematics was a subject that was not in demand by most of the students.

According to Susanti (2018), one of the reasons that mathematics needed to be taught from primary to university is to teach students how to think mathematically. To be able to think mathematically, the cognitive skills and communication skills of students are needed to be acquired by students. According to Lim and his friends (2007), while doing problem-solving, students need to connect the concrete information with the abstract situation of the problem given. If the students have good communication skills, they are able to organize, consolidate and explain their mathematical thinking clearly to peers and teachers. Moreover, they are able to analyse and evaluate the mathematical thinking of others. Therefore, the deficiency of using the cognitive abilities and mathematical skills during the mathematics learning process will hinder the problem-solving skill. In addition, the inabilities of students to make accurate perceptions and interpretation, memorizing and retrieving facts using their logic thinking is also a hindrance to promote problem-solving among the students (Tarzimah & Meerah, 2010). According to the reports from TIMSS (2007), Malaysian students had shown a significant low achievement in cognitive dimension which emphasized on thinking skills and problem-solving skills. Moreover, the Examination Board of Malaysia (2009) has also stated that Malaysian students had low problem-solving skills (Tarzimah & Meerah, 2010).

Generally, the learning process of mathematics requires students to actively participate in constructing new solution by using their existing knowledge to solve a problem given by teacher (Noraini and Liew, 2017). In Malaysia, primary students are encouraged to acquire mathematical skills and knowledge through actively participating in the teaching and learning process during mathematic lesson (Noraini, 2017). However, a recent studied has stated that the ineffective teaching and learning process in the classroom has led to students' low ability in problem-solving (Mushlihuddin & Irvan, 2018). This can be understood as many teachers today still using the conventional method in their mathematics classroom. The conventional method such as chalk and talk method used by teacher is more likely a teacher-centred method where teachers give explanation, procedures and instruction to the students and students listen to the explanation, follow the instruction given and memorized the facts (Najihah, 2016). As a result, students become passive and teachers become the information provider. Furthermore, most of the conventional teaching method emphasize more on drilling the questions and rote learning than meaning learning (Najihah, 2016). This makes students feel burden while learning mathematics.

According to Toni (2008) study, mathematical thinking skills was related to the teaching strategies used by teachers. Many teachers found that memorization of facts and concepts for learning is better to be conducted in the lesson instead of using other teaching approaches in the classroom (Shalini, 2017). Although most of the teachers are aware that the implementation of new teaching methods is a need for students especially in 21st century classroom. However, packed and tight syllabus and traditional rote teaching methods led to insufficient time and uninterested of teacher to implement new teaching approach in their classroom.

Apart from the teaching strategies used by teachers in the classroom, teachers felt unmotivated and frustrated towards students' attitudes and behaviours during the problem-solving situation. In Malaysia classroom, teachers encountered with the problems that when teachers give a problem, students are supposed to read and understand the problems, discuss among each other about the problems, analyse the problems and come out with solutions. However, most of the students reluctantly to try the questions and they are rather wait for the solutions given by teacher. A previous

studied was conducted to assess primary 5 students on their mathematical thinking level (Madiah, 2017). These students were given five problem-solving questions that required mathematical thinking. At the end of the studied, the researcher found that these students are not get used to think and analyse the questions and also, they could not explain their answers clearly and were afraid of getting wrong answers and were easily give up to answer.

According to Cesare and Barbara (2015), a lot of primary students are struggling with arithmetical problem solving and this has led to the low motivation of students in learning problem-solving. With this negative effect, teachers find it difficult to teach and promote students' ability in problem-solving and thus teachers tend to use the conventional method and repeated exercises to drill students' ability in problem-solving task. Gupta (2017) has stated that teaching method can be defined as the type of principal and method used for instruction. It depends on what information or skill the teaching is trying to convey. Previous studied has stated that many teachers prefer teaching facts and students have to listen to teachers' explanation instead of finding the answers or solution by themselves. Hence, students followed blindly teachers' instruction in doing their mathematics question, and thus students do not have chances to improve their problem-solving skills as they do not have chances to find the answers through their thinking process. In addition, teachers often teach students to memorize the theories and less involved students in activities that require thinking process (Apriliyanto & Dewi, 2018).

In Malaysia, although the mathematics syllabus was revised to enhance students' ability in reasoning, thinking and communication skills, however, the presence of public examination such as UPSR, PT3 and SPM which encouraged teacher-centred classroom was pre-occupied in preparing students for the examination. As we know, the stakeholders such as parents or school authorities are focused on the results rather than the mathematical thinking and problem-solving skills that possessed by the students. They would not bother about the conceptual understanding, the ability of reasoning and thinking skills of students have in the problem-solving (Madiah, 2017). Furthermore, teachers have been used to cramming students with drilling a lot

of exercises or past year papers so that they can scores flying colour in the public examination.

Although the Ministry of Education has encouraged teachers to implement more 21st century teaching in their teaching process, however many teachers are still in the dark on how to incorporate the mathematical thinking into their lesson and also, they do not know the proper way to plan their task or activities which involve both mathematical thinking and problem-solving situation to train students to think mathematically. Madihah (2017) has mentioned that, teachers play important roles in creating an intellectual classroom and creating the problems that needed to be used to enhance student's ability in using mathematical thinking to solve the problem given by teachers. Besides, lacking of mathematical knowledge and concept is also one of the reasons that cause students' low ability in problem-solving.

There was a previous studied stated that exposing students with daily problem-solving activities encouraged students to solve problem involving the use of their thinking skills and reasoning skills. In this research, the researcher exposed her students with mathematics word problems daily which required students to solve the problem through thinking process. Throughout the study, the process of mathematical thinking process was observed by the researcher and the researcher has found that through the daily problem-solving activities, the students' ability in problem-solving was enhanced (Julie, 2008). When students are engaged in problem-solving, they explore the problem, from the exploration, they develop the methods and solution of thinking about the problem. Therefore, mathematical thinking process can be observed throughout the lesson.

There were a lot of studies about the enhancing problem-solving using different kind of teaching approaches. However, there are fewer studied about the using of mathematical thinking approach to enhance the students' ability in problem-solving. Bahar and Maker (2015) stated that problem-solving involved high level of thinking process which consist of intellectual and cognitive processes. Therefore, a good problem solver uses the strategies or procedures formulated by Polya (1945), that is, understand the problem, plan a solution, carry out the plan and looking back to the

solutions obtained to solve the problem. However, students are encouraged to follow blindly to Polya's steps in problem-solving lead to students' dependency to remember the facts and follow the rule in the procedure.

1.3 Problem Statement

Problem-solving is not only a part of the mathematics education that students should learn but it is also an ability that students should possess especially in their real-life situation. As we know, we are facing problems and challenges every day. It forces us to search for the solution or strategies to solve the problems. When students start to think about a solution or strategy to solve the problem, they involve the thinking process. In the thinking process, they need think of the problem, analyse the information given and finally come out with the solution. As a result, students able to develop their problem-solving skills through mathematical thinking process. (Noraini and Liew, 2017). However, the poor performance of Malaysian students in national test results have indicated that Malaysian students has failed to use their existing knowledge in problem-solving questions (Hisyam & Mat Salleh, 2017). This is also explained that students understood the basic concept of mathematics but they are lacking of skills in term of applying the knowledge while doing problem-solving question.

Students lacking of opportunity to apply mathematical thinking in the mathematics lesson. There were several local studies has stated that teacher-centred with procedural approach has become the main teaching strategies for most of the mathematics teachers in Malaysia (Chap Sam, 2008). In the previous studied done by Jamaliah (2001) has stated that Malaysian teachers prefer to present their lesson in the form of question-answers or provide brief explanation of the topics through examples taken from reference book or textbook and then followed by exercises to drill students' understanding of the topic. By reviewing the whole process, we can see that there is no involvement of any mathematical thinking process throughout the learning process.

In Malaysia, the main teaching resources for primary school mathematics teachers are the textbook with well organised syllabus provided by the Ministry of Education. The contents of textbooks are totally based on the syllabus and each topic will be provided with a lot of diagrams, explanation, calculation methods and also pictures in order to assist and guide teachers in their teaching process, lesson planning and classroom activities, however it is less emphasising on the mathematical thinking and reasoning skills (Noraini and Liew, 2017).

In mathematics education, the term ‘problem-solving’ refers to mathematics task that provide intellectual challenges to enhance students’ mathematical understanding. As stated by the NCTM, problem-solving skills should be developed to students when they were in primary school (Rita, 2012). Problem-solving requires students to connect the mathematical concepts with the real-life situation or problems, so that they are able to solve the problem. However, a lot of Malaysia primary students are unable to utilize their knowledge in to solve a problem and also, they are unable to related the knowledge to their real-life situation. For instance, in a mathematics classroom, teacher asks students to draw a circle but the teacher did not give any instruction to students on how they are going to complete the task. Hence, we will notice that students who are good problem-solvers will try to search for any objects that have a circle shape to draw the circle. However, there might have certain students do not have any ideas on how to draw the circle and they do not know what should they do and this end up with they are just sitting and waiting for teacher’s further explanation or instructions.

Phonapichat (2014) has stated that many students were lack of mathematical problem-solving skills and this has been proven by the low achievement of students’ performance in the national test results. Phonapichat (2014) also stated that students faced difficulties in understanding the mathematics question and thus they do not able to know which mathematical concepts or operations should be applied to solve the problems. Besides, Pearce and Bruun (2016) also showed that students’ abilities to read and understand the mathematics problem solving questions was the most frequently cited difficulty. In addition, the research also suggested that both reading and mathematics skills are needed overall for students’ reasoning abilities.

A recent studied done by Noraini and Liew (2017) to determine the mathematical thinking level of primary school students have revealed that Malaysian Year 4 students have unsatisfactory level of mathematical thinking with only 2.5% of the students achieved satisfactory level of mathematical thinking based on the 'cut off' score of 30.00. In addition, this studied has further revealed that most of the students were unable to show their reasoning skills in making decision, and they were unable to make generalisation based on their observation of the patterns and were unable to find more than one solution for a problem, and moreover they were found contented with only having one solution (Noraini & Liew, 2017).

This studied was conducted on 516 years 4 students from 7 primary schools in the state of Terengganu, Johor, Kedah and Federal Territory. In this study, a paper and pencil test of 9 items which included the decision making, justifying decision, spatial reasoning, conceptual understanding, conjecturing and generalization were chosen to assess students' mathematical thinking level. The findings of this study revealed that Malaysian primary school students were unable to show their mathematical thinking in all nine items of the test with the mean scores of all the nine items were lower than the "cut-off" point of the item. Moreover, further analysis of this study also shown that majority of primary school students were unable to provide reasoning for their decision. On the other hand, the ability of students to obtain more than one solution of the problems was not found in most of the students upon the analysis.

In Malaysia, the emphasising on developing mathematical thinking is fairly new and the extent of the development of mathematical thinking among the primary students is also unknown (Noraini & Liew et. Al, 2017). Therefore, there is a need to continuous the exploration and development of mathematical thinking in mathematical education (Halil, 2017). Furthermore, Malaysia is also understood it was a need to develop and enhance students' thinking (Ministry of Education Malaysia, 2013) in order to actualize the vision of the national education philosophy. This study aims to determine the effectiveness of using mathematical approach to improve problem-solving among primary students and also how did the students perform according to the type of mathematical thinking processes after going through mathematical thinking approach.

In this study, Mathematical thinking approach is adopted to determine whether students' problem-solving can be improved after going through mathematical thinking processes. Mathematical thinking is a process of work which is abstracted and hard to be determined through observation or formal test. However, it is included in every stages of our lives. In mathematics subject, mathematics is not only a subject that teach students about the numbers, formula and operation but it supports students by giving them significant achievement for instance thinking, establishing relationships between incidents, reasoning, estimating and also solving problem in every day challenges. Individuals use mathematical thinking with or without conscious to solve a situation, problem or incident they encounter at every stage of their lives. Therefore, mathematical thinking is not only a technique or an approach that can only be conducted in the classroom but it can be occurred at any time. Besides, problem solving is one of the processes or concepts that focuses on the mathematical thinking of an individual. Therefore, it is assumed that problem-solving among the students can be developed and improved with mathematical thinking approach.

1.4 Research Objectives

The objectives of this study are:

- i) To determine the effectiveness of using mathematical thinking approach to improve problem solving.
- ii) To determine how did the students perform according to type of mathematical thinking-processes.

1.5 Research Question

The research questions of this study are as follow:

- i) Is there any significant difference on students' problem-solving skills after using the mathematical thinking approach?
- ii) How did the students perform according to type of mathematical thinking processes?

1.6 Research Hypothesis

The null hypothesis of this study is as follow:

- i) There is no significant difference on students' problem-solving skills after using the mathematical thinking approach.

1.7 Conceptual Framework

The aim of Malaysian mathematics curriculum was to develop students who are able to think mathematically and able to solve problems effectively using existing mathematical knowledge (Chap Sam, 2008). Mathematical thinking involves the effective thinking with conceptual understanding and ability to apply mathematics knowledge in problem solving. Conceptual understanding refers to the understanding of more than one mathematics concepts and mathematics facts and able to generate new ideas based on the ideas they have known. Generally, effective problem-solving occurs when students able to solve problems through a series of thinking process. Therefore, this study aims to determine the effectiveness of using mathematical thinking approach to improve problem-solving skills and how students perform according to the type of mathematical thinking and which type of mathematical thinking is dominant by the students after going through mathematics approach.

According to Stacey (2014), both mathematical thinking and problem-solving skills require a wide range of skills, which are deep mathematical knowledge, general reasoning abilities and heuristic strategies. Mathematics is a complex subject and involve a lot of concepts and facts. Therefore, having deep mathematical knowledge is important in doing problem-solving as students able to apply or implement their mathematical knowledge to the problems. The reasoning abilities refer to the process of thinking in making conclusion or obtain solution for problems. While heuristic strategies refer to the strategies that students able to use to solve complex word problems.

The framework of this study was adopted from the framework of Stacey (2014) which stated that mathematical thinking consisted of four fundamental processes that occur alternatively while solving problems through the mathematical thinking process. The four processes are Specialising, Generalising, Conjecturing and Convincing (Stacey, 2014). In specialising process, specialising refers to trying out similar examples or cases when encounter with problems. Specialising is said to be the starting point of the problem-solving process (Mason, 2010). Generalising is the shifting from examples to conjecture about and expressing the wider patterns of the problems (Mason, 2010). In generalising, students will search for the patterns of the problems and their relationships. Conjecturing is the statement that used to express the generalisation. while convincing is the testing of conjecturing, finding and communicating reasons why the ideas or solution is true (Stacey, 2014).

In this study, specialising, generalising, conjecturing and convincing processes will be used to assist students in developing their mathematical thinking skills when doing problem solving questions. The effectiveness of mathematical thinking approach in enhancing students' problem solving will be determined by using pre-test and post-test. Pre-test will be given before the implementation of mathematical thinking approach while post-test will be given to students after the implementation of mathematical thinking approach in the mathematics classroom. The test result will be analysed to determine the effectiveness of mathematical thinking approach to improve problem-solving among students.

Besides, this study also wants to determine to how students perform according to the type of mathematical thinking processes after going through the mathematical thinking approach. In order to determine the mathematical thinking processes that students have performed going the mathematical thinking approach, Stacey (2014) framework is adopted to observe the performance of students according to the type of mathematical thinking processes, which are specialising, generalising, conjecturing and convincing after going the mathematical thinking embedded lessons.

Whilst, Polya's four steps which are understand the problem, devise a plan, carry out the plan and look back will be used by students to solve mathematics word problems. The conceptual framework of this study is shown in Figure 1.1.

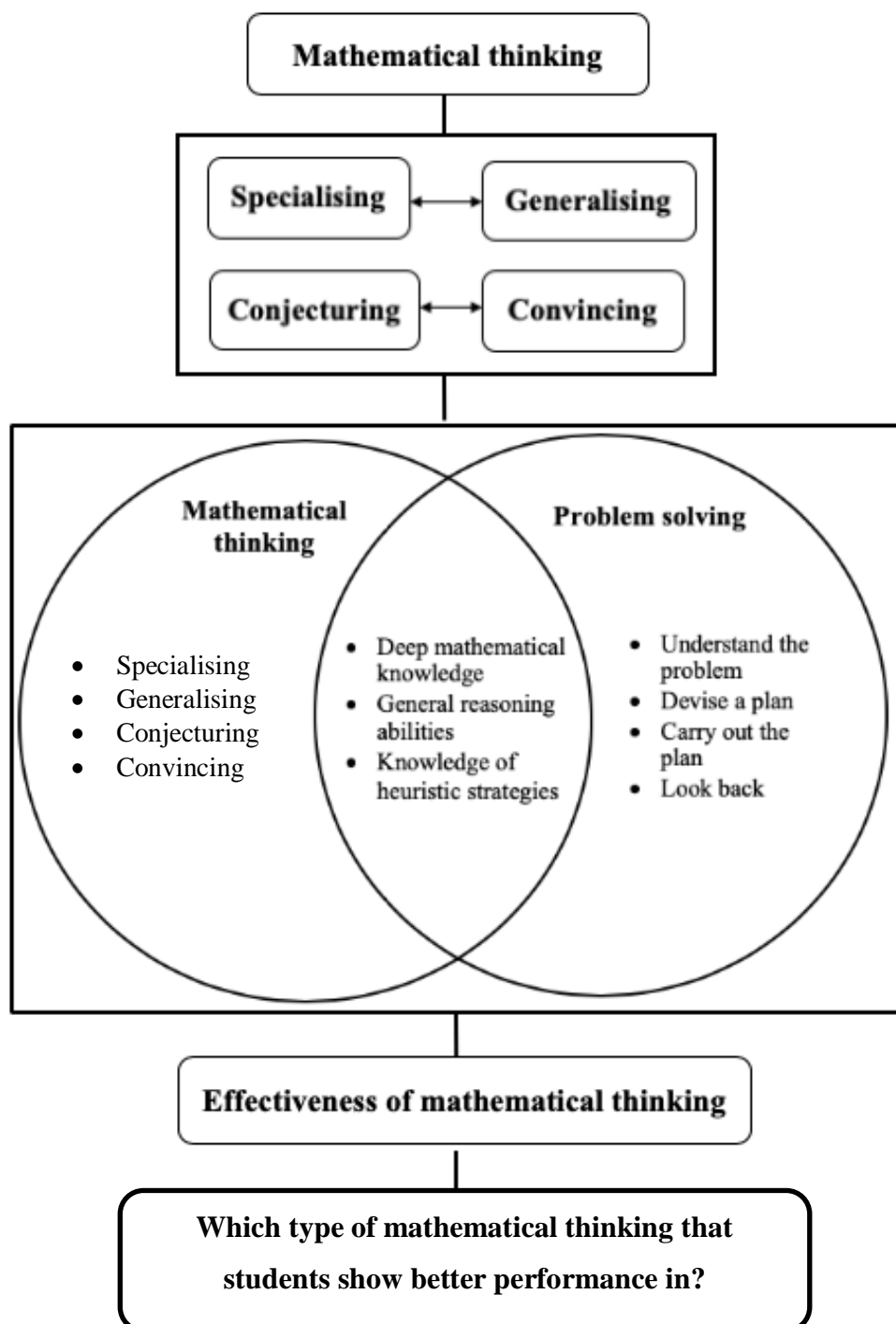


Figure 1.1 Conceptual framework

1.8 Significant of the Study

The aim of this study is to determine the effectiveness of mathematical thinking approach to improve problem-solving among primary students and also to identify to what mathematical thinking processes that students develop going through mathematical thinking approach. As shown in the international assessment PISA and TIMSS (2012), Malaysian students have low ability in the problem-solving skills. Besides, the survey done by the Organisation for Economic Co-operation and Development (OECD) in the year 2012 were shown that there were more than one in five Malaysian students could not even reach the basic levels (level 1) of problem-solving which referred to the low-level of thinking ability (Susanti, 2018). Therefore, this study tends to use the mathematical thinking approach to enhance primary students' ability in problem-solving.

For Ministry of Education, the findings of this study provide information about the effectiveness of using mathematical thinking approach in enhancing problem-solving skills in primary school. Besides, this study also provides information about the level of mathematical thinking among the primary school students and thus allow the ministry of education to discover suitable teaching strategies for teachers to improve their teaching skills and teaching strategies.

For school, school principals and teachers are able to aware the level of mathematical thinking and problem-solving ability among primary students and understand that there is a need to change to implement mathematical thinking approach in their mathematics teaching process. In addition, teachers able to reflect their won teaching approach or strategies used so that they can improve themselves to assist students in enhancing the level of mathematical thinking and problem-solving abilities.

For researcher, this study helps the researcher to uncover the gaps and areas in the mathematical thinking and problem-solving that many researchers were not able to explore.

1.9 Definition of Terminology

1.9.1 Mathematical Thinking Approach

Mathematical thinking approach in this study refers to the method that teachers use to implement in the classroom to enhance the ability of students in problem-solving. It includes a series of thinking processes which enables students develop their thinking skill in problem solving process. Moreover, by using their mathematical knowledge, concepts, understanding and logical reasoning that students able to cultivated through the mathematics learning process.

1.9.2 Problem-Solving in Mathematics

Problem-solving in mathematics requires students to apply their mathematical knowledge, reasoning skills and thinking skills to solve problems. In this study, mathematics problem-solving refers to the act of defining problems, identify the problems and select an appropriate solution for the problems. The ability of problem-solving in the study will be studied after the implementation of mathematical thinking approach in the classroom. Diagram 1.2 shows the problem-solving process chart which involves four stages.

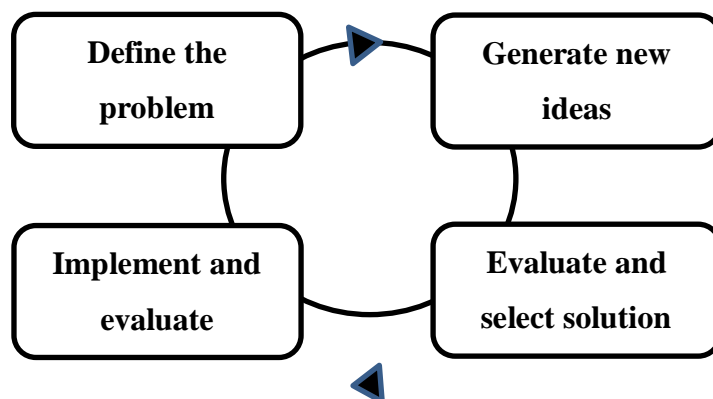


Figure 1.2 Problem-solving process chart

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